



ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS

Preliminary Examination Paper 1

9758/01

3 hours

Additional Material(s): Printed Answer Booklet

25 Aug 2025

CANDIDATE
NAME

CLASS

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READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

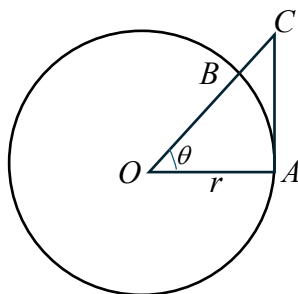
- 1** A cubic curve passes through the points $(2, 3)$ and $(-3, -22)$. Find the equation of the curve if it has a stationary point at $(1, -6)$. [4]

- 2 (a)** Show that $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$. [3]

- (b)** Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{6}} \sin 2\theta \cos 3\theta \, d\theta$ exactly. [4]

- 3 (a)** Given that θ is small, show that $\sec \theta \approx 1 + \frac{1}{2}\theta^2$. [2]

- (b)** The diagram below shows a circle, centre O and radius r , with points A and B on the circumference such that $\angle AOB = \theta$ radians, where θ is small. AC is a tangent to the circle at A and OBC is a straight line.



- (i)** Show that the length of chord AB can be approximated by $r\theta$. [2]
- (ii)** Hence show that the perimeter of triangle ABC can be approximated by $r\theta(a + b\theta)$, where a and b are constants to be determined. [4]
- 4** The Folium of Descartes is a curve, defined by the equation $x^3 + y^3 = 3axy$, where a is a real constant. It is given that $a \neq 0$ for this question.
- (a)** Show that $\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$. [2]
- (b)** The point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ lies on this curve. Show that the equation of the normal at this point on the curve is independent of the value of a . [2]
- (c)** Given that the curve has a stationary point at (pa, qa) , where p and q are positive constants, find the exact values of p and q . You need not determine the nature of this stationary point. [4]

5 The curve C has equation given by $y = \frac{1+3x-18x^2}{9x+3}$.

(a) Without the use of a calculator, find the range of values that y can take. [4]

(b) Sketch the graph of C indicating clearly its asymptotes and stationary points. [3]

(c) State a sequence of transformations that will transform the curve C to the curve with equation $y = \frac{1+3(2x-1)-18(2x-1)^2}{9(2x-1)+3}$. [2]

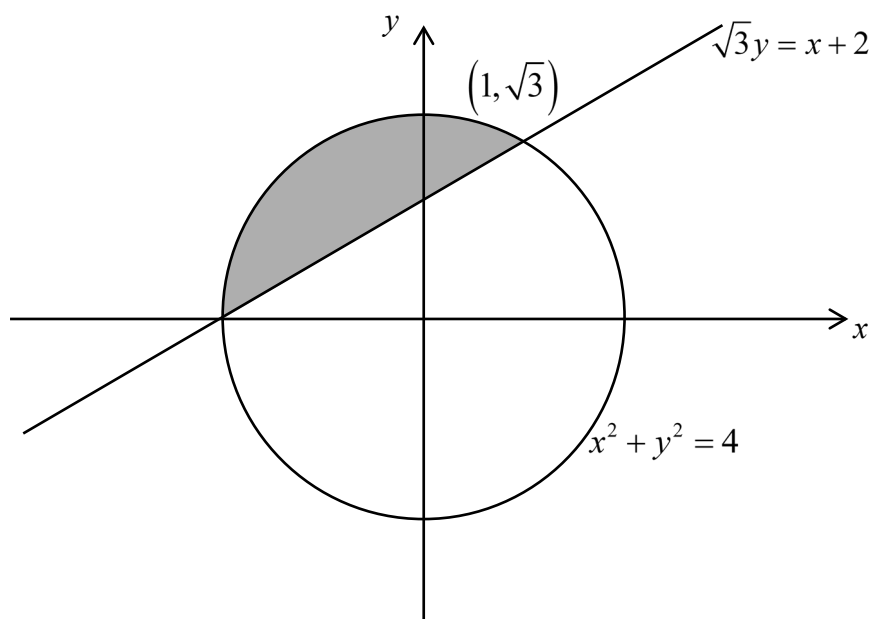
6 It is given that $\sum_{r=1}^n \frac{7r+4}{r(r+1)(r+2)} = \frac{9}{2} - \frac{2}{n+1} - \frac{5}{n+2}$.

(a) Find $\sum_{r=7}^{2n} \frac{7r-3}{r^3-r}$ giving your answers in terms of n . [4]

(b) Show algebraically that $(r+1)^3 > r(r+1)(r+2)$ for all positive integers r . [2]

(c) Hence show that $\sum_{r=1}^n \frac{7r+4}{(r+1)^3} < \frac{9}{2}$. [3]

7 Curve C is a circle with radius 2 and center at the origin with equation $x^2 + y^2 = 4$. Line L has the equation $\sqrt{3}y = x + 2$. The diagram below shows the shaded region A which is enclosed between C and L .



- (a) Use the substitution $x = 2 \sin \theta$ to show that the area of region A can be written in the expression $\int_b^a 4 \cos^2 \theta \, d\theta - c$, where a , b and c are exact constants to be determined. Hence evaluate this area exactly. [6]
- (b) The region B is bounded by the curve C , the line L and the line $x = 2$. Find the volume generated when region B is rotated through 2π radians about the y -axis. Give your answer to two decimal places. [3]
- 8 (a) (i) The ninth, fifth and second terms of an arithmetic progression are successive terms of a geometric progression with first term a and common ratio r , where $r \neq 1$ and $a > 0$. Find the value of r and deduce that the geometric series is convergent. [3]
- (ii) Using the value of r found in (i), find the least value of n such that the sum of all the terms after the n^{th} term of the geometric progression is less than 1% of its sum of the first n terms. [3]
- (b) The sum, S_n , of the first n terms of another sequence is given by $S_n = n \ln 2^{q(n+1)}$, where q is a constant. Prove that the sequence follows an arithmetic progression. [4]

- 9 The line ℓ has the equation $\frac{x-a}{2} = y-2 = \frac{1-2z}{b}$, where a and b are real constants. The plane π has the equation $3x - y + 4z = 10$.

(a) Given that the line ℓ and the plane π do not intersect, show that $a \neq \frac{10}{3}$ and $b = \frac{5}{2}$.

[5]

(b) It is given that $a = 1$ and $b = 3$. Find the point of intersection between the line ℓ and the plane π .

[3]

(c) It is given now that $a = 1$, $b = \frac{5}{2}$.

(i) Find the distance between the line ℓ and the plane π .

[2]

(ii) Determine whether the line ℓ and the origin O lie on the same side of the plane π . State an equation of the other plane that is equidistant from line ℓ and parallel to plane π .

[3]

- 10 The function f is defined by

$$f: x \rightarrow 2x - \frac{1}{2x}, \quad 0 < x < 2.$$

It is given that f^{-1} exists.

(a) Define f^{-1} in a similar form.

[3]

(b) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram.

[2]

(c) The region R is bounded by the curve $y = f^{-1}(x)$, $y = x$ and the y -axis. Find the exact area of R .

[3]

(d) Another function g is defined by

$$g: x \rightarrow \begin{cases} \frac{3}{2} + \frac{3}{3x-11} & \text{for } x \leq 3, \\ \left| x - \frac{1}{3}x^2 \right| & \text{for } 3 < x \leq 5. \end{cases}$$

Show that the composite function gf exists and find the range of gf .

[2]

- 11** In a chemical reaction, compounds X and Y react to form a product. Let x and y denote the concentrations, in mol per kilolitre (mol/kL), of X and Y respectively, at time t minutes after the start of the experiment.

The initial concentrations of X and Y are given by x_0 and y_0 mol/kL.

- (a)** In a particular experiment where Y is present in excess, the reaction rate can be modeled as a pseudo-first-order reaction, leading to the differential equation $\frac{dx}{dt} = -ax$, where a is a positive constant.

(i) Solve this differential equation, expressing x in terms of t , x_0 and a . [3]

(ii) The half-life of the reaction, denoted by $t_{0.5}$, is defined as the time taken for the concentration of X to decrease to half its initial value. Show that $t_{0.5} = \frac{\ln 2}{a}$. [2]

- (b)** In another experiment conducted by a chemist, the rate of the reaction is directly proportional to the product of the concentration of X and the square of the concentration of Y . It is also known that at any instance during the reaction, every 1 mol of X reacts with every 2 mol of Y , giving the equation $y_0 - y = 2(x_0 - x)$.

The initial concentrations of X and Y are 1 mol/kL and 4 mol/kL respectively.

(i) Show that $\frac{dx}{dt} = -bx(x+1)^2$, where b is a positive constant. [2]

(ii) It is given that the concentration of X is 0.5 mol/kL at the instance 1 min after the start of the experiment. Find the concentration of X at the instance 2 min after the start of the experiment, giving your answer to 3 significant figures. [6]

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