Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2018

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

Wednesday 10th October 2018

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		9	
2			
3		10	
4			
5			
6		11	
7			
8			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

This question paper consists of 11 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The hours of sunlight in each day of a year, $h(t)$, where t is the day of the year, is given by $h(t) = 2.4\sin(0.0172t - 1.38) + 12, \quad 1 \le t \le 365$.

(a) Find the maximum hours of sunlight and the minimum hours of sunlight in a day.

(b) Find the number of days where there is less than 12 hours of sunlight.

2. [Maximum mark: 4]

Planes π_1, π_2 and π_3 have equations $x + 2y + 3z = 9$, $2x - y + z = k$ and $3x + kz = 3$ respectively, where k is a constant.

(a) Given that $k = 2$, find the point of intersection of π_1, π_2 and π_3.

(b) Given instead that $k = 3$, describe the relationship between π_1, π_2 and π_3.

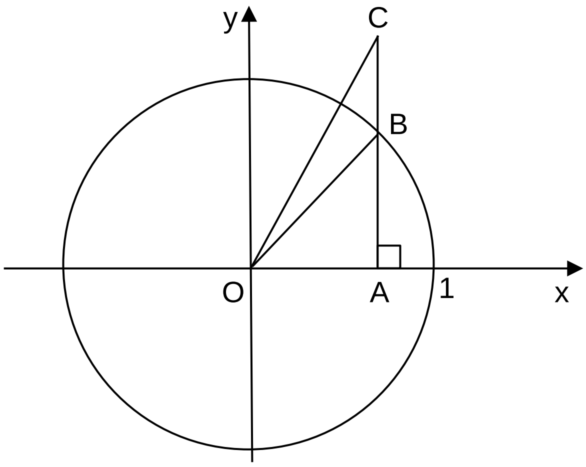
3. [Maximum mark: 5]

(a) Find the complex roots z_1 and z_2 of the equation $z^2 + (7+i)z + 24 + 7i = 0$.

(b) The points z_1, z_2, z_3 and z_4 form a square centred at the origin. Find the complex numbers represented by z_3 and z_4.

4. [Maximum mark: 7]

A clock has an hour hand of unit length OB and a longer minute hand OC. At a certain time, the hands of the clock are in the position shown below where C, B and A lie on a straight line, $B\hat{O}A = \alpha$ and $C\hat{O}A = 2\alpha$.



[Diagram is not drawn to scale.]

(a) Write down the lengths of OA and BA in terms of α .

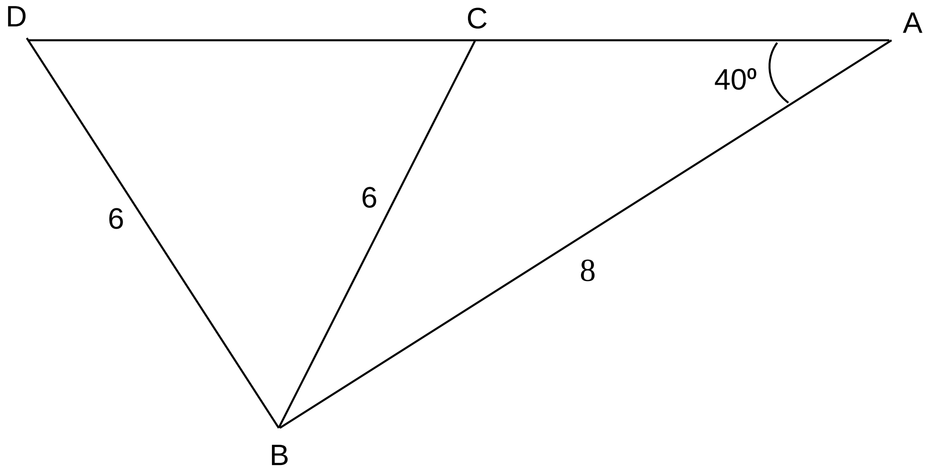
(b) Given that the length of OC is $\sqrt{3}$ units, find the time on the clock face when C, B and A lie on a straight line, for $0 \le \alpha \le \frac{\pi}{2}$.

5. [Maximum mark: 6]

Given ΔABC, with lengths shown in the diagram below,

(a) find $A\hat{C}B$.

(b) Find the length of line segment CD.



6. [Maximum mark: 7]

The function f is defined by $f(x) = x\sqrt{9-x^2} + 2\arcsin\left(\frac{x}{3}\right)$.

(a) State the largest possible domain for f .

(b) Find $f'(x)$ in simplified form.

7. [Maximum mark: 7]

Consider the functions $f(x) = x^3 + 1$ and $g(x) = \dfrac{1}{x^3 + 1}$. The graphs of $y = f(x)$ and $y = g(x)$ meet at the point (0, 1) and one other point, P.

(a) Find the coordinates of P.

(b) Calculate the size of the acute angle between the tangents to the two graphs at the point P.

8. [Maximum mark: 8]

(a) Sketch the graph of $y = \left|\frac{4x^2+8x}{x^2-2}\right|$, for $-15 \le x \le 15$, showing and stating clearly any asymptotes.

(b) Hence find the set of values for x when $\left|\frac{4x^2+8x}{x^2-2}\right| > 3$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 13]

Let f be a function defined by $f(x) = x - \arctan 3x + \sin x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) Show that f is an odd function. [2]

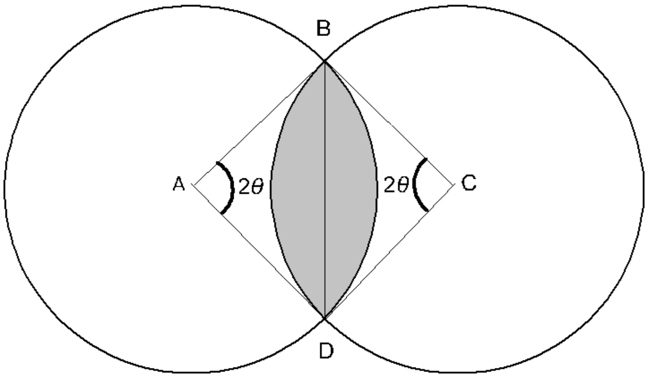
(b) Find the coordinates of the points of inflexion of the function f. [5]

(c) Determine if each point found in (b) is a stationary or non-stationary point of inflexion and justify your answer. [3]

(d) State the values of x for which f is concave up. [3]

10. [Maximum mark: 13]

Two circles, of fixed radius one centimetre, intersect and the chord on both circles subtends an angle of 2θ radians at either circle as shown in the diagram below. $0 < \theta \le \frac{\pi}{2}$.



[Diagram is not drawn to scale.]

(a) Show that the area of the shaded region common to both circles can be expressed by $S = 2\theta - \sin 2\theta$. [2]

(b) Find the value of θ, in radians, when the shaded region S is half the area of one circle. [3]

(c) As the centers of the circles, A and C, move towards each other, AC decreases at a rate of 2 cm/min. Show that θ is increasing at a rate of $\frac{1}{\sin\theta}$ rad/min. [4]

(d) Hence show that the rate at which S is increasing can be expressed by $\frac{dS}{dt} = 4\sin\theta$. [4]

[Turn the page over]

11. [Maximum mark: 24]

The following lines L_1 and L_2, have equations:

$$L_1 : \underset{\sim}{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, t \in \mathbb{R} \qquad \text{and} \qquad L_2 : \underset{\sim}{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, s \in \mathbb{R}$$

(a) Determine where L_1 meets the x-z plane. [2]

(b) Determine whether the lines L_1 and L_2 are intersecting, parallel or skew. [5]

(c) P is a point with coordinates $(-17, 9, 9)$.

Find the coordinates of the foot of the perpendicular from P to L_1. Hence find the coordinates of the image of P reflected in L_1. [8]

A plane π_1 has vector equation

$$\underset{\sim}{r} = (-2\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}) + \lambda(2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}) + \mu(6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}), \quad \lambda, \mu \in \mathbb{R}.$$

(d) Show that the cartesian equation of the plane π_1 is $3x + 2y - 6z = 12$. [3]

(e) Find the angle between the plane π_1 and the line L_1. [3]

(f) Given another plane with equation, $\pi_2 : x + 2y - z = -3$, find the vector equation of the line of intersection of π_1 and π_2. [3]

END OF PAPER