Mathematical Formula

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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1 Solve the equation $3\left(9^x\right) = 15 - 4\left(3^x\right)$. [4]

2 Given $\log_5 3 = k$, express the following in terms of k.

(i) $\log_5 15$ [2]

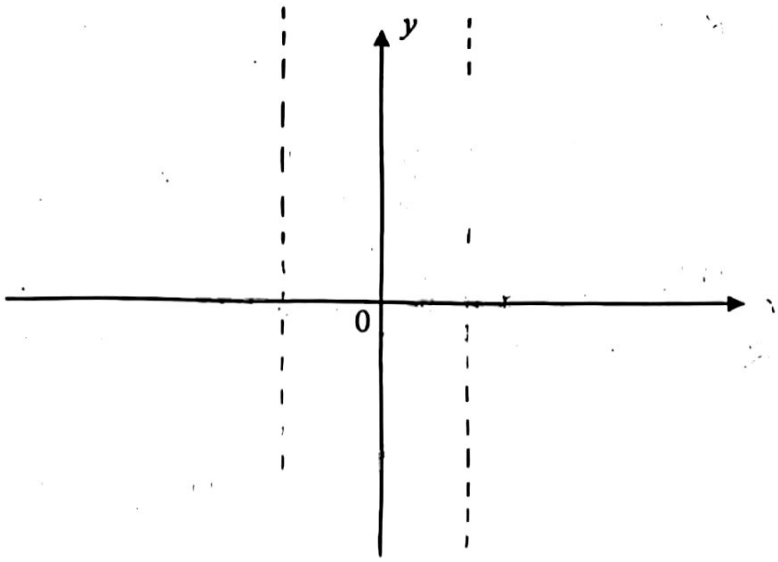
(ii) $\log_3 \frac{9}{5}$ [2]

3 Determine if the equation $\log_3 (x-4) = 2\log_3 x - \log_3 3$ has any real roots. [3]

4 (i) Sketch the graph of $y = \ln(x-2)$ on the given axes, indicating clearly the intercept(s) and asymptote(s). [2]

(ii) In order to solve the equation $e^x + e^4(2-x) = 0$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = \ln(x-2)$. Find the equation of the straight line. Add the straight line to your diagram and find the number of solutions to the equation. [3]

(i)

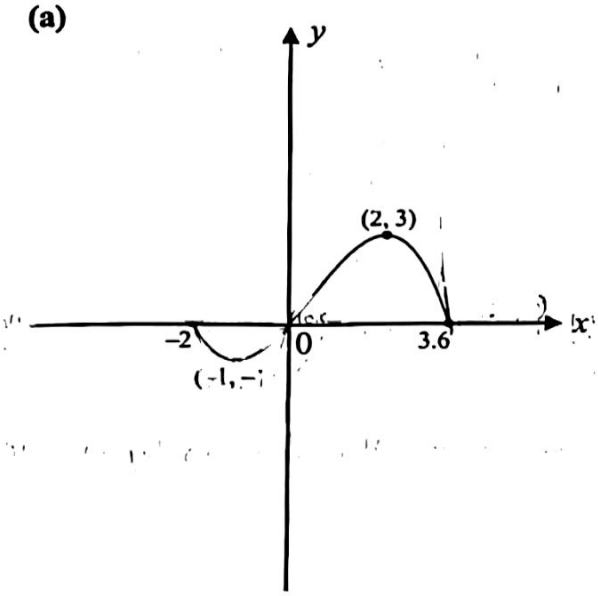


5 The graph of $y = f(x)$ is shown in the diagram below. On the same axes, sketch and label the following graphs, showing clearly the new positions of the points indicated in the diagram:

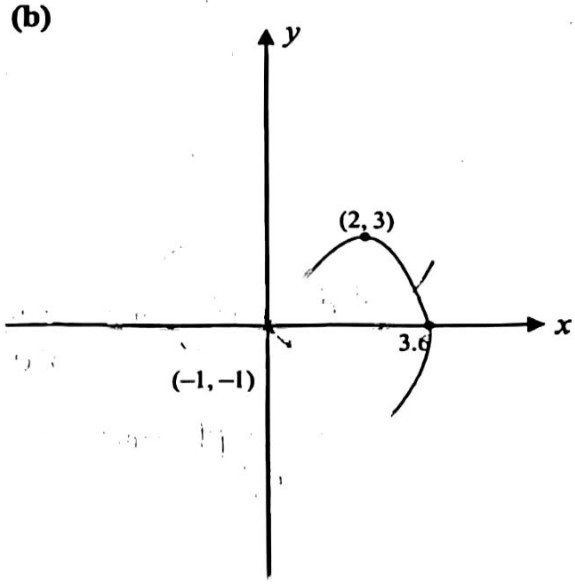
(a) $y = 2f(x)$; → scale [2]

(b) $y = f(-x)$. → ref [2]

(a)



(b)



6 **(a)** The graph of $y = x^2 + 3x - 1$ can be obtained from the graph of $y = (x+1)(x+2)$ under a single transformation. Describe this transformation. [2]

(b) The graph of $y = g(x)$ undergoes two successive transformations:

I Scaling parallel to the x-axis by factor $\frac{1}{2}$.

II Reflection in the x-axis.

The resulting graph is $y = -4x^2 + 10x + 1$. Find an expression for $g(x)$. [2]

7 Chilled dessert is reheated to a temperature of 85°C. It subsequently cools in such a way that its temperature, M °C, t minutes after removal from the heating source, is given by $M = 23 + Ae^{-kt}$, where A and k are constants. 2 minutes after removing from the heating source, the temperature of the dessert is found to be 55°C.

(i) Show that $A = 62$. [1]

(ii) Find the value of k. [2]

(iii) The dessert should only be served when its temperature is less than 50°C. Determine, with clear working, whether the dessert is suitable to be served 3 minutes after removing from the heating source. [3]

End of Test