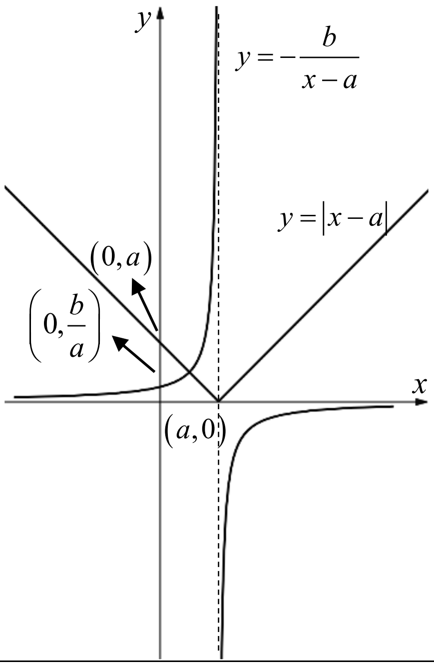
2025 NJC Preliminary Examination H2 Mathematics Paper 1 Solutions

1	Suggested Solution
	Let the exchange rate for US Dollar, Japanese Yen and Chinese Yuan be 1 to U, 1 to J and 1 to C Singapore Dollars respectively. $150U + 5500J + 1000C = 419.30$ $250U + 9500J + 2200C = 797.20$ $425U + 1000J + 2000C = 913.10$ By GC, $U = 1.2856, \quad J = 0.00862, \ C = 0.17905$ For Maybelline, $1.2856a + 1200(0.17905) = 568.40$ $a = 275$

2	Suggested Solutions
(i)	$(1+\mathrm{i})z+2w=-2+4\mathrm{i}$ $3z-w=4+2\mathrm{i}$ $\Rightarrow 6z-2w=8+4\mathrm{i}$ Solving simultaneously by elimination: $(6+1+\mathrm{i})z=-2+4\mathrm{i}+8+4\mathrm{i}$ $(7+\mathrm{i})z=6+8\mathrm{i}$ $z=\dfrac{6+8\mathrm{i}}{7+\mathrm{i}}\times\dfrac{7-\mathrm{i}}{7-\mathrm{i}}$ $=\dfrac{42-6\mathrm{i}+56\mathrm{i}+8}{50}$ $=\dfrac{50+50\mathrm{i}}{50}$ $=1+\mathrm{i}$ w $=3z-4-2\mathrm{i}$ $=3(1+\mathrm{i})-4-2\mathrm{i}$ $=-1+\mathrm{i}$
(ii)	$\dfrac{w}{z}$ $=\dfrac{-1+\mathrm{i}}{1+\mathrm{i}}$ $=\dfrac{-1+\mathrm{i}}{1+\mathrm{i}}\times\dfrac{1-\mathrm{i}}{1-\mathrm{i}}$ $=\dfrac{-1+\mathrm{i}+\mathrm{i}+1}{1^2+1^2}$ $=\mathrm{i}$ $w=\mathrm{i}z$ **Alternative** $\dfrac{w}{z}$ $=\dfrac{-1+\mathrm{i}}{1+\mathrm{i}}$ $=\dfrac{\mathrm{i}^2+\mathrm{i}}{1+\mathrm{i}}$ $=\dfrac{\mathrm{i}(\mathrm{i}+1)}{1+\mathrm{i}}$ $=\mathrm{i}$ $w=\mathrm{i}z$

	Rotating line segment OZ by $\frac{\pi}{2}$ anti-clockwise about the origin will give OW. Or line segment OW is a anti-clockwise rotation about the origin by $\frac{\pi}{2}$ from line segment OZ.

<table>
<tr><th>3</th><th>Suggested Solutions</th></tr>
<tr><td>(i)</td><td>
</td></tr>
<tr><td>(ii)</td><td>$-\dfrac{b}{x-a}=-(x-a)$
$(x-a)^2=b$
$x=a-\sqrt{b}$ or $x=a+\sqrt{b}$ (rejected since $x<a$)

$x<a-\sqrt{b}$ or $x>a$</td></tr>
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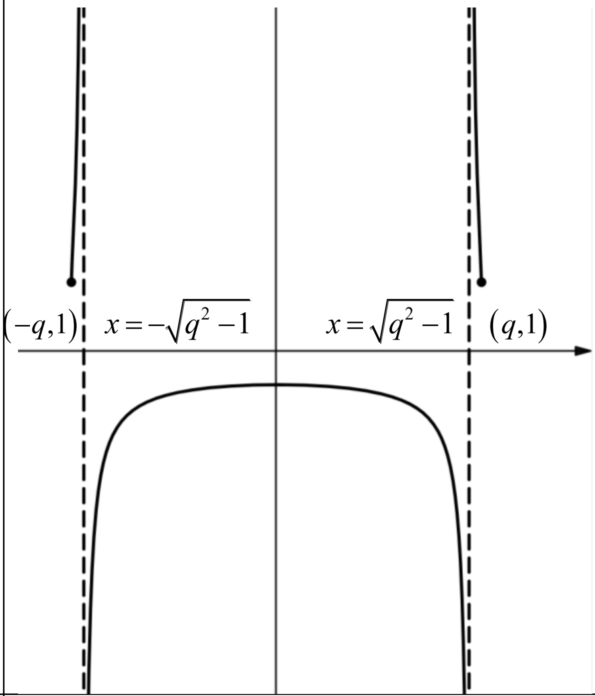
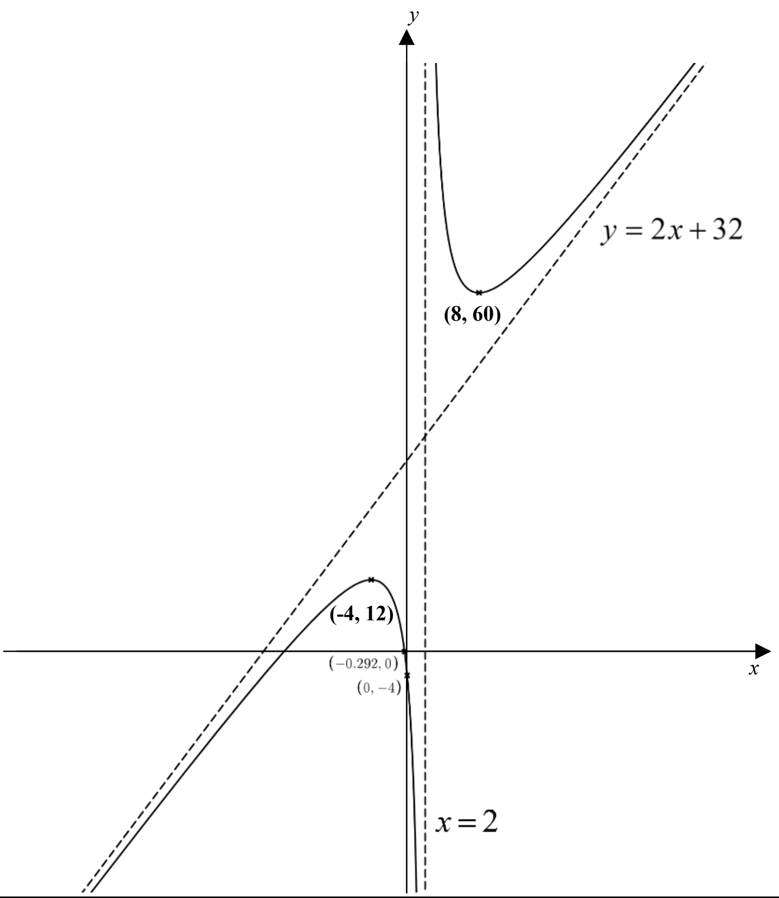
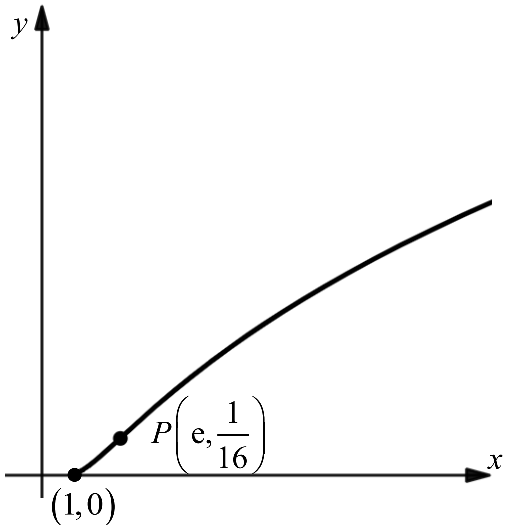
4	Suggested Solutions
	$y=1-\sqrt{q^2-x^2}$, $q>1$ $(y-1)^2=q^2-x^2$ $x^2+(y-1)^2=q^2$ $y=\mathrm{f}(x)$ is a semi-circle.
(i)	
(ii)	$y=1-\sqrt{q^2-x^2}$ Replace y with $y+1$: $y+1=1-\sqrt{q^2-x^2}$ $y=-\sqrt{q^2-x^2}$ Replace x with qx: $y=-\sqrt{q^2-(qx)^2}$ $y=-q\sqrt{1-x^2}$ Replace y with qy: $qy=-q\sqrt{1-x^2}$ $y=-\sqrt{1-x^2}$ Translate 1 unit in the negative y direction. Scale by a factor of $\frac{1}{q}$ parallel to the x-axis. Scale by a factor of $\frac{1}{q}$ parallel to the y-axis.

5	Suggested Solutions
(i)	Given vertical asymptote at $x=2$, $p=-2$ $\therefore y=\dfrac{2x^2+kx+8}{x-2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x-2)(4x+k)-(2x^2+kx+8)}{(x-2)^2}$ Given $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\vert_{x=-4}=0$, $((-4)-2)(4(-4)+k)-(2(-4)^2+k(-4)+8)=0$ $(-6)(-16+k)-(32-4k+8)=0$ $\therefore k=28$ By long division or otherwise, $\therefore y=\dfrac{2x^2+28x+8}{x-2}=2x+32+\dfrac{72}{x-2}$ Equation of oblique asymptote of C is $\therefore y=2x+32$
(ii)	

6	Suggested Solutions
(a) (i)	$S_\infty = \dfrac{1}{1-r}$ and $S_3 = \dfrac{\left(1-r^3\right)}{1-r}$, $\lvert r\rvert<1$ **Method 1** $\dfrac{1}{1-r} = \left(\dfrac{1-r^3}{1-r}\right)^2$ $1 = \dfrac{\left(1-r^3\right)^2}{1-r}$, since $\dfrac{1}{1-r} \neq 0$ $1-r = \left(1-r^3\right)^2$ $1-r = 1-2r^3+r^6$ $r^6-2r^3+r=0$ $r^5-2r^2+1=0$, since $r \neq 0$ $(r-1)\left(r^4+r^3+r^2-r-1\right)=0$ Since $r \neq 1$, $r^4+r^3+r^2-r-1=0$ **Method 2** $\dfrac{1}{1-r} = \left(r^2+r+1\right)^2$ $1 = \left(r^2+r+1\right)^2(1-r)$ $\left(r^2+(r+1)\right)^2(1-r)-1=0$ $\left[r^4+(1+r)^2+2r^2(1+r)\right](1-r)-1=0$ $r^4(1-r)+(1+r)\left(1-r^2\right)+2r^2\left(1-r^2\right)-1=0$ $\left(r^4-r^5\right)+\left(1-r^2\right)+r-r^3+\left(2r^2-2r^4\right)-1=0$ $r+r^2-r^3-r^4-r^5=0$ $r^4+r^3+r^2-r-1=0$, since $r \neq 0$
(a) (ii)	By G.C., since $\lvert r\rvert<1$ $r=-0.661$ (3 s.f.) or $r=0.848$ (3 s.f.)

(b)	Sum of first $4n$ terms $=\frac{4n}{2}\left[2(7)+(4n-1)(d)\right]$ $=28n+8dn^2-2dn$ Terms to be removed, $7+3d, 7+7d, 7+11d, \ldots,$ (First term = $7+3d$; common difference = $4d$; No. of terms removed = n terms. Sum of removed terms $=\frac{n}{2}\left[2(7+3d)+(n-1)(4d)\right]$ $=\frac{n}{2}(14+6d+4dn-4d)$ $=\frac{n}{2}(14+2d+4dn)$ $=7n+nd+2dn^2$ Sum of remaining terms $=28n+8dn^2-2dn-7n-nd-2dn^2$ $=6dn^2-3dn+21n$
	Method 2 Sum every three consecutive terms as a single term i.e. $7+(7+d)+(7+2d),\ (7+4d)+(7+5d)+(7+6d), \ldots$ $\Rightarrow 21+3d,\ 21+15d, 21+27d$ In the progression above, first term is $21+3d$, common difference is $12d$, and there are n terms. Sum $=\frac{n}{2}\left(2(21+3d)+(n-1)12d\right)$ $=21n+3dn+n(n-1)6d$ $=6dn^2-3dn+21n$

7	Suggested Solutions
(i)	$x = \mathrm{e}^{4t}$, $y = t^2$, $t \geq 0$. Sub $t = \dfrac{\ln x}{4}$ into $y = t^2$: $y = \left(\dfrac{\ln x}{4}\right)^2$ $\Rightarrow y = \dfrac{1}{16}(\ln x)^2$
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{8x}(\ln x)$ Steepest gradient: $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ $= \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $= \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{8x}(\ln x)$ $= \dfrac{1}{8}\dfrac{\mathrm{d}}{\mathrm{d}x}x^{-1}(\ln x)$ $= \dfrac{1}{8}\left[-x^{-2}(\ln x) + \left(\dfrac{1}{x}\right)^2\right]$ $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$ $\dfrac{1}{8}\left[-\dfrac{1}{x^2}(\ln x) + \left(\dfrac{1}{x}\right)^2\right] = 0$ $\left(\dfrac{1}{x}\right)^2\left[-(\ln x) + 1\right] = 0$ $-(\ln x) + 1 = 0$ $\Rightarrow x = \mathrm{e}$ $\quad\left(\text{since } \dfrac{1}{x} \neq 0 \text{ for } x \in \mathbf{R}\right)$ $P\left(\mathrm{e}, \dfrac{1}{16}\right)$

(iii)	Point $\left(\mathrm{e}, \frac{1}{16}\right)$ is the point with the greatest gradient. When $t=0$, $x=\mathrm{e}^{4(0)}=1$, $y=0$ Point $(1,0)$ is the initial point. For $t \geq 0$, $x>1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8x}(\ln x)>0$. 

8	Suggested Solutions
(i)	$\frac{a}{a-x}-1$ $=a(a-x)^{-1}-1$ $=\frac{a}{a}\left(1-\frac{x}{a}\right)^{-1}-1$ $=\left(1-\frac{x}{a}\right)^{-1}-1$ $=\left(1+(-1)\left(-\frac{x}{a}\right)+\frac{(-1)(-2)}{2!}\left(-\frac{x}{a}\right)^{2}+\ldots\right)-1$ $=\frac{x}{a}+\frac{x^2}{a^2}+\ldots$ Validity range: $\left\|-\frac{x}{a}\right\|<1$ $\|-1\|\left\|\frac{x}{a}\right\|<1$ $\frac{\|x\|}{\|a\|}<1$ $\|x\|<a \ (\text{since } a>0)$ $-a<x<a$
(ii)	$e^{\frac{a}{a-x}-1}=e^{\frac{x}{a}+\frac{x^2}{a^2}+\ldots}$ $=1+\left(\frac{x}{a}+\frac{x^2}{a^2}+\ldots\right)+\frac{1}{2}\left(\frac{x}{a}+\frac{x^2}{a^2}+\ldots\right)^2$ $\approx 1+\left(\frac{x}{a}+\frac{x^2}{a^2}+\ldots\right)+\frac{1}{2}\frac{x^2}{a^2}$ $=1+\frac{x}{a}+\frac{3}{2}\frac{x^2}{a^2}+\ldots$

(iii)	$\int_0^{\frac{a}{2}} e^{\frac{a}{a-x}-1}\ dx$ $\approx \int_0^{\frac{a}{2}} 1+\frac{x}{a}+\frac{3}{2}\frac{x^2}{a^2}\ dx$ $=\left[x+\frac{x^2}{2a}+\frac{1}{2}\frac{x^3}{a^2}\right]_0^{\frac{a}{2}}$ $=\left(\frac{a}{2}\right)+\frac{\left(\frac{a}{2}\right)^2}{2a}+\frac{1}{2}\frac{\left(\frac{a}{2}\right)^3}{a^2}$ $=\frac{11}{16}a$ The neglected terms in the expansion of $e^{\frac{a}{a-x}-1}$ for $0<x<\frac{a}{2}$ are all positive terms. The approximating curve $1+\frac{x}{a}+\frac{3}{2}\frac{x^2}{a^2}+...$will be below $y=e^{\frac{a}{a-x}-1}$. Hence area under the approximated curve will be smaller than area under $y=e^{\frac{a}{a-x}-1}$ for $0<x<\frac{a}{2}$.

9	Solution
(a)	$\int \cos(3\ln x)\,dx$ $= x\cos(3\ln x) - \int x\left(-\sin(3\ln x)\frac{3}{x}\right)dx$ $= x\cos(3\ln x) + 3\int \sin(3\ln x)\,dx$ $= x\cos(3\ln x) + 3\left[x\sin(3\ln x) - \int x\cos(3\ln x)\frac{3}{x}\,dx\right]$ $= x\cos(3\ln x) + 3x\sin(3\ln x) - 9\int \cos(3\ln x)\,dx$ Hence $10\int \cos(3\ln x)\,dx = x\cos(3\ln x) + 3x\sin(3\ln x) + c$ $\int \cos(3\ln x)\,dx = \frac{x}{10}\cos(3\ln x) + \frac{3x}{10}\sin(3\ln x) + C$
(b) (i)	$\int \frac{1-x}{1-\sqrt{x}}\,dx = \int \frac{(1-\sqrt{x})(1+\sqrt{x})}{1-\sqrt{x}}\,dx$ $= \int 1+\sqrt{x}\,dx$ $= x + \frac{2}{3}x^{\frac{3}{2}} + c$ OR $\int \frac{1-x}{1-\sqrt{x}}\,dx = \int \frac{(1-x)(1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})}\,dx$ $= \int \frac{(1-x)(1+\sqrt{x})}{1-x}\,dx$ $= \int 1+\sqrt{x}\,dx$ $= x + \frac{2}{3}x^{\frac{3}{2}} + c$
(ii)	$u = 1-\sqrt{x} \Rightarrow \frac{du}{dx} = -\frac{1}{2\sqrt{x}} = -\frac{1}{2(1-u)}$ $\int \frac{1}{1-\sqrt{x}}\,dx$ $= \int \frac{1}{u}\cdot -2(1-u)\,du$ $= 2\int 1 - \frac{1}{u}\,du$ $= 2(u - \ln u) + c$, since $0 < x < 1, u > 0$ $= 2\left(1-\sqrt{x} - \ln\left(1-\sqrt{x}\right)\right) + c$ $= -2\sqrt{x} - 2\ln\left(1-\sqrt{x}\right) + C$

	$\int \frac{x}{1-\sqrt{x}}\,\mathrm{d}x = -\int \frac{1-x}{1-\sqrt{x}}\,\mathrm{d}x + \int \frac{1}{1-\sqrt{x}}\,\mathrm{d}x$ $= -x - \frac{2}{3}x^{\frac{3}{2}} + 2\left(1-\sqrt{x}-\ln\left(1-\sqrt{x}\right)\right) + c$ $= -x - \frac{2}{3}x^{\frac{3}{2}} + 2 - 2\sqrt{x} - 2\ln\left(1-\sqrt{x}\right) + c$ $= -x - \frac{2}{3}x^{\frac{3}{2}} - 2\sqrt{x} - 2\ln\left(1-\sqrt{x}\right) + C$

10	Suggested Solution
(i)	$u_1 = S_1 = 1$ $1 = A - \frac{2}{2!}$ $A = 2$
(ii)	$u_n = S_n - S_{n-1}$ $= \left[2 - \frac{2}{(n+1)!}\right] - \left[2 - \frac{2}{n!}\right]$ $= \frac{2}{n!} - \frac{2}{(n+1)!}$ $= \frac{2(n+1)}{(n+1)!} - \frac{2}{(n+1)!}$ $= \frac{2n}{(n+1)!}$ $= \frac{2n}{(n+1)(n)\left[(n-1)!\right]}$ $= \frac{2}{(n+1)\left[(n-1)!\right]}$ So $u_n = \frac{2}{(n+1)\left[(n-1)!\right]}$. Check for u_1: $\frac{2}{(1+1)\left[(1-1)!\right]} = \frac{2}{2(0!)} = 1 = u_1$ For $n \geq 1$, $u_n = \frac{2}{(n+1)\left[(n-1)!\right]}$.
(iii)	$u_{n+1} = \frac{2}{(n+2)(n!)}$. $\frac{u_{n+1}}{u_n} = \frac{\frac{2}{(n+2)(n!)}}{\frac{2}{(n+1)\left[(n-1)!\right]}}$ $= \frac{2}{(n+2)(n!)} \times \frac{(n+1)\left[(n-1)!\right]}{2}$ $= \frac{n+1}{n(n+2)}$ So $u_{n+1} = \frac{n+1}{n(n+2)} u_n$

<table>
<tr><td>(iv)</td><td>As $n \to \infty$, $(n+1)! \to \infty, \dfrac{2}{(n+1)!} \to 0$.

Therefore S_n converges to 2.</td></tr>
<tr><td>(v)</td><td>$u_m + u_{m+1} + u_{m+2} + ...$
$= \sum_{r=1}^{\infty} u_r - \sum_{r=1}^{m-1} u_r$
$= 2 - \left[2 - \dfrac{2}{(m-1+1)!} \right]$
$= \dfrac{2}{m!} \le 10^{-10}$

By GC,
<table><tr><td>m</td><td>$\dfrac{2}{m!}$</td></tr><tr><td>…</td><td>…</td></tr><tr><td>13</td><td>3.21×10^{-10}</td></tr><tr><td>14</td><td>2.29×10^{-11}</td></tr><tr><td>…</td><td>…</td></tr></table>
Least value of *m* is 14.</td></tr>
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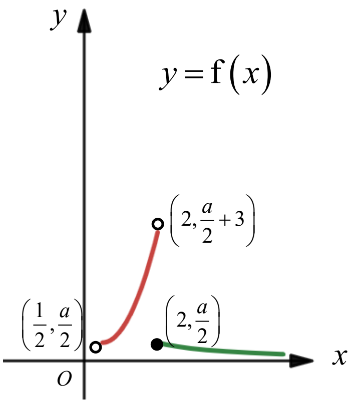
11	Suggested Solution
(i)	 $\mathrm{R_f}=\left(0,\frac{a}{2}+3\right)$
(ii)	For $\frac{1}{2}<x<2$, $y=\frac{a}{2}+\frac{4}{3}\left(x-\frac{1}{2}\right)^2$ $\frac{1}{2}\pm\sqrt{\frac{3}{4}\left(y-\frac{a}{2}\right)}=x$ $x=\frac{1}{2}+\sqrt{\frac{3}{4}\left(y-\frac{a}{2}\right)}$ (since $x>\frac{1}{2}$) $\therefore \mathrm{f}^{-1}(x)=\frac{1}{2}+\sqrt{\frac{3}{4}\left(x-\frac{a}{2}\right)}$ For $x\geq 2$, $y=\frac{a}{x}$ $x=\frac{a}{y}$ $\therefore \mathrm{f}^{-1}(x)=\frac{a}{x}$ $\mathrm{f}^{-1}(x)=\begin{cases}\frac{a}{x} & \text{for } x\in\mathbf{R},\ 0<x\leq\frac{a}{2}\\ \frac{1}{2}+\sqrt{\frac{3}{4}\left(x-\frac{a}{2}\right)} & \text{for } x\in\mathbf{R},\ \frac{a}{2}<x<\frac{a}{2}+3\end{cases}$
(iii)	Since $\mathrm{R_g}=(3,\infty)\subseteq\left(\frac{1}{2},\infty\right)=\mathrm{D_f}$, fg exists

(iv)	**Method 1:** $\text{fg}(k)=\dfrac{a}{7}$ $\text{g}(k)=\text{f}^{-1}\left(\dfrac{a}{7}\right)$ $3+\text{e}^k=\dfrac{a}{\left(\dfrac{a}{7}\right)}\left(\because 0<\dfrac{a}{7}\le\dfrac{a}{2}\right)$ $\text{e}^k=4$ $k=\ln 4$ **Method 2:** $\text{fg}(k)=\text{f}\left(3+\text{e}^k\right)$ $\quad=\dfrac{a}{3+\text{e}^k}\quad\left(\because 3+\text{e}^k>3>2\right)$ $\dfrac{a}{3+\text{e}^k}=\dfrac{a}{7}$ $7=3+\text{e}^k$ $4=\text{e}^k$ $k=\ln 4$

12	Suggested Solutions
(i)	$y=0$
(ii)	$\overrightarrow{OM}=\overrightarrow{OA}+\lambda\left(\overrightarrow{OB}-\overrightarrow{OA}\right),0\le\lambda\le1$ $\overrightarrow{OM}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\left(\begin{pmatrix}10\\0\\4\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}\right),0\le\lambda\le1$ $\overrightarrow{OM}=\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix},0\le\lambda\le1$ OR $\overrightarrow{OM}=\begin{pmatrix}10-10\mu\\0\\4-\mu\end{pmatrix},0\le\mu\le1$
(iii)	$\overrightarrow{MC}=\overrightarrow{OC}-\overrightarrow{OM}=\begin{pmatrix}6\\4\\2\end{pmatrix}-\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}=\begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}$ **<u>Method 1:</u>** $\overrightarrow{MC}\bullet\overrightarrow{EC}=0 \quad\Rightarrow\quad \begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}=0$ $-1-\lambda=0 \quad\Rightarrow\quad \lambda=-1$ Hence, since $0\le\lambda\le1$ for a point on line segment AB, cables will not be perpendicular. **<u>Method 2:</u>** $\overrightarrow{OM}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}10\\0\\1\end{pmatrix}=\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}$ $\overrightarrow{MC}=\overrightarrow{OC}-\overrightarrow{OM}=\begin{pmatrix}6\\4\\2\end{pmatrix}-\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}=\begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}$

$$\cos\angle MCE = \frac{\overrightarrow{MC}\bullet\overrightarrow{EC}}{\left|\overrightarrow{MC}\right|\left|\overrightarrow{EC}\right|}$$

$$= \frac{\begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{(6-10\lambda)^2+4^2+(-1-\lambda)^2}}$$

$$= -\frac{1+\lambda}{\sqrt{(6-10\lambda)^2+4^2+(1+\lambda)^2}}$$

From GC, for $0 \le \lambda \le 1$, $\angle MCE$ is always greater than $\frac{\pi}{2}$. (i.e. between $97.9°$ and $109.4°$). Hence it is impossible for *MC* and *EC* to be perpendicular.

<u>Method 3:</u>

If both cables are perpendicular, it is necessary that *M* is a point that is vertically 2 units from the horizontal ground.

z-component of $\overrightarrow{OM}$ is $3+\lambda$, where $0 \le \lambda \le 1$. This means that the *z*-component is always greater than 2.

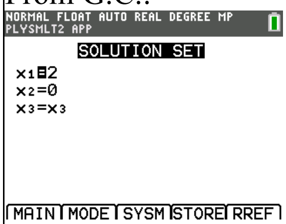
Both cables are never perpendicular to each other.

(iv) Solving *ECFG* : $x - y = 2$ and *OAB:* $y = 0$:

Method 1a: Intersection of planes (using GC)

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\0\end{pmatrix}=2 \text{ and } \begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}0\\1\\0\end{pmatrix}=0$$

From G.C.:



$$\text{r} = \begin{pmatrix}2\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\0\\1\end{pmatrix}, \lambda \in \text{R}$$

Method 1b: Intersection of planes (w/o GC)

	Solving simultaneously, we get $x = 2$, $y = 0$ and $z \in \mathbb{R}$. Equation of line of intersection l_{FG}: $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$ Points F and G lie on l so $\overrightarrow{FG}$ is parallel to $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and is parallel to $\overrightarrow{CE}$. Hence, $ECFG$ is a trapezium. **<u>Method 2: Solve for points *F* and *G*</u>** To find F: Solve $ECFG$ with $x - y = 2$ and line AB with $\mathbf{r} = \begin{pmatrix} 10\lambda \\ 0 \\ 3+\lambda \end{pmatrix}$: $10\lambda - 0 = 2 \Rightarrow \lambda = \frac{1}{5}$ $\overrightarrow{OF} = \begin{pmatrix} 10\left(\frac{1}{5}\right) \\ 0 \\ 3+\frac{1}{5} \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 16/5 \end{pmatrix}$ To find G: Solve $ECFG$ with $x - y = 2$ and line OD with $r = \mu \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$: $\mu - 0 = 2 \Rightarrow \mu = 2$ $\overrightarrow{OG} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

	$\overrightarrow{GF}=\overrightarrow{OF}-\overrightarrow{OG}=\begin{pmatrix}2\\0\\16/5\end{pmatrix}-\begin{pmatrix}2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\16/5\end{pmatrix}=k\begin{pmatrix}0\\0\\1\end{pmatrix}$, for non-zero k. $\overrightarrow{GF}$ is parallel to $\begin{pmatrix}0\\0\\1\end{pmatrix}$ and is parallel to $\overrightarrow{CE}$.
(v)	**Method 1:** Apply $\overrightarrow{EG}\perp\overrightarrow{EC}$ Since $\overrightarrow{EC}$ is perpendicular to $z=0$, $\overrightarrow{EC}\perp\overrightarrow{EG}$. $\overrightarrow{OG}=\begin{pmatrix}2\\0\\0\end{pmatrix},\overrightarrow{OE}=\begin{pmatrix}6\\4\\0\end{pmatrix}$ (FG and CE are vertical poles) Perpendicular height of trapezium $ECFG$ $\left\|\overrightarrow{EG}\right\|=\left\|\begin{pmatrix}2\\0\\0\end{pmatrix}-\begin{pmatrix}6\\4\\0\end{pmatrix}\right\|=\left\|\begin{pmatrix}-4\\-4\\0\end{pmatrix}\right\|=4\sqrt{2}$ (or $\sqrt{32}$) **Method 2:** Use of $\left\|\mathbf{a}\times\frac{\mathbf{b}}{\|\mathbf{b}\|}\right\|$ To find $\overrightarrow{OF}$: Solve equation of l_{AB} and l_{FG} simultaneously: $\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}10\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\0\end{pmatrix}+\mu\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}=\begin{pmatrix}2\\0\\\mu\end{pmatrix}\Rightarrow\lambda=0.2,\mu=3.2$ Sub $\mu=3.2$: $\overrightarrow{OF}=\begin{pmatrix}2\\0\\0\end{pmatrix}+3.2\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\3.2\end{pmatrix}$ Hence shortest distance from C to line FG is

	$\left\lvert \overrightarrow{CF} \times \dfrac{\begin{pmatrix}0\\0\\1\end{pmatrix}}{\left\lvert\begin{pmatrix}0\\0\\1\end{pmatrix}\right\rvert} \right\rvert = \left\lvert \left[\begin{pmatrix}2\\0\\3.2\end{pmatrix} - \begin{pmatrix}6\\4\\2\end{pmatrix}\right] \times \begin{pmatrix}0\\0\\1\end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix}-4\\-4\\1.2\end{pmatrix} \times \begin{pmatrix}0\\0\\1\end{pmatrix} \right\rvert$ $= \sqrt{32}\ \left(\text{or } 4\sqrt{2} \text{ or } 5.66\ (5.65685)\right)$
	Method 1: Area of trapezium formula $EC = 2, FG = 3.2$ Area of $ECFG$ $= \frac{1}{2}(3.2+2)\left(\sqrt{32}\right)$ $= \frac{1}{2}(2+3.2)\left(4\sqrt{2}\right)$ $= (10.4)\sqrt{2}$ $= \frac{52}{5}\sqrt{2}$ sq units **Method 2:** Use of area of triangles Area of triangle ECG $= \frac{1}{2}\left\lvert \overrightarrow{EC} \times \overrightarrow{EG} \right\rvert = \frac{1}{2}\left\lvert \begin{pmatrix}0\\0\\2\end{pmatrix} \times \begin{pmatrix}-4\\-4\\0\end{pmatrix} \right\rvert$ $= \frac{1}{2}\left\lvert \begin{pmatrix}0\\0\\2\end{pmatrix} \times \begin{pmatrix}-4\\-4\\0\end{pmatrix} \right\rvert = \frac{1}{2}\lvert 2 \rvert \lvert -4 \rvert \left\lvert \begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\1\\0\end{pmatrix} \right\rvert$ $= 4\left\lvert \begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\1\\0\end{pmatrix} \right\rvert = 4\sqrt{2}$ Area of triangle CFG $= \frac{1}{2}\left\lvert \overrightarrow{CF} \times \overrightarrow{CG} \right\rvert$ $= \frac{1}{2}\left\lvert \begin{pmatrix}-4\\-4\\1.2\end{pmatrix} \times \begin{pmatrix}-4\\-4\\-2\end{pmatrix} \right\rvert = \frac{1}{2}\lvert -2 \rvert \left\lvert \begin{pmatrix}-4\\-4\\1.2\end{pmatrix} \times \begin{pmatrix}2\\2\\1\end{pmatrix} \right\rvert$

	$= \frac{1}{2}\lvert -2 \rvert \left\lvert \begin{pmatrix} -6.4 \\ -6.4 \\ 0 \end{pmatrix} \right\rvert = \sqrt{2\left(6.4\right)^2} = 6.4\sqrt{2}$ Area of $ECFG$ $= 4\sqrt{2} + 6.4\sqrt{2}$ $= 10.4\sqrt{2}$ $= \frac{52}{5}\sqrt{2}$ sq units