Curve Sketching

1. **IJC/II/1**

The curve C has equation $y = \dfrac{x^2+9x+16}{x+2}$.

(i) Find the equations of the asymptotes of C. [2]

(ii) Find, leaving your answers in exact form, the range of values of k for which the equation $k = \dfrac{x^2+9x+16}{x+2}$ has no real roots. [4]

(iii) Sketch C. Show, on your diagram, the equations of the asymptotes, the coordinates of the stationary points, and the coordinates of the points of intersection of C with the x- and y- axes. [3]

2. **IJC/2015/I/7**

The curve C_1 has equation $y = \dfrac{1}{2a-x} + \dfrac{1}{x}$, where a is a positive constant.

(i) By using differentiation, or otherwise, find the coordinates of the stationary point and determine the nature of the stationary point. [4]

(ii) Sketch C_1, indicating clearly the equations of any asymptotes and the coordinates of the stationary point. [2]

Another curve C_2 has equation $\dfrac{(x-a)^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where b is a positive constant.

(iii) By considering the graph of C_2, find an inequality satisfied by b such that C_1 and C_2 intersect at more than one point. [2]

(i) Given that $a = 2$ and $b = 6$, find the range of values of x such that

$$\frac{1}{2a-x} + \frac{1}{x} > b\left[\sqrt{\left(1 - \frac{(x-a)^2}{a^2}\right)}\right].$$ [2]

3. **NYJC/I/7**

The curve C has equation $y = \dfrac{2x^2+4}{3(x+\lambda+1)}$. It is given that C has a vertical asymptote $x = 0$.

(i) Determine the value of λ. [1]

(ii) Find the equation of the other asymptote of C. [2]

(iii) Prove, using an algebraic method, that C cannot lie between $-\dfrac{4\sqrt{a}}{3} < y < \dfrac{4\sqrt{a}}{3}$, $a > 0$, where a is to be determined. [3]

(iv) When $\lambda = 0$, the graph of $y = \dfrac{2x^2+4}{3(x+\lambda+1)} + k$ cuts the x-axis at 2 distinct points where k is a positive integer.

(a) Sketch the graph of $y = \dfrac{2x^2+4}{3(x+\lambda+1)} + k$ when $\lambda = 0$, indicating clearly all asymptotes and coordinates of the axial intercepts. [4]

(b) Deduce the least possible integer of k. [1]

4. **PJC/I/12**

A curve C has the equation $y=\dfrac{ax^2+bx+c}{x+d}$ for some constants a, b, c and d. Given that the asymptotes are $x=5$ and $y=x+7$, find the values of a, b and d. [3]

Given that the curve has a stationary point at $x=-1$, find the value of c. [2]

Draw a sketch of C, indicating clearly the asymptotes, the coordinate(s) of the stationary points and the point(s) of intersection with the axes. [3]

By drawing a sketch of another suitable curve on the same diagram, state, with a reason, the number of roots of the equation

$$(1-\lambda)x^2+(2+5\lambda)x+1=0$$

(i) when $\lambda=1$,
(ii) when $0<\lambda<1$. [3]

5. **AJC/2011/I/8**

The curve C has the equation $y=\dfrac{ax^2}{x+a}$ where a is a negative constant.

(i) Show that the curve C has two stationary points for all negative values of a. [2]

(ii) Sketch the curve C, showing clearly all the asymptotes, axial intercepts and turning points. [3]

(iii) Using the sketch in (ii), find the range of values of k for which $x^4=\left(k-x^2\right)(x-1)^2$ has exactly two roots. [3]

6. **EJC Prelim/2020/02/Q2**

The curve C_1 has equation $y=\dfrac{3x^2-2x+3}{x-1}$. The curve C_2 has equation $(x-1)^2-\dfrac{(y-4)^2}{b^2}=1$, where $b>0$.

(i) Sketch C_1, stating the equations of any asymptotes and the coordinates of any turning points and points where the curve crosses the axes. [4]

(ii) Hence, find the range of values of b such that there is no point of intersection between C_1 and C_2. Using the maximum value of b found, sketch C_2 on the same diagram as **part (i)**. [3]

7. **NJC/2012/I/10a**

The curve C has equation

$$y=\frac{2x^2-a^2x+4a^2}{x^2-a^2},\ x\neq\pm a,$$

where a is a positive constant.

(i) Find the equations of the asymptotes, leaving your answers in terms of a. [2]

(ii) Show that if C has two turning points, then $0<a<6$. [3]

(iii) For $a=4$, sketch the curve C, stating the equations of any asymptotes, the coordinates of the turning points and axial intercept(s) if any. [2]

(iv) Hence, find the range of values of h such that the graph of $h^2\left(x-6+2\sqrt{5}\right)^2+\left(y+1\right)^2=h^2$, where h is a positive integer, intersects C more than once. [2]

8. **MJC/2013/I/Q8**

The curve C_1 has equation $y=3+\dfrac{x}{x^2-2x-8}$.

(i) Express y in the form $3+\dfrac{A}{x+c}+\dfrac{B}{x+d}$ and show by differentiation that C_1 has no stationary points. [3]

(ii) Sketch C_1, stating the axial intercepts and the equations of any asymptotes. [3]

(iii) Find the exact area bounded by the curve C_1, the line $x=2$ and the axes. [3]

(iv) The curve C_2 has equation $\dfrac{\left(x-5\right)^2}{3^2}+\dfrac{y^2}{2^2}=1$. Sketch C_2 on the same diagram as C_1 and find the coordinates of any points of intersection between C_1 and C_2.

9.**VJC/2013/II/2**

The curve C has equation $y=\dfrac{ax^2+bx+c}{x-1}$, where a, b and c are non-zero constants.

(a) It is given that C passes through the point $\left(3,\dfrac{23}{2}\right)$ and has a minimum point at $\left(2,10\right)$.

(i) Find the values of a, b and c. [3]

(ii) Sketch C, giving the coordinates of any turning points, points of intersection with the axes and the equations of any asymptotes. [3]

(iii) Find the set of values of $\dfrac{m}{k}$, where m and k are positive constants, such that the curve with equation $\dfrac{x^2}{k^2}-\dfrac{\left(y-1\right)^2}{m^2}=1$ does not intersect C. [2]

(b) In the case when $a=-1$ and $b=-1$, find the set of values of c, where $c\neq 2$, such that C has no stationary point. [3]

10. **PJC/2013/II/5**

The curve C has equation $y=\dfrac{x^2+ax+b}{c-x}$. The vertical asymptote of C is $x=-2$, and the coordinates of the turning points are $(-4,2)$ and $(0,-6)$.

(i) Find the values of a, b and c. [3]

(ii) Sketch C, stating the equations of the asymptotes. [2]

(iii) By drawing an appropriate graph on the sketch of C, find the range of values of k $(k>0)$ such that the equation $(x+4)^2+\left(\dfrac{x^2+ax+b}{c-x}\right)^2=k^2$ has no real roots. [2]

(iv) When C undergoes a translation in the direction of y-axis by p units, C intersects the line $y=-1$ at one negative value of x. State the set of values for p. [2]

11. **PJC/2014/1/10**

The curve C has equation

$$y=\frac{px+q}{x^2+r},$$

where p, q and r are constants.

It is given that C has an asymptote at $x=2$ and a turning point at $\left(4,\dfrac{1}{4}\right)$.

(i) Find the values of p, q and r. [4]

(ii) Sketch C, stating clearly equations of asymptotes, and the coordinates of turning points and intersections with the axes. [3]

(iii) Hence, solve the inequality $\left|\dfrac{px+q}{x^2+r}\right|\le 1$. [3]

12. **SAJC/2014/I/6**

The equation of a curve C is $y=2x+\dfrac{8k^2}{x-2}$, where $k>0$.

(i) Prove, using an algebraic method, that y cannot lie between two values (to be determined in terms of k). [3]

(ii) Sketch C for the case where k = 1, stating the equations of any asymptotes and the coordinates of any turning points. [3]

(iii) State, with a reason, the range of values of b, where $b>0$, such that the graph of $b^2x^2-4y^2=4b^2$ does not intersect C. [2]

13. **VJC/2018/I/9**

The curve C has equation $y=\dfrac{2x+5}{4-x^2}$.

(i) Sketch C, giving the coordinates of the axial intercepts, turning points and equations of any asymptotes. [4]

(ii) Find the numerical value of the area bounded by C and the line $y=2$. [3]

(iii) Find the set of values of k such that $\left|\dfrac{2x+5}{x^2-4}\right|=k$ has 3 distinct negative real solutions. [3]

14. **SAJC/2019/I/Q2**

(i) On the same axes, sketch the curves with equations $y=\left|2x^2+6x+4\right|$ and $y=3-4x$, indicating any intercepts with the axes and points of intersection. Hence solve the inequality $3-4x<\left|2x^2+6x+4\right|$. **[4]**

(ii) Find the exact area bounded by the graphs of $y=3-4x$, $y=\left|2x^2+6x+4\right|$, $x=-3$ and $x=-1$. **[3]**

15. **ACJC/2021/I/Q4**

The curve C has equation $y=\mathrm{f}(x)$, where $\mathrm{f}(x)=\dfrac{x(x+a)}{x-a}$ and a is a positive real constant.

(i) Find algebraically, in terms of a, the set of values that y can take. [4]

(ii) Sketch C, indicating clearly the equations of any asymptotes and the coordinates of the points where the curve crosses the axes. [3]

(iii) By adding a suitable graph to your same diagram in **(ii)** and labelling it clearly, solve the inequality $\dfrac{x(x+a)}{x-a}<\left|3x\right|$. [3]

16. **ASRJC Prelim 9758/2024/01/Q7**

(a) State a sequence of transformations that will transform the curve with equation $y^2-x^2=1$ on to the curve with equation $9y^2-54y-x^2-2x+79=0$. [4]

(b) A curve C has equation $9y^2-54y-x^2-2x+79=0$.

(i) For real values x, use a non-graphical method to determine that y cannot lie between a and b, where a and b are exact real constants to be determined. [3]

(ii) Sketch the curve C, indicating clearly the equations of all asymptotes and the coordinates of the turning points. [3]

(iii) By adding a suitable curve, determine the number of real roots of the equation,

$$9\left[(x+1)^2+3\right]^2-54\left[(x+1)^2+3\right]-x^2-2x+79=0. \qquad [2]$$

17. **EJC Prelim 9758/2024/01/Q4**

A curve C has equation $y=\frac{x^2+25x+114}{x+1}$.

(a) Sketch C, indicating on the diagram the equations of any asymptotes, and the coordinates of any turning points and any axial intercepts. [4]

A curve D has parametric equations

$$x=23\sin 2\theta-1,\ y=23+23\cos 2\theta \text{ for} -\frac{\pi}{2}<\theta<\frac{\pi}{2}.$$

(b) Find the cartesian equation of curve D. [2]

(c) Hence, on the same diagram as in part **(a)**, sketch the curve D and determine the number of intersection points between curve C and curve D. [3]

18. **ASRJC Prelim 9758/2025/P2/Q4**

The curve C is defined by the parametric equations

$$x=\cos t+\frac{1}{2}\cos 5t,\quad y=\sin t+\frac{1}{2}\sin 5t \quad \text{for } 0\le t\le \frac{\pi}{2}.$$

(a) Find the coordinates of the point on the curve C corresponding to $t=0$. [1]

Another curve D has the following parametric equations.

$$x=2+\sqrt{k}\cos\theta,\ y=\frac{3}{2}\sin\theta \quad \text{for } 0\le\theta\le 2\pi \text{ and } k>0.$$

(b) Find the cartesian equation for curve D. [2]

(c) Sketch the curves C and D on the same diagram. [2]

(d) Hence determine the range of values of k such that the equation

$$\frac{(\cos t+0.5\cos 5t-2)^2}{k}+\frac{4(\sin t+0.5\sin 5t)^2}{9}=1$$

has no real solutions. [1]

19. **EJC Prelim 9758/2025/P2/Q3**

The curve C_1 has equation $y=\frac{x^2-8x+28}{2-x}$.

The curve C_2 has equation $\frac{(x-6)^2}{16}+\frac{y^2}{25}=1$.

(a) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any vertices, turning points, intersections with the x-axis and the equations of any asymptotes. [7]

(b) Find the volume of solid obtained when the smaller region bounded by C_1 and C_2 is rotated through 2π radians about the x-axis. [4]

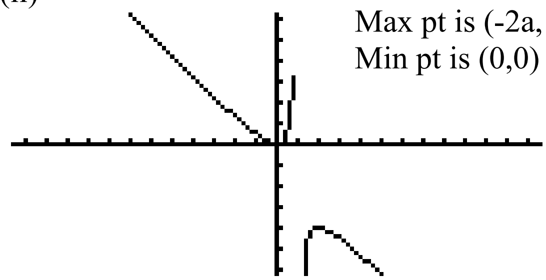
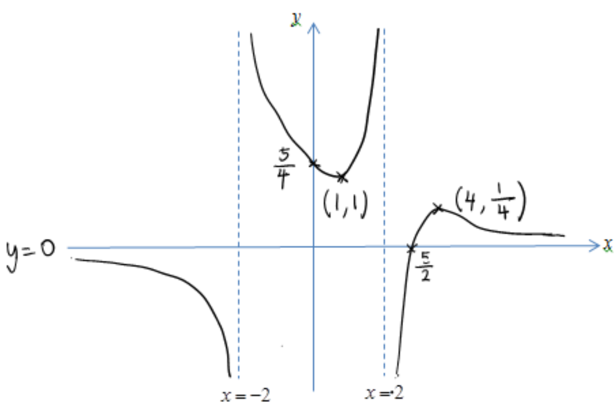
20. **DHS Prelim 9758/2025/P2/Q1**

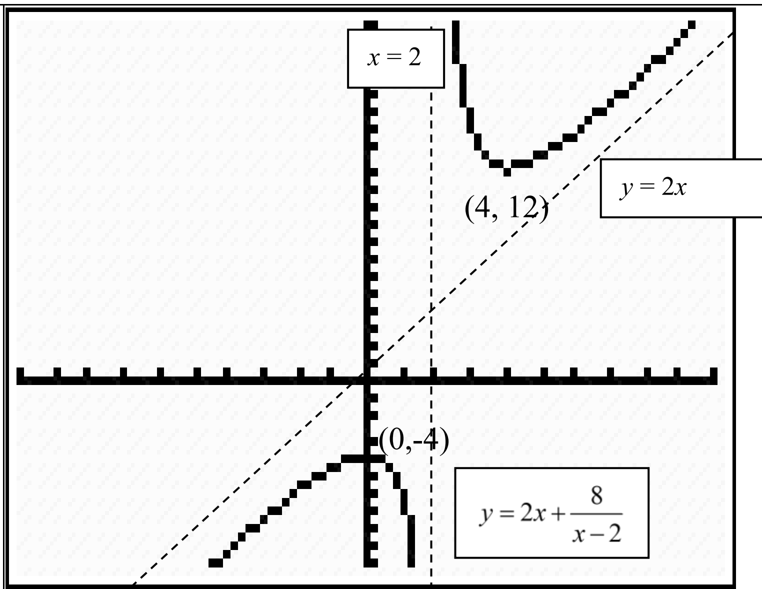
A curve C has equation $y=x+\frac{9}{x+1}$.

(a) Using differentiation, find the range of values of x such that C concaves upwards. [2]

(b) Sketch C. [3]

Answers:

1	(i) $y=x+7$, $x=-2$ (ii) $5-2\sqrt{2}<k<5+2\sqrt{2}$
2	(i) $\left(a\ ,\ \dfrac{2}{a}\right)$ min point (iii) $b>\frac{2}{a}$ (iv) $0<x<0.330$ or $3.67<x<4$
3	(i) -1 (ii) $y=\dfrac{2}{3}x$ (iii) x – intercepts are $-\dfrac{3k}{4}+\sqrt{\dfrac{9k^2}{16}-\dfrac{3k}{2}-2}$ and $-\dfrac{3k}{4}-\sqrt{\dfrac{9k^2}{16}-\dfrac{3k}{2}-2}$ (iv) 4
4	$a=1$, $b=2$, $c=1$, $d=-5$ (i) one real root (ii) two real roots
5	(ii)  Max pt is (-2a, $-4a^2$) Min pt is (0,0) (iii) $0<k<20$
6	(ii) $0<b\le 3$
7	(i) Vertical asymptotes: $x=-a$ or $x=a$, Horizontal asymptote: $y=2$ $h\ge 3$
8	(i) $y=3+\dfrac{x}{x^2-2x-8}=3+\dfrac{2}{3(x-4)}+\dfrac{1}{3(x+2)}$; $6+\dfrac{1}{3}\ln 2-\dfrac{1}{3}\ln 4$ units^2 or $6-\dfrac{1}{3}\ln 2$ units^2
9	(a)(i) $a=3,\ b=-2,\ c=2.$ (iii) $\left\{\frac{m}{k}\in\mathbb{R}:0<\frac{m}{k}\le 3\right\}.$ (b) $\{c\in\mathbb{R}:c\ge 2\}.$
10	(i) $a=6,\ \ b=12,\ \ c=-2$ (iii) $0<k<2$ (iv) Translate C by -3 in the direction of y-axis, hence $p=-3$ Or Translate C by more than 5 units in the direction of y-axis, $p>5$
11	(i) $p=2, q=-5, r=-4$ (ii)  (iii) $x\le -4.16$ or $x=1$ or $x\ge 2.16$
12	(i) y cannot lie between 4 – 8k and 4 + 8k (ii)

	 (iii) $0 < b \le 4$
13	(ii) 1.95 (iii) $\left\{ k \in \mathbb{R} : 0 < k < \dfrac{1}{4} \text{ or } 1 < k < \dfrac{5}{4} \right\}$
14	(i) $x < -4.90$ or $x > -0.102$ (3sf) (ii) 20
15	(i) $y \ge \left(3 + 2\sqrt{2}\right)a$ or $y \le \left(3 - 2\sqrt{2}\right)a$ (iii) $x < 0$ or $0 < x < a$ or $x > 2a$
16	(a) Translation of 1 unit in the negative x direction Translation of 9 units in the positive y direction Scaling parallel to the $y-$axis by a factor of $\dfrac{1}{3}$ (bi) y cannot lie between $\dfrac{8}{3}$ and $\dfrac{10}{3}$ (iii) 2 real roots
17	(b) $\left(x+1\right)^2 + \left(y-23\right)^2 = 23^2$ (c) 4
18	(a) $\left(\frac{3}{2}, 0\right)$ (b) $\frac{(x-2)^2}{k} + \frac{4y^2}{9} = 1$ (d) $0 < k < \frac{1}{4}$
19	(b) 58.7
20	(a) $x > -1$