CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2022

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS **4048/1**

Paper 1 **29 August 2022**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answers in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Simplify $2(5a+3b)-3(a-2b)$.

$2(5a+3b)-3(a-2b)$
$=10a+6b-3a+6b$ [M1 – correct expansion]
$=7a+12b$ [A1]

Answer .... [2]

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2 Factorise completely

(a) $2x^2+5x-3$,

$2x^2+5x-3$
$=(2x-1)(x+3)$ [B1]

Answer .... [1]

(b) $3ax+6ay-x-2y$.

$3ax+6ay-x-2y$
$=3a(x+2y)-(x+2y)$ [M1]
$=(x+2y)(3a-1)$ [A1]

Answer .... [2]

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3 If Abigail sells a bicycle for $270, she would make a loss of 25%.
How much must she sell the bicycle for to make a profit of 40%?

75% of the cost price = $270

$\therefore$ 140% of the cost price = $\dfrac{270}{75\%}\times 140\%$ [M1]

$= \$504$ [A1]

Answer $ .... [2]

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4 Cerra changed S$500 to US dollars in January at a rate of S$1 = USD 0.753.
In October, she changed the US dollars back to Singapore dollars at a rate of S$1 = USD 0.722.
Find the difference in the amount of Singapore dollars she received.
Give your answer correct to the nearest cent.

S$1 = USD 0.753

$\therefore$ S$500 $= \dfrac{0.753}{1}\times 500 =$ USD 376.50

Since USD 0.721 = S$1,

$\therefore$ USD 376.50 $= \dfrac{1}{0.721}\times 376.50$ [M1]

= S$521.47

Hence, difference = 521.47 − 500 = S$21.47 [A1]

Answer S$ .... [2]

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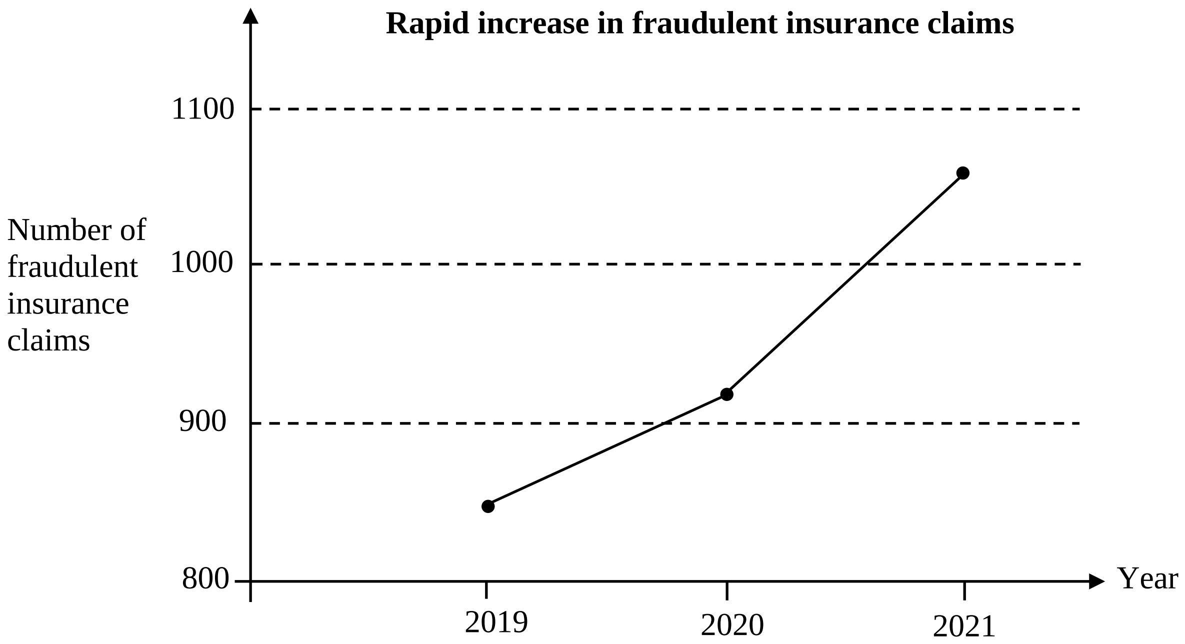
5 The graph shows the total number of fraudulent insurance claims received by a company in each of the years from 2019 to 2021.



(a) State one misleading feature of the graph.

Answer The vertical axis of the graph does not start from zero. [B1]
(Accept other plausible answers)

[1]

(b) Explain how this feature affects the reader's interpretation of the graph.

Answer The reader might think that there is a sudden spike/significant increase in the number of fraudulent insurance claims in the year 2021. [B1]
(Accept other plausible answers)

[1]

6 **(a)** Solve the equation $\frac{x}{3}-\frac{2x-5}{2}=1$.

$\frac{x}{3}-\frac{2x-5}{2}=1$

$2x-3(2x-5)=6$ [M1 – Getting rid of the denominator correctly]

$2x-6x+15=6$

$-4x=-9$

$\therefore x=2\frac{1}{4}$ [A1]

Answer .... [2]

(b) It is given that $q=25-2p^2$.

(i) Find q when $p=-3$.

$q=25-2(-3)^2=7$ [B1]

Answer .... [1]

(ii) Rearrange the formula to make p the subject.

$q=25-2p^2$

$2p^2=25-q$

$p^2=\frac{25-q}{2}$ [M1 – Isolating p correctly]

$\therefore p=\pm\sqrt{\frac{25-q}{2}}$ [A1]

Answer p = ... [2]

7 Given that $x - y = 7$ and $xy = 18$, find the values of $x^2 - y^2$.

$x - y = 7$

$\therefore (x-y)^2 = 49$

$x^2 - 2xy + y^2 = 49$

$\therefore x^2 + y^2 = 49 + 2xy$

$= 49 + 2(18)$ [M1]

$= 85$

$(x+y)^2 = x^2 + 2xy + y^2 = 85 + 2(18) = 121$

$\therefore (x+y) = 11$ or -11 [A1 for both answers]

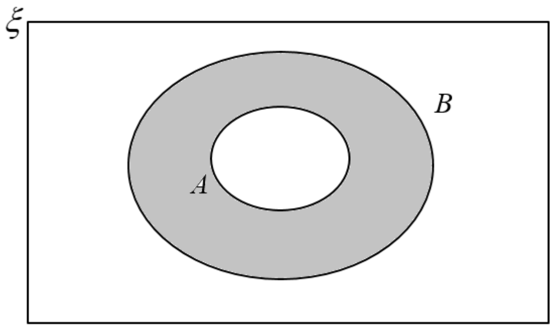
$\therefore x^2 - y^2 = (x+y)(x-y)$

$= 11(7)$ or $-11(7)$

$= 77$ or -77 [A1, A1]

Answer ……………… or ……….…… [4]

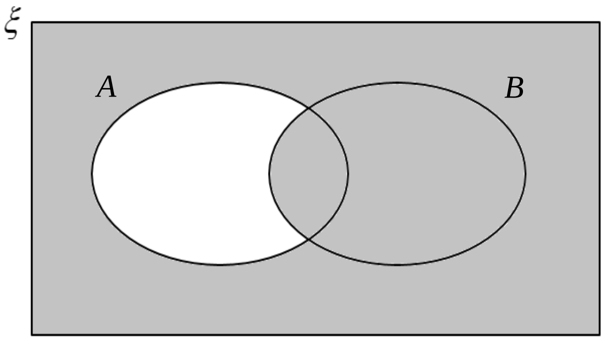
8 **(a)** On the Venn diagram, shade the region which represents $A' \cap B$.



[B1]

[1]

(b) Write down the sets represented by the shaded region.



$B \cup A'$ [B1]

Answer .... [1]

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9 The diagram shows a regular octagon, *ABCDEFGH*, and three sides, *PH*, *HG* and *GQ*, of a second regular polygon. Angle *PHA* = 81°.
Find the number of sides in the second regular polygon.

$\angle AHG = \dfrac{(8-2)\times 180^\circ}{8}$ [M1]

$= 135^\circ$

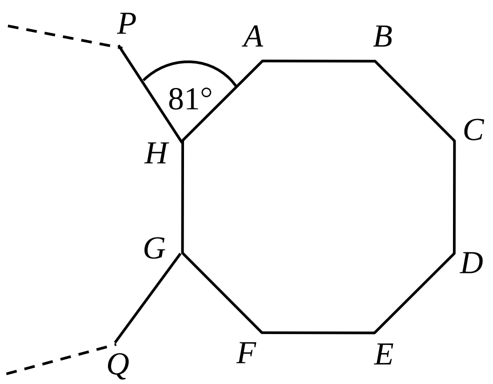
$\angle PHG = 360^\circ - 135^\circ - 81^\circ = 144^\circ$ (Angles at a pt)

$\dfrac{(n-2)\times 180^\circ}{n} = 144^\circ$ [M1]

$180^\circ n - 360^\circ = 144^\circ n$

$36^\circ n = 360^\circ$

$\therefore n = \dfrac{360^\circ}{36^\circ} = 10$ [A1]



Answer .... [3]

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10 **(a)** Simplify $\dfrac{6x^3}{5y^2} \div \dfrac{14xy^3}{15}$.

$$\frac{6x^3}{5y^2} \div \frac{14xy^3}{15} = \frac{6x^3}{5y^2} \times \frac{15}{14xy^3}$$

$= \dfrac{9x^2}{7y^5}$ [B1 – Correct numerator]
[B1 – Correct denominator]

Answer .... [2]

(b) $4^k = \dfrac{1}{4^9} + \dfrac{1}{4^9} + \dfrac{1}{4^9} + \dfrac{1}{4^9}$

<u>Without the use of a calculator</u>, find the value of *k*.

$4^k = \dfrac{1}{4^9} + \dfrac{1}{4^9} + \dfrac{1}{4^9} + \dfrac{1}{4^9}$

$4^k = 4\left(\dfrac{1}{4^9}\right)$

$4^k = 4 \times 4^{-9}$ [M1 – express RHS as base 4 correctly]

$= 4^{1-9} = 4^{-8}$

$\therefore k = -8$ [A1]

Answer .... [2]

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11 The values of seven numbers are

$$(x-3),\ (x+3),\ (x-4),\ (x+7),\ (x+4),\ (x-3),\ (x+2).$$

Find the value of *x* if the sum of the mode and the median of the numbers is 17.

$(x-4),\ (x-3),\ (x-3),\ (x+2),\ (x+3),\ (x+4),\ (x+7)$

Mode $= (x-3)$

Median $= (x+2)$

$\therefore (x-3)+(x+2) = 17$ [M1 – Constructing the eqn correctly]

$2x = 18$

$\therefore x = 9$ [A1]

Answer .... [2]

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12 A map has a scale of 1 : 50 000.

(a) The distance from Jurong and Changi is 26 km.
Calculate the distance, in centimetres, between Jurong and Changi on the map.

1 cm : 50000 cm

1 cm : 0.5 km [M1 – converting cm to km correctly]

Since 0.5 km is represented by 1 cm,

$\therefore$ 26 km is represented by $\dfrac{1}{0.5}\times 26$

$= 52$ cm [A1]

Answer .... cm [2]

(b) The East Coast Park is the largest park in Singapore with an area of 1.85 km^2.
Calculate the area, in square centimetres, of East Coast Park on the map.

0.5 km : 1 cm

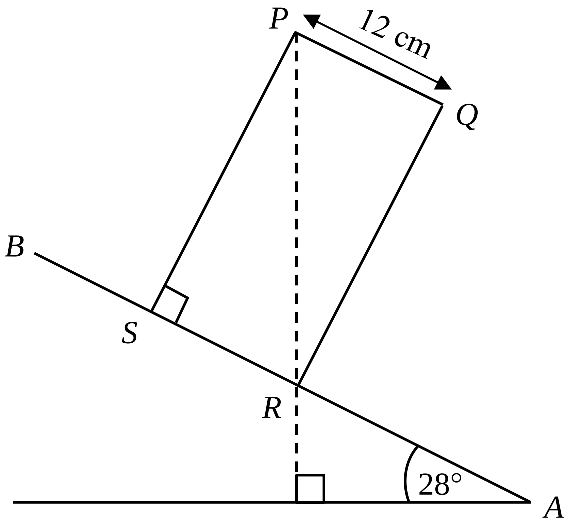
$\therefore$ 0.25 km^2 : 1 cm^2 [M1 – converting km^2 to cm^2 correctly]

$\therefore$ 1.85 km^2 is represented by $\dfrac{1}{0.25}\times 1.85$

$= 7.4$ cm^2 [A1]

Answer .... cm^2 [2]

13 AB is a plane inclined at an angle of 28° with the horizontal ground.
$PQRS$ is the side view of a cuboid. The diagonal PR produced is perpendicular to the ground.
Given that $PQ = 12$ cm and $AR = \frac{3}{4}PR$, find



(a) PS,

$\angle PRS = 90^\circ - 28^\circ = 62^\circ$

$\tan 62^\circ = \frac{PS}{12}$

$\therefore PS = 12 \tan 62^\circ$ [M1]

$= 22.56871758$

≈ 22.6 cm (3 s.f.) [A1]

Answer .... cm [2]

(b) the angle of elevation of Q from A.

$PR = \sqrt{(22.56871758)^2 + (12)^2}$ [M1]

$= 25.56065362$ cm

$\therefore AR = \frac{3}{4} \times 25.56065362 = 19.17049021$ cm

$\tan \angle QAR = \frac{22.56871758}{19.17049021}$

$\therefore \angle QAR = \tan^{-1}\left(\frac{22.56871758}{19.17049021}\right)$ [M1]

$= 49.6545107^\circ$

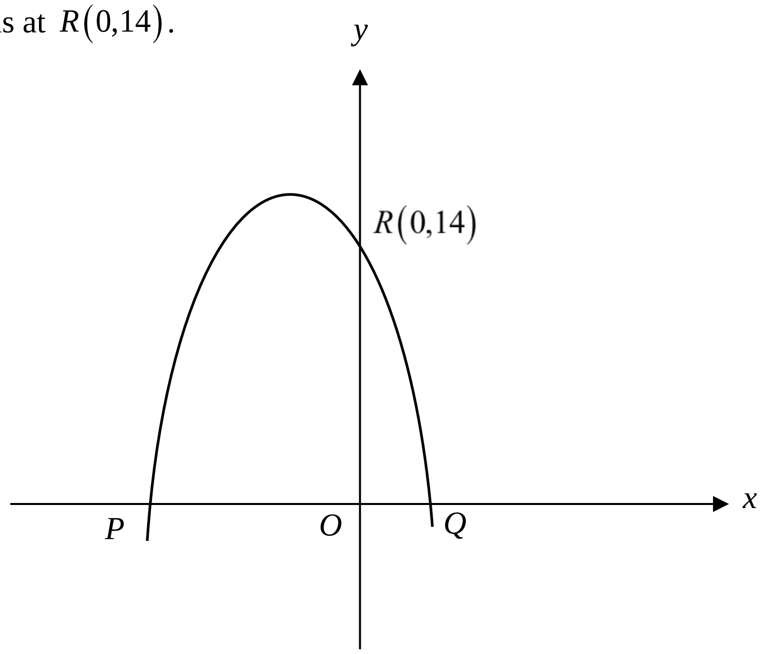
$\therefore$ Angle of elevation $= 49.6545107^\circ + 28^\circ$

$= 77.6545107^\circ$

$= 77.7^\circ$ (1 d.p.) [A1]

Answer .... ° [3]

14 The graph of $y=(x+a)(2-x)$, where a is a constant, cuts the x-axis at P and Q and it cuts the y-axis at $R(0,14)$.



(a) Find the value of a.

When $x=0,\ y=14,$ we have,

$14=(0+a)(2-0)$

$14=2a$

$\therefore a=7$ [B1]

Answer $a=$ .... [1]

(b) State the equation of the line of symmetry for $y=(x+a)(2-x)$.

$P(-7,0)$ and $Q(2,0)$

$\therefore$ equation of the line of symmetry is $x=\dfrac{-7+2}{2}$

$x=-2\dfrac{1}{2}$ [B1 – Accept also $x=-2.5$ or $x=-\dfrac{5}{2}$]

Answer .... [1]

15 **(a)** Express $x^2-6x+11$ in the form $(x-h)^2+k$.

$x^2-6x+11$

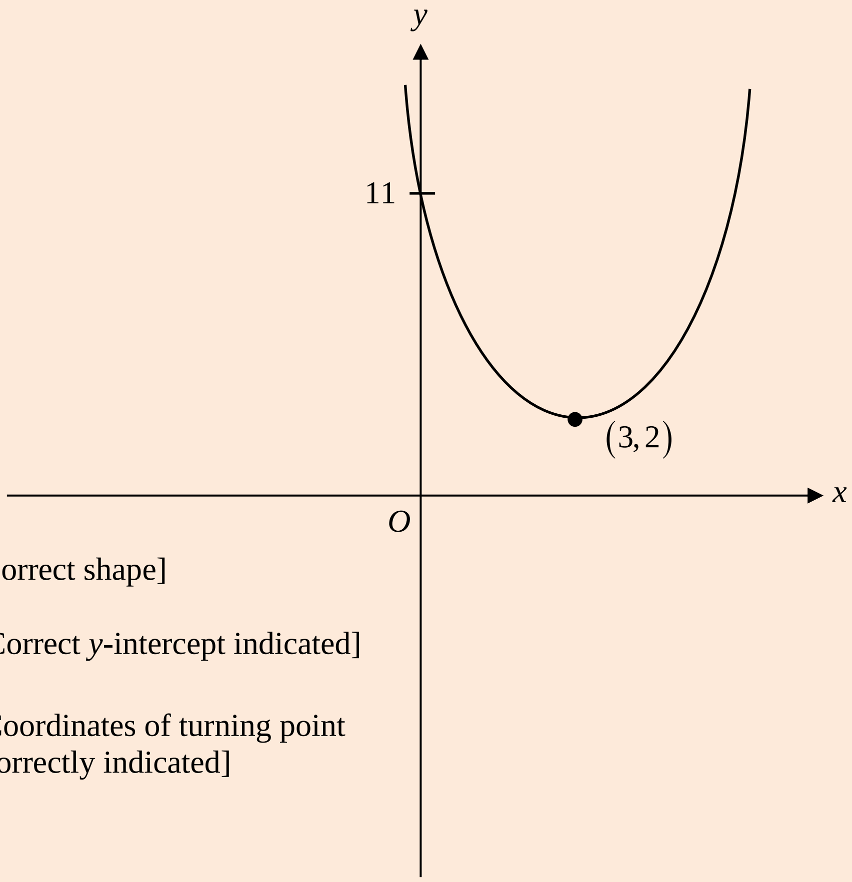
$=x^2-6x+\left(\frac{-6}{2}\right)^2-\left(\frac{-6}{2}\right)^2+11$

$=(x-3)^2+2$ [B1 for $(x-3)^2$] [B1 for $+2$]

Answer .... [2]

(b) Sketch the graph of $y=x^2-6x+11$, indicating clearly where the graph crosses the axes (if any) and the coordinates of the turning point of the curve.

[3]



[P1 – Correct shape]

[B1 – Correct *y*-intercept indicated]

[B1 – Coordinates of turning point correctly indicated]

16 **(a)** Find the prime factors of 35000, giving your answer in index form.

$35000 = 2^3 \times 5^4 \times 7$ [B1]

Answer .... [1]

(b) Two integers P and Q, can be written as products of prime factors.

$$P = a^3 \times b^c \times 7 \qquad Q = a \times b^{c+1} \times 7$$

The lowest common multiple (LCM) of P and Q is 35000.

(i) Write down the value of a, b and c.

$35000 = 2^3 \times 5^4 \times 7$

$P = a^3 \times b^c \times 7$

$Q = a \times b^{c+1} \times 7$

$\therefore a = 2,$ [B1]

$b = 5,$ [B1]

$c + 1 = 4$

$\therefore c = 3$ [B1]

Answer a = ...

b = ...

c = ... [3]

(ii) Find the highest common factor (HCF) of P and Q.

$P = 2^3 \times 5^3 \times 7$

$Q = 2 \times 5^4 \times 7$

$\therefore \text{HCF} = 2 \times 5^3 \times 7 = 1750$ [B1]

Answer .... [1]

17 Eunice bought a 3-Room HDB apartment for $120 000. She paid 15% downpayment on the apartment and took a loan on the outstanding balance from a bank, to be paid over 10 years. Given that the bank charges an interest rate of r% compounded annually, calculate the value of r if her monthly instalment over the 10 years was $1270.

Amt borrowed $= 0.85 \times \$120000$ [M1]

$= \$102000$

Total amt of instalments paid $= \$1270 \times 12 \times 10 = \$152400$

$\therefore\ 152400 = 102000\left(1+\frac{r}{100}\right)^{10}$ [M1]

$\left(1+\frac{r}{100}\right)^{10} = \frac{152400}{102000} = \frac{127}{85}$

$1+\frac{r}{100} = \sqrt[10]{\frac{127}{85}}$

$\therefore r = \left(\sqrt[10]{\frac{127}{85}}-1\right)\times 100 = 4.097063731 \simeq 4.10\%$ [A1]

Answer …………………………… % [3]

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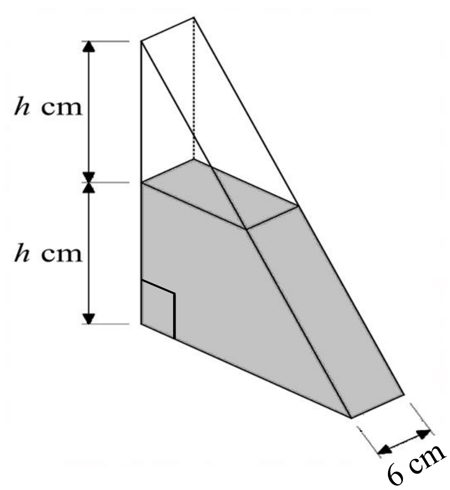
18 The diagram shows a triangular container with a uniform cross-section and a thickness of 5 cm.

The container is filled with liquid to half its perpendicular height. Find the ratio of the volume of water in the container to the volume of the container.



Let the base of the empty space in the container = b cm.

$\therefore$ Ratio of the volume of water : volume of container

$= \dfrac{\left(\frac{1}{2}\times 2b\times 2h\times 6\right)-\left(\frac{1}{2}\times b\times h\times 6\right)}{\left(\frac{1}{2}\times 2b\times 2h\times 6\right)}$ [M1]

$= \dfrac{9bh}{12bh}$

$= \dfrac{3}{4}$

$= 3:4$ [A1]

Answer ……………… : ……………… [2]

19 In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are

$$17 \quad p \quad q \quad r \quad -7$$

(a) Find the value of r.

Common difference $= \dfrac{17-(-7)}{4} = 6$

$\therefore r = -7+6 = -1$ [B1]

Answer .... [1]

(b) Find an expression, in terms of n, for the n^{th} term of the sequence.

$T_n = -6n + 23$ [B1]

Answer .... [1]

(c) Explain why –345 is not a term of this sequence.

$-6n + 23 = -345$

$-6n = -368$

$n = 61\frac{1}{3}$

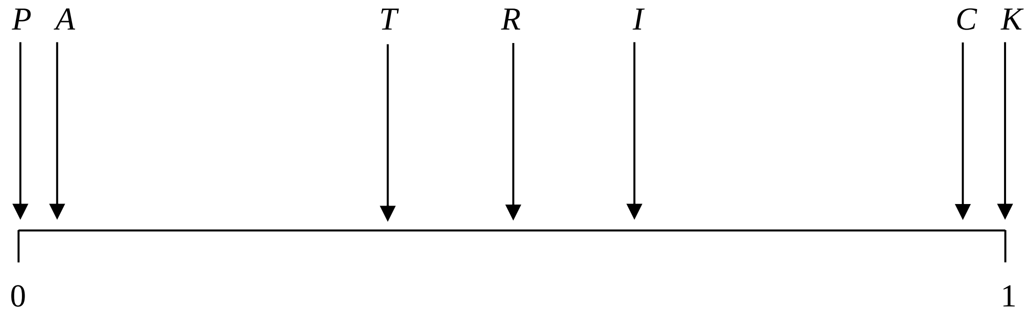
Since $n = 61\frac{1}{3}$ **<u>is not an integer</u>**, –345 is not a term of the sequence. [B1]

Answer..

....

.... [1]

20 The number line shows the probabilities of 7 events represented by the arrows *P*, *A*, *T*, *R*, *I*, *C* and *K*.



Write down the letter of the correct arrow that best represents each of these events:

(a) Choosing a marble randomly from a box that contains 5 red marbles and 3 green marbles and obtaining a green marble.

$\mathrm{P}(\text{green marble}) = \frac{3}{8} \Rightarrow \text{arrow } T$ [B1]

Answer .... [1]

(b) Rolling a fair 6-sided die and obtaining a prime number.

$\mathrm{P}(\text{prime number}) = \frac{3}{6} = \frac{1}{2} \Rightarrow \text{arrow } R$ [B1]

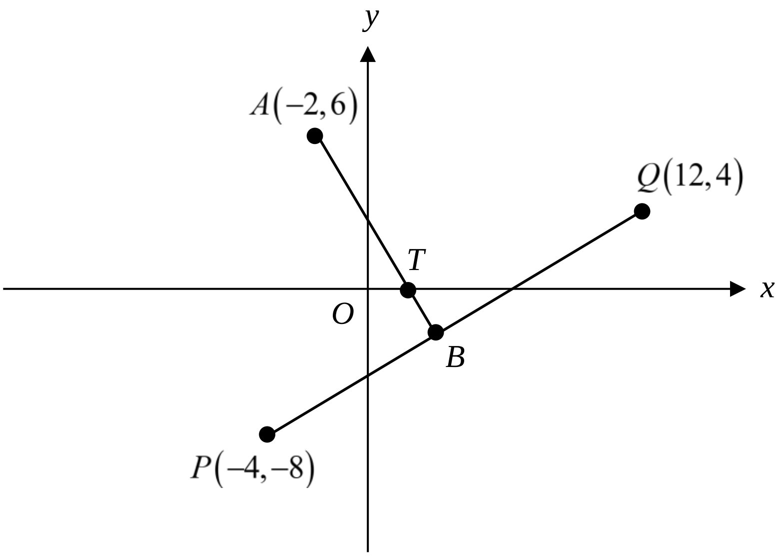
Answer .... [1]

(c) Selecting a number at random from the set { 3, 5, 7, 9 } and obtaining an even number.

$\mathrm{P}(\text{even number}) = 0 \Rightarrow \text{arrow } P$ [B1]

Answer .... [1]

21



The diagram shows the points $P(-4,-8)$, $Q(12,4)$ and $A(-2,6)$.
B is a point on *PQ* so that *AB* is perpendicular to *PQ*.
The line segment *AB* cuts the *x*-axis at point *T*.

The product $(\text{gradient of } AB)\times(\text{gradient of } PQ)=-1$.

Use this information to find the coordinates of *T*.

$m_{PQ}=\dfrac{4-(-8)}{12-(-4)}$ [M1]

$=\dfrac{3}{4}$

$\therefore m_{AB}=\dfrac{-1}{\frac{3}{4}}$ [M1]

$=-\dfrac{4}{3}$

$y=-\dfrac{4}{3}x+c$

When $x=-2,\ y=6,$ we have,

$6=-\dfrac{4}{3}(-2)+c$

$\therefore c=\dfrac{10}{3}$

$\therefore$ equation of *AB* is $y=-\dfrac{4}{3}x+\dfrac{10}{3}$ [A1]

When $y=0,$ we have,

$0=-\dfrac{4}{3}x+\dfrac{10}{3}$ [M1]

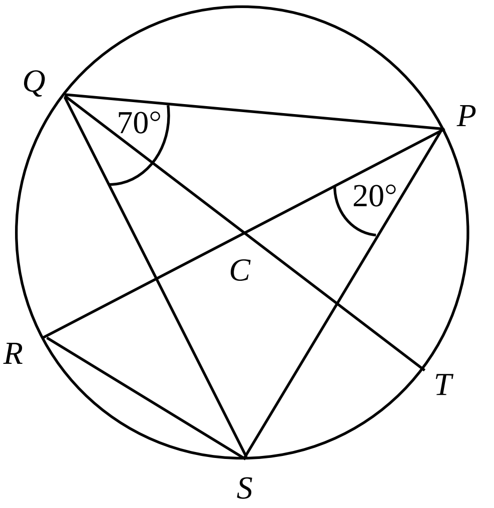
$\therefore x=2\dfrac{1}{2}$

$\therefore$ Coordinates of T is $\left(2\dfrac{1}{2},0\right)$ [A1]

* [Also accept $(2.5,0)$ or $\left(\dfrac{5}{2},0\right)$]

Answer (................ , ...) [5]

22 In the diagram, QT is the diameter of the circle.
R, S and P are points on the circle. Angle PQS = 70° and angle RPS =20°.
QT intersects PR at point C.



(a) Find angle PRS.

$\angle PRS = 70^{\circ}$ (Angles in the same segment) [B1]

Answer .... ° [1]

(b) Hence, prove that the point C is the centre of the circle.

$\angle RSP = 180^{\circ} - 70^{\circ} - 20^{\circ} = 90^{\circ}$ (Angle sum of a triangle)
This implies that PR is a diameter (Angle in a semicircle). [B1]

Since QT is also a diameter and point C is the point of intersection of the diameters PR and QT, C is the centre of the circle. [B1]

Answer..

....

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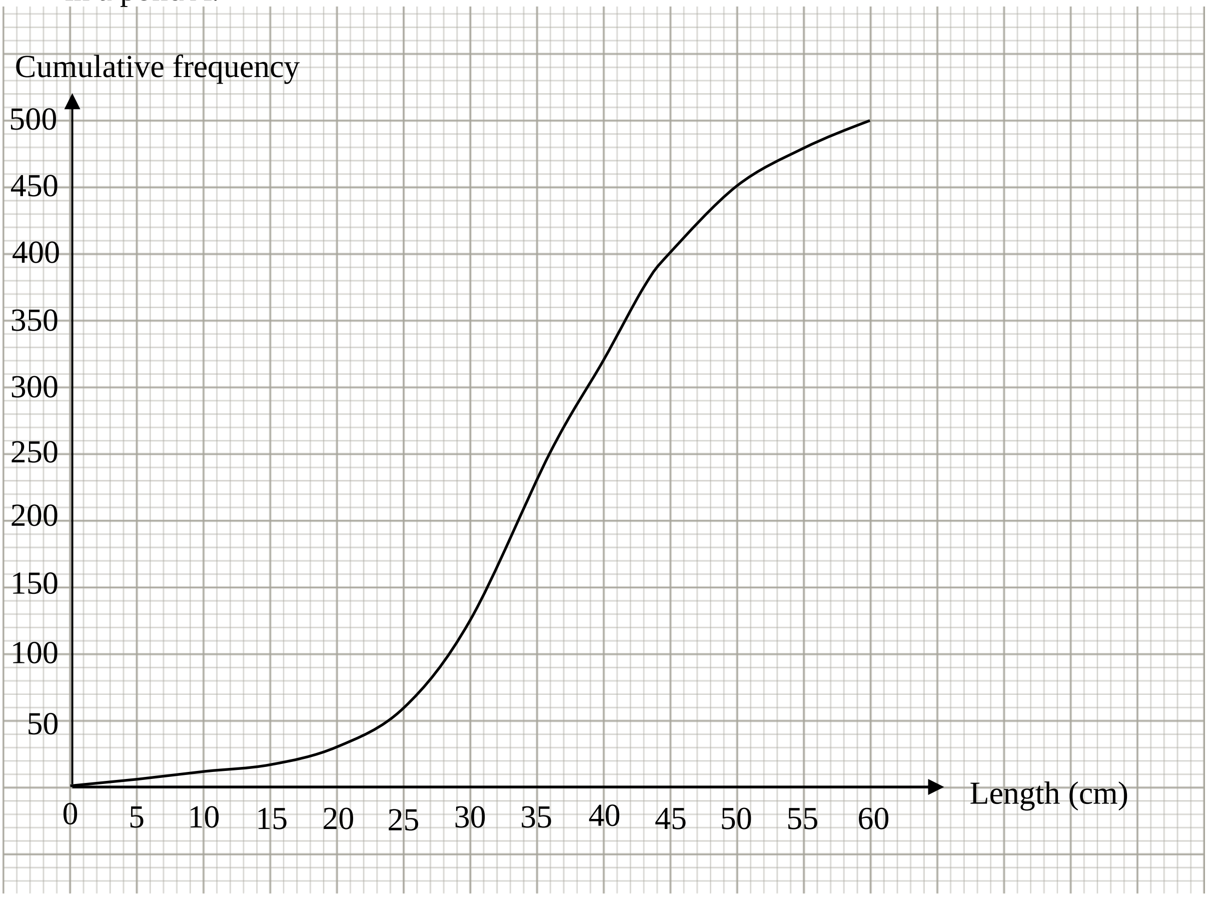
.... [2]

23 The diagram below shows the cumulative frequency curve for the lengths of 500 fishes in a pond *A*.



(a) Use the graph to find the

(i) the percentage of fishes that are more than 45 cm long,

$\frac{500-400}{500} \times 100\% = 20\%$ [B1]

Answer .... % [1]

(ii) the median length of the fishes,

36 [B1]

Answer .... cm [1]

(iii) the interquartile range of the lengths of the fishes.

I.Q. range $= 43 - 30 = 13$ [B1]

Answer .... cm [1]

(b) In another pond *B*, the interquartile range of the lengths of the fishes there is 5 cm. Explain what this tells us about the fishes in pond *B* compared to those in pond *A*.

Answer..

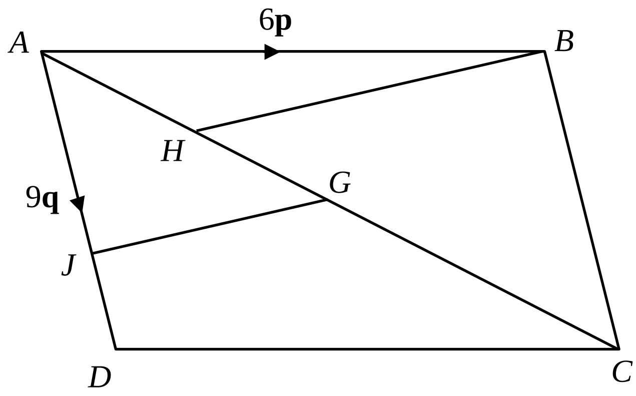
The lengths of the fishes in Pond *B* are more consistent than the lengths of the fishes in Pond *A* as the Inter-Quartile range for *B* < Inter-Quartile range for *A*. [B1]

....

.... [1]

24 *ABCD* is a parallelogram. *G* is the mid-point of *AC* and *H* is the mid-point of *AG*.
J is a point on *AD* such that $AD = 3JD$.

$\overrightarrow{AB} = 6\mathbf{p}$ and $\overrightarrow{AD} = 9\mathbf{q}$.



(a) Express, as simply as possible, in terms of **p** and/or **q**,

(i) $\overrightarrow{AC}$,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 6\mathbf{p} + 9\mathbf{q}$ [B1]

Answer .... [1]

(ii) $\overrightarrow{BH}$,

$\overrightarrow{BH} = \overrightarrow{BA} + \overrightarrow{AH}$

$= -6\mathbf{p} + \frac{1}{4}(6\mathbf{p} + 9\mathbf{q}) = -\frac{9}{2}\mathbf{p} + \frac{9}{4}\mathbf{q}$ [B1]

Answer .... [1]

(iii) $\overrightarrow{GJ}$.

$\overrightarrow{GJ} = \overrightarrow{GA} + \overrightarrow{AJ}$

$= \frac{1}{2}(-6\mathbf{p} - 9\mathbf{q}) + \frac{2}{3}(9\mathbf{q}) = -3\mathbf{p} + \frac{3}{2}\mathbf{q}$ [B1]

Answer .... [1]

(b) What do your answers to **(a)(ii)** and **(a)(iii)** tell you about *BH* and *GJ*?
Explain your answers clearly.

$\overrightarrow{BH} = -\frac{9}{2}\mathbf{p} + \frac{9}{4}\mathbf{q} = \frac{3}{2}(-3\mathbf{p} + \frac{3}{2}\mathbf{q}) = \frac{3}{2}\overrightarrow{GJ}$

Since $\overrightarrow{BH} = k(\overrightarrow{GJ})$, where $k = \frac{3}{2}$, *BH* is parallel to *GJ*. [B1]

$|\overrightarrow{BH}| = \frac{3}{2}|\overrightarrow{GJ}|$. [B1]

.... [2]

~~ End of Paper 1 ~~