Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2017
Secondary Three

INTEGRATED MATHEMATICS 2 **2 hours**

Tuesday **10 Oct 2017** **0845 – 1045**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class on all the work you hand in.
2. Answer questions 1 - 12 before attempting question 13 (Bonus Question).
3. Write your answers and working on the separate writing paper provided.
4. Write in dark blue or black pen on both sides of the paper.
5. You may use an HB pencil for any diagrams or graphs.
6. Do not use staples, paper clips, glue or correction fluid.
7. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
8. You are reminded of the need for clear presentation in your answers.

INFORMATION FOR CANDIDATES

1. At the end of the examination, fasten all your work securely together.
2. The number of marks is given in brackets [ ] at the end of each question or part question.
3. The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

3. MENSURATION

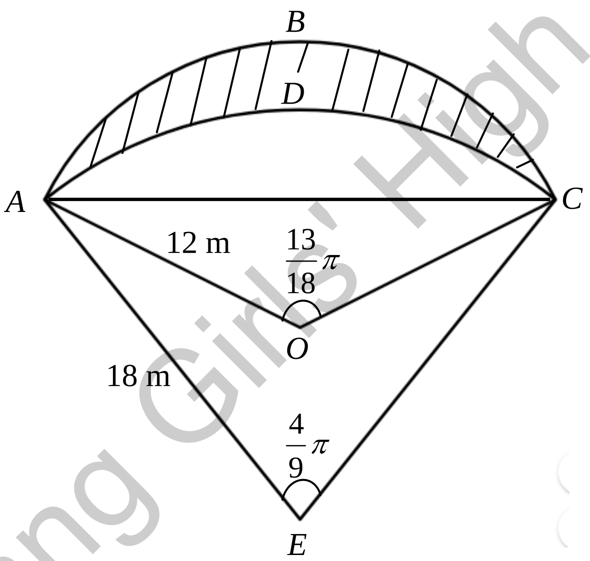
Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

1 By rationalising the denominators, simplify $\dfrac{1+\sqrt{3}}{2-\sqrt{3}}+\dfrac{2-\sqrt{3}}{\sqrt{3}}$. [3]

2 Given that $2x^2-3x-4>mx-6$ for all real values of x, find the range of values of m. [3]

3 The diagram shows the layout plan of a garden. *ABC* is an arc of a circle, centre *O* and radius 12 m. *ADC* is an arc of a circle, centre *E* and radius 18 m. $\angle AOC=\dfrac{13}{18}\pi$ radians and $\angle AEC=\dfrac{4}{9}\pi$ radians.



(a) The segment *ADC* is a fish pond. Calculate the area of this region. [3]

(b) The shaded region *ABCD* is allocated for growing plants. Calculate the perimeter of this region. [2]

4 Sound is measured in decibels according to the formula $d = 10\left(\lg \dfrac{P}{P_0}\right)$, where d represents the noise rating in decibels, P represents the intensity of the sound and P_0 represents the intensity of the weakest sound that the human ear can hear. A thunderclap has a noise rating of 120 decibels and a vacuum cleaner has a noise rating of 80 decibels. Calculate the ratio of the intensity of a thunderclap to that of a vacuum cleaner. [3]

5 The roots of the equation $x^2 + 2x - 6 = 0$ are α and β.

(i) Show that $\alpha^3 = 10\alpha - 12$. [2]

(ii) Find a quadratic equation whose roots are $\dfrac{\beta}{\alpha^2}$ and $\dfrac{\alpha}{\beta^2}$. [5]

6 **(a)** Sketch the graph of $y = \ln(x-2)$, labelling the asymptote(s) and intercept(s) clearly. Find the equation of the straight line that needs to be added onto the same axes so as to solve the equation $e^{3x} = \dfrac{x-2}{e}$. **It is not necessary to draw the line on your sketch.** [4]

(b) Sketch the graph of $y = \left|\sin\left(\dfrac{x}{2}\right) - 1\right|$ for $0 \le x \le 2\pi$. [3]

7 **(a)** Solve the equation $3^{3x-1} = 2\left(3^{2x}\right) - 3\left(3^{x}\right)$. [4]

(b) Solve the equation $\ln e + \ln x = 1 - \ln(2x-1)$. [3]

(c) Given that $\lg 4 = m$, express each of the following in terms of m.

(i) $\lg \sqrt[3]{400}$, [2]

(ii) $\log_{\frac{1}{16}} 10$. [3]

8 **<u>Answer the whole of this question on the INSERT provided.</u>**

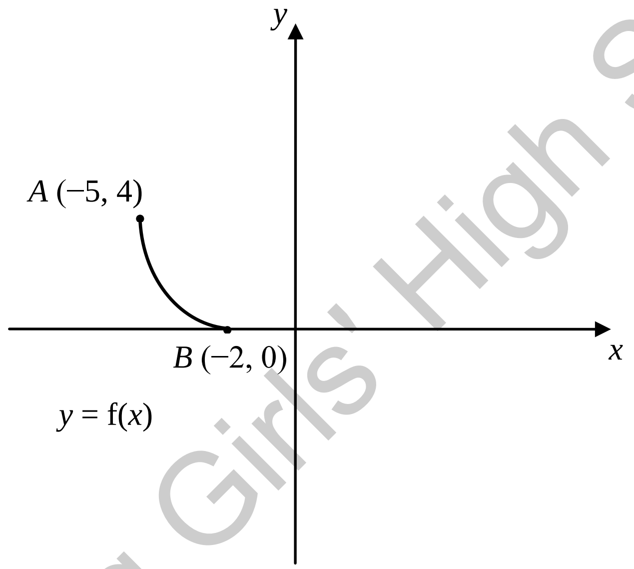
The diagram shows the graph of $y = \mathrm{f}(x)$ for $-5 \le x \le -2$. On **<u>separate</u>** axes, sketch the graphs of

(a) $y = \mathrm{f}\left(x+4\right)$, [2]

(b) $y = \dfrac{3}{2}\mathrm{f}\left(x\right)$, [2]

(c) $y = \mathrm{f}^{-1}\left(x\right)$, [2]

labelling clearly the images of the points A and B as A' and B' respectively under the respective transformations.



9 **(a)** Given that A and B are in different quadrants, $\cos A = -p$, $\tan B = 2p$, $90° \le A \le 360°$, $90° \le B \le 360°$ and p is a positive integer. Express the following in terms of p.

(i) $\tan A$, [2]

(ii) $\cos B$. [2]

(b) Solve the equation

$$3\sin x + 9 = 10\cos^2 x \text{ for } 0° \le x \le 360°.$$ [4]

10 **(a)** The function f is defined by $\mathrm{f}: x \mapsto 2x^2 - 8x + 3$, for $0 \le x \le 4$.

(i) By completing the square, express $2x^2 - 8x + 3$ in the form $a(x+b)^2 + c$. [2]

(ii) Find the range of f. [2]

(iii) Does f have an inverse? Explain your answer. [1]

(iv) The function g is defined by $\mathrm{g}: x \mapsto 2x^2 - 8x + 3$, for $x \ge k$. State the smallest value of k for which the function g^{-1} exists. [1]

(b) A function h is defined as $\mathrm{h}: x \mapsto \dfrac{2x}{3x-1}$ for $x \ne \dfrac{1}{3}$. Find the values of x which map onto themselves under the function h. [3]

11 **(a)** The function $\mathrm{f}(x) = 10x^3 + ax^2 + bx - 4$ where a and b are constants, is exactly divisible by $x - 2$ and leaves a remainder of -21 when divided by $x - 1$.

(i) Find the values of a and b. [4]

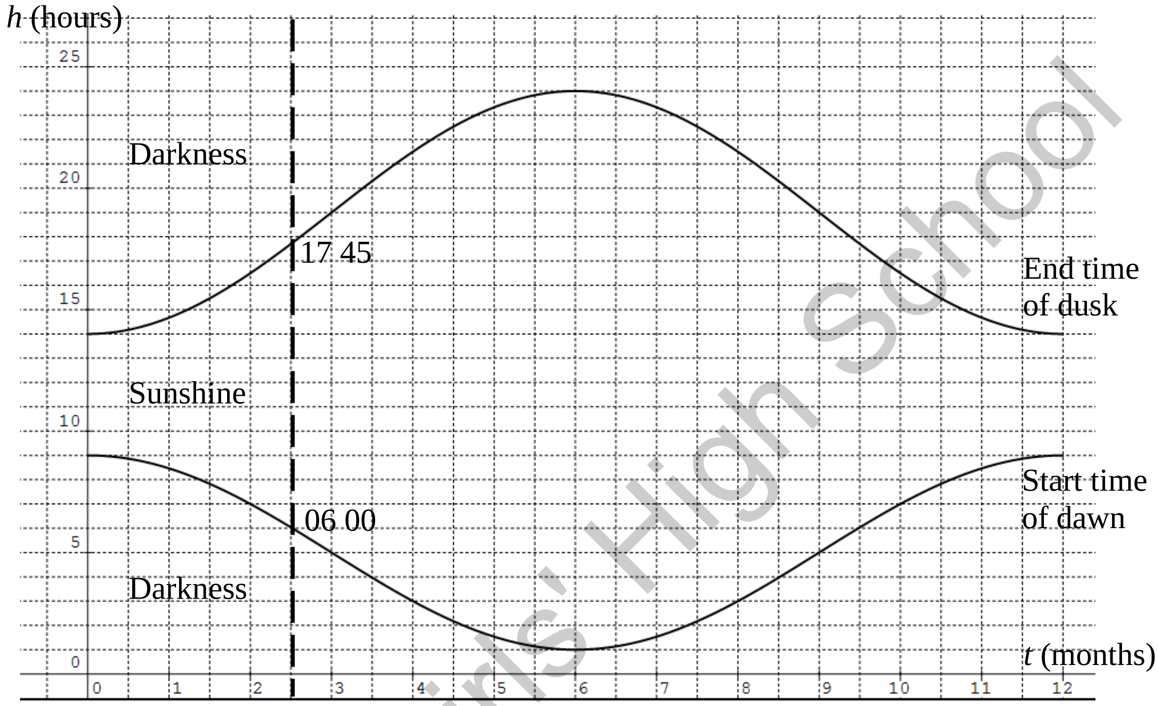
(ii) Hence, factorise $\mathrm{f}(x)$ completely and solve the equation $\mathrm{f}(x) = 0$. [3]

(b) Express $\dfrac{2x^2 - 3x - 1}{(x+2)(x+1)^2}$ as partial fractions. [4]

12 Figure I shows the start time of dawn and end time of dusk in Helsinki, Finland, for the 12 months of the whole year (January to December). For example, on the marked day in March, as shown by the dotted line, the start time of dawn is 06 00, there is sunshine from 06 00 to 17 45, the end time of dusk is at 17 45 and there is darkness after 17 45.

(Source: https://www.gaisma.com/en/location/helsinki.html)

Figure I



It is given that the start time of dawn is modelled by the function $h_1 = 4\cos\left(\frac{\pi t}{6}\right) + 5$, while the end time of dusk is modelled by the function $h_2 = 19 - 5\cos\left(\frac{\pi t}{6}\right)$. h_1 and h_2 are the number of hours after midnight and t is the time in months from the start of the year.

(i) State the amplitude of h_2. [1]

(ii) It is believed that the best period of the year for tourists to visit Helsinki is when the days are extremely long, that is, the difference in start time of dawn and end time of dusk is **at least 14 hours**. Use Figure I to find the best period of the year to visit Helsinki. [2]

(iii) It is also known that the start time of dawn in Beijing, China, is modelled by the function $h_3 = \frac{11}{2} + \frac{3}{2}\cos\left(\frac{\pi t}{6}\right)$. Calculate the values of t when dawn starts at 06 15 in Beijing, China. [3]

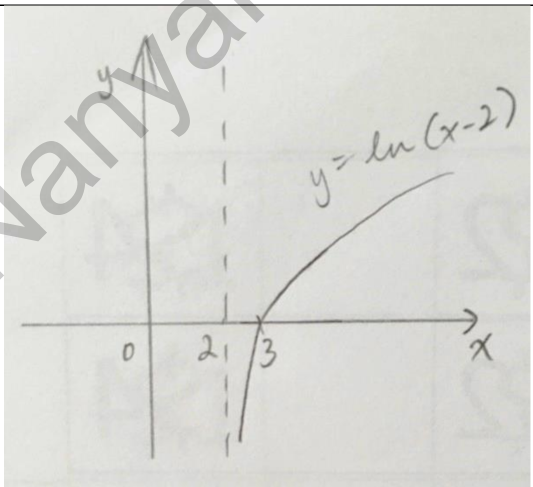
[Turn over

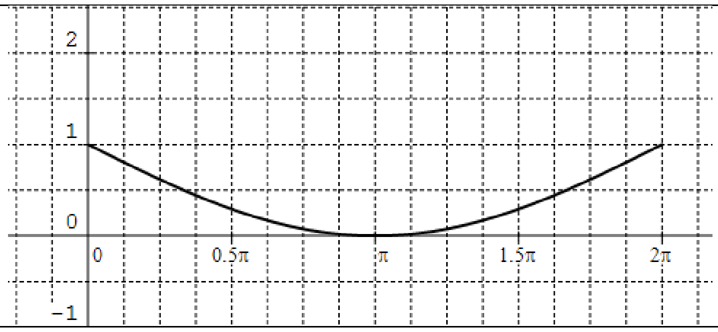
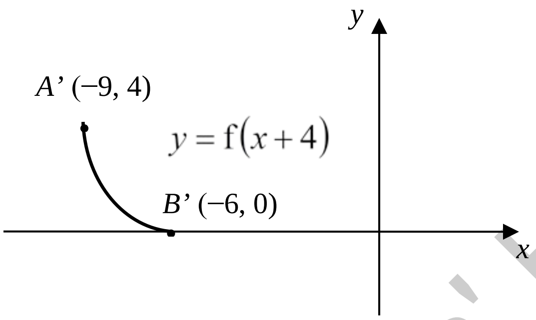
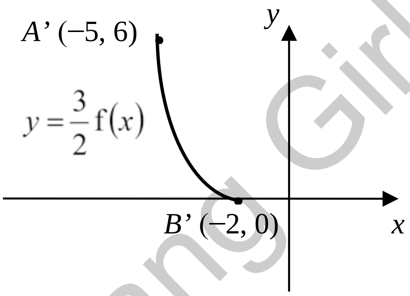
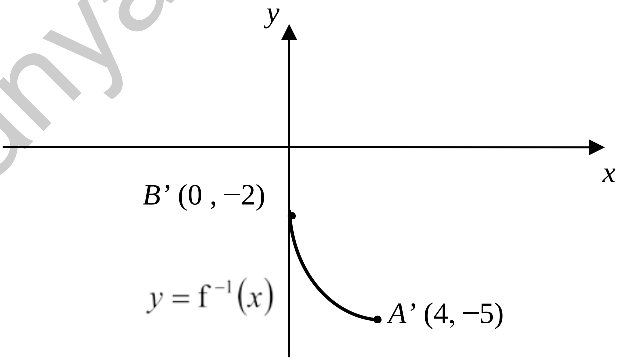
Bonus Question

13 Given that $(\log_{ab} a)(\log_c b) = \frac{1}{4}\log_c ab$, prove that $a = b$. [3]

END OF PAPER

Sec 3 IM2 EOY Answers only

Qn	Solution	
1	$\frac{11\sqrt{3}}{3}+4$	
2	$-7<m<1$	
3a	Area of pond $66.7 \quad m^2 \quad (3\,sf)$	
3b	Total perimeter $52.4 \quad m\,(3\,sf)$	
4	The ratio is 10 000 : 1.	
5i	$\alpha^2+2\alpha-6=0$ $\alpha^3+2\alpha^2-6\alpha=0$ $\alpha^3+2(6-2\alpha)-6\alpha=0$ $\alpha^3+12-4\alpha-6\alpha=0$ $\alpha^3=10\alpha-12$ Or $\beta=-2-\alpha$ $\alpha(-2-\alpha)=-6$ $-2\alpha-\alpha^2=-6$ $\alpha^2=6-2\alpha$ $\alpha^3=6\alpha-2\alpha^2$ $\alpha^3=6\alpha-2(6-2\alpha)$ $\alpha^3=10\alpha-12$	
5ii	New equation $x^2+\frac{14}{9}x-\frac{1}{6}=0$ $18x^2+28x-3=0$	
6a	 $y=3x+1$	
6b	Graph of $y=\left\|\sin\frac{x}{2}-1\right\|$	

		
7a	$x=1$	
7b	$x=1$	
7ci	$\frac{1}{3}m+\frac{2}{3}$	
7cii	$\frac{1}{-2m}$	
8a		
8b		
8c		
9ai	$\tan A=-\frac{\sqrt{1-p^2}}{p}$	
9aii		

	$\cos B = -\dfrac{1}{\sqrt{4p^2+1}}$	
9b	$\therefore x = 11.5°, 168.5°, 210° \text{ or } 330°$	
10ai	$2(x-2)^2 - 5$	
10aii	Range of f $-5 \le \mathrm{f}(x) \le 3$	
10aiii	f does not have an inverse because it is not a one to one function.	
10aiv	2	
10b	$x = 0 \quad or \quad x = 1$	
11ai	$a = -11$ $b = -16$	
11aii	$\mathrm{f}(x) = 10x^3 - 11x^2 - 16x - 4$ $= (x-2)(10x^2 + 9x + 2)$ $= (x-2)(2x+1)(5x+2)$ $(x-2)(2x+1)(5x+2) = 0$ $x = 2 \quad or \quad -\dfrac{1}{2} \quad or \quad -\dfrac{2}{5}$	
11b	$\dfrac{2x^2 - 3x - 1}{(x+2)(x+1)^2} = \dfrac{13}{x+2} - \dfrac{11}{x+1} + \dfrac{4}{(x+1)^2}$	
12i	5	
12ii	From last day of March to last day of September. Accept From: End of March, first day of April, beginning of April To: first day of October, beginning of October, end of September	
12iii	$t = 2 \quad or \quad 10$	
13	$(\log_{ab} a)(\log_c b) = \dfrac{1}{4}\log_c ab$ $\dfrac{\log_c a}{\log_c ab}(\log_c b) = \dfrac{1}{4}\log_c ab$ $(\log_c a)(\log_c b) = \dfrac{1}{4}(\log_c ab)^2$ $4(\log_c a)(\log_c b) = (\log_c a + \log_c b)^2$ $4(\log_c a)(\log_c b) = (\log_c a)^2 + 2(\log_c a)(\log_c b) + (\log_c b)^2$ $(\log_c a)^2 - 2(\log_c a)(\log_c b) + (\log_c b)^2 = 0$ $(\log_c a - \log_c b)^2 = 0$ $\log_c a = \log_c b$ $a = b$ (proven)	