Practice Paper A
Sec 3 Solutions for Students

Qn	Solutions
1(i)	$\alpha+\beta=\frac{5}{2}$ and $\alpha\beta=\frac{1}{2}$
1(ii)	$\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ $=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$ $=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ $=21$ $\frac{1}{\alpha^2}\times\frac{1}{\beta^2}=4$ Hence equation is $x^2-21x+4=0$
2(a)	$2-4x^2+4x=-4(x^2-x)+2$ $=-4\left(x-\frac{1}{2}\right)^2+3$ Since $\left(x-\frac{1}{2}\right)^2\geq 0$, $-4\left(x-\frac{1}{2}\right)^2+3\leq 3$ so $2-4x^2+4x$ cannot be greater than 3.
2(b)	Need $4p>0$ and $[-4(p+2)]^2-4(4p)(6p+5)<0$ i.e. $p>0$ and $5p^2+p-4>0$ $\Rightarrow (5p-4)(p+1)>0$ $\Rightarrow p<-1 \text{ or } p>\frac{4}{5}$ Hence $p>\frac{4}{5}$

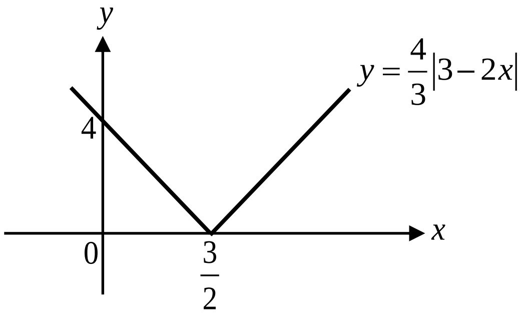
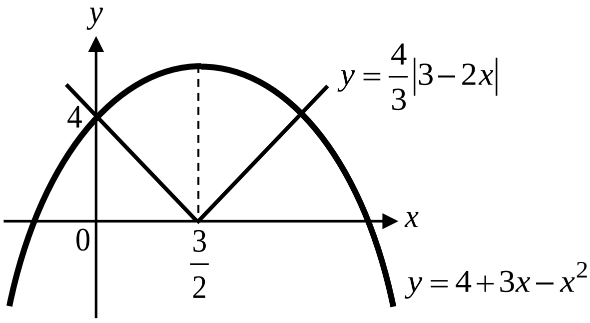
Qn	Solutions
3(a)	$s=\dfrac{4(2p^2+1)}{u^2}$ $\Rightarrow su^2=8p^2+4$ $\Rightarrow p^2=\dfrac{su^2-4}{8}$ $\Rightarrow p=\pm\sqrt{\dfrac{su^2-4}{8}}$
3(b)	$\dfrac{x^3+8y^3}{x^2-4y^2}-\dfrac{16xy^2-4x^3}{(4y-2x)^2}$ $=\dfrac{(x+2y)(x^2-2xy+4y^2)}{(x+2y)(x-2y)}-\dfrac{4x(2y-x)(2y+x)}{4(2y-x)^2}$ $=\dfrac{x^2-2xy+4y^2}{x-2y}-\dfrac{2xy+x^2}{2y-x}$ $=\dfrac{2x^2+4y^2}{x-2y}$
4(a)	$x^2-4x=y^2-4y\Rightarrow(x-y)(x+y)=4(x-y)$ Since $x\neq y$, $x-y\neq 0\Rightarrow x+y=4$ Hence $3\sqrt{x+y}=3\sqrt{4}$ $=6$
4(b)	$2(27^x)=45(3^x)+\dfrac{243}{3^x}\Rightarrow 2(81^x)=45(9^x)+243$ Let $y=9^x$, equation becomes $2y^2=45y+243\Rightarrow 2y^2-45y-243=0$ Solving, $y=27$ ($y=-\dfrac{9}{2}$ is rejected since 9^x cannot be negative) Hence $9^x=27\Rightarrow x=\dfrac{3}{2}$

Qn	**Solutions**
5(i)	Largest possible value of k is 4.
5(ii) (a)	$\mathrm{f}^{-1}(x)=4-(2-x)^2$ or $x(4-x)$, domain of f^{-1} is $x\le 1$
5(ii) (b)	$\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ $\Rightarrow \mathrm{f}^{-1}(x)=x$ $\Rightarrow 4x-x^2=x$ $\Rightarrow x(x-3)=0$ hence $x=0$ since $x=3\notin$ domain of f^{-1}
6(a)	$\log_3 x=\ln 2$ $x=3^{\ln 2}\approx 2.14$ (3 s.f.)
6(b)	$1+\log_{\sqrt{2}}(y-1)=\log_2(22-7y)$ $\Rightarrow \log_2 2+\log_2(y-1)^2=\log_2(22-7y)$ Hence $2(y-1)^2=22-7y\Rightarrow 2y^2+3y-20=0$ Solving, $y=\frac{5}{2}$ or -4 (rej.)

Qn	Solutions
7(i)	$\mathbf{C}=\begin{pmatrix}87\\49\end{pmatrix}$ the numbers represent the cost of producing one table and one chair ($87 and $49 respectively).
7(ii)	$\mathbf{D}=(20 \quad 50)$ $\mathbf{DC}=(4190)$, it represents the total cost ($4190) of producing 20 tables and 50 chairs.
8(i)	[See Graph Paper] Incorrect value of y recorded is 6.81.
8(ii)	[See Graph Paper] The gradient represents a, so $a \approx 2.8$ (accept $2.73 \le a \le 2.87$) The vertical intercept represents b, so $b \approx 1.5$ (accept $1.3 \le b \le 1.7$) $\frac{y^2}{x} \approx 12.7 \Rightarrow y \approx 7.13$ (accept $7.07 \le y \le 7.18$)

Qn	Solutions
9(a)	$2(e^{3x}+2)=e^{x}(7e^{x}+5)\Rightarrow 2y^3-7y^2-5y+4=0$ where $y=e^x$ $y=-1$ is a root so $(y+1)(2y-1)(y-4)=0\Rightarrow e^x=-1$ (rej.) or $\frac{1}{2}$ or 4 Hence $x=-\ln 2$ or $\ln 4$
9(b)	$f(2)=2\Rightarrow 8+8p+q=0$ and $f\left(-\frac{3}{2}\right)=9\Rightarrow -27p+8q=300$ Solving, $p=-4$ and $q=24$ Hence $f(x)=8x^4-4x^3-30x^2+x+24$ By long division, remainder is $6-2x$
10(i)	Midpoint of $AB=M\left(\frac{1}{2},-1\right)$ and gradient of $AB=-2$ Hence equation of l_2: $y-(-1)=\frac{1}{2}\left(x-\frac{1}{2}\right)\Rightarrow 4y+5=2x$ or $y=\frac{1}{2}x-\frac{5}{4}$
10(ii)	Equation of l_1: $y-8=\frac{1}{2}(x-5)\Rightarrow 2y=x+11$ or $y=\frac{1}{2}x+\frac{11}{2}$

Qn	Solutions
10(iii)	Equation of perpendicular to l_1 at $\left(0, \dfrac{11}{2}\right)$: $y=-2x+\dfrac{11}{2}$ Intersecting with l_2: $-2x+\dfrac{11}{2}=\dfrac{1}{2}x-\dfrac{5}{4}$ $\Rightarrow x=2.7$ and $y=0.1$ Hence distance between the two lines $=\sqrt{2.7^2+(5.5-0.1)^2}$ units $=2.7\sqrt{5}$ or 6.04 (3 sf.) units **<u>OR</u>** Equation of AB: $y-4=-2\left(x-(-2)\right)\Rightarrow y=-2x$ Intersecting with l_1: $\dfrac{1}{2}x+\dfrac{11}{2}=-2x$ $\Rightarrow x=-2.2$ and $y=4.4$ Hence distance between the two lines $=\sqrt{(0.5+2.2)^2+(4.4+1)^2}$ units $=2.7\sqrt{5}$ or 6.04 (3 sf.) units
11(i)	Angle of elevation $=\tan^{-1}\dfrac{25}{80}$ $=17.4°$ (1 d.p.)
11(ii)	$\measuredangle DMP$ $=50°-(200°-180°)$ $=30°$ Distance he walks $=80\cos 30°$ m $=40\sqrt{3}$ or 69.3 (3 s.f.) m

Qn	Solutions
11(iii)	By Sine Rule, $\dfrac{DP}{\sin 30°} = \dfrac{80\text{ m}}{\sin(180° - 30° - 110°)}$ $\Rightarrow DP = \dfrac{80}{2\sin 40°}$ m $\Rightarrow DP = 62.2$ (3 s.f.) m
12	 $y = 4 + 3x - x^2 \Rightarrow y = -(x+1)(x-4)$  $12 + 9x - 3x^2 > 4\lvert 3-2x \rvert \Rightarrow 4 + 3x - x^2 > \dfrac{4}{3}\lvert 3-2x \rvert$ By symmetry, $0 < x < 3$.
13	$\sec x + \tan^2 x = 0 \Rightarrow \sec^2 x + \sec x - 1 = 0$ Hence $\sec x = \dfrac{-1 \pm \sqrt{5}}{2} \Rightarrow \cos x = \dfrac{2}{-1 \pm \sqrt{5}}$ But $-1 \le \cos x \le 1$ hence $\cos x = -\dfrac{2}{1+\sqrt{5}}$
13(i)	$\dfrac{\operatorname{cosec} x - \sec x}{\operatorname{cosec} x + 1} + \dfrac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x - 1}$ $= \dfrac{2\operatorname{cosec}^2 x + 2\sec x}{\operatorname{cosec}^2 x - 1}$

	$=\dfrac{2\operatorname{cosec}^2 x+2\sec x}{\cot^2 x}$ $=2\operatorname{cosec}^2 x\tan^2 x+2\sec x\tan^2 x$ $=2\sec^2 x+2\sec x\tan^2 x$
13(ii)	$\dfrac{\operatorname{cosec} x-\sec x}{1+\operatorname{cosec} x}=\dfrac{\operatorname{cosec} x+\sec x}{1-\operatorname{cosec} x}$ $\Rightarrow \dfrac{\operatorname{cosec} x-\sec x}{\operatorname{cosec} x+1}+\dfrac{\operatorname{cosec} x+\sec x}{\operatorname{cosec} x-1}=0$ Hence $2\sec^2 x+2\sec x\tan^2 x=0$ $\Rightarrow \sec x(\sec x+\tan^2 x)=0$ Since $\sec x\neq 0$, $\sec x+\tan^2 x=0\Rightarrow \cos x=-\dfrac{2}{1+\sqrt{5}}$ Hence $x=180°-51.8°$ (1 d.p.) or $180°+51.8°$ (1 d.p.) $=128.2°$ (1 d.p.) or $231.8°$ (1 d.p.)

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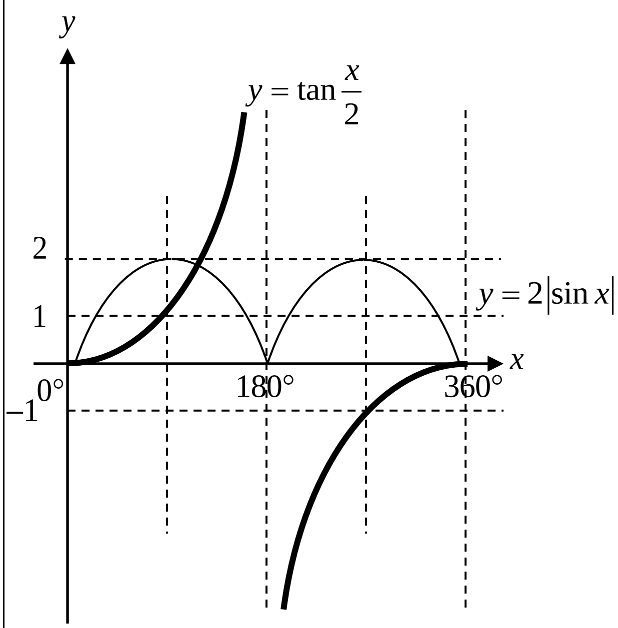
14	 $\tan\frac{x}{2} = 2\|\sin x\|$ By including the graph of $y = 2\|\sin x\|$ it can be seen that there are three solutions between 0° and 360° inclusive.