華僑中學
Hwa Chong Institution
Secondary 3 Common Test 2021

CANDIDATE NAME: Wan Run Hao

CLASS: 3 I 1

REGISTER NUMBER: 2 7

Integrated Mathematics **1 h 30 min**

Additional materials:

Writing papers

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work that you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 60.

This document consists of **3** printed pages.

1 Find the values of a and b in the identity $4x^3 - 5x + 3 \equiv (x-2)(4x^2 + ax + 11) + b$. [4]

2 The cubic function has the equation $f(x) = 2x^3 + 9x^2 + 7x - 6$.

(i) Evaluate $f(-2)$. Hence, write down a linear factor of $f(x)$. [2]

(ii) Using long division, find the other quadratic factor of $f(x)$. [2]

(iii) Hence, solve the equation $2x^3 + 9x^2 + 7x - 6 = 0$. [3]

3 h is a function such that $h(x) = \dfrac{3}{x-2}$, $x < 2$.

(i) Find an expression for $h^{-1}(x)$ and state its domain. [3]

(ii) Hence, determine the value of x such that $h^{-1}(x) = h(x)$. [2]

4 When a polynomial $f(x)$ is divided by $(x-3)$ and $(x+2)$, the remainders are 4 and -11 respectively. When $f(x)$ is divided by $(x^2 - x - 6)$, the remainder is a polynomial $R(x)$.

(i) Explain why $R(x)$ is of the form $ax + b$, where a and b are constants. [1]

(ii) Hence, find the polynomial $R(x)$. [4]

5 **(a)** Express $x^5 = e^{9x+6}$ in the form $\ln x = ax + b$, where a and b are constants. [3]

(b) Using a suitable substitution or otherwise, solve $e^{2x} - 2e^{-2x} = 1$. [4]

(c) Solve $\log_6(1-2x) - 2\log_6 x = 1 - \log_6(2-5x)$. [5]

6 **On the same axes,** sketch the graph of $y = e^x + 1$ and its inverse function. Label the straight line $y = x$, the intercepts and asymptotes clearly. [4]

7 The graph of a cubic polynomial crosses the x-axis at the origin, 2 and -1.

(i) How many such cubic polynomials satisfy the above conditions? Justify your answer. [2]

(ii) Given also that the graph passes through point (1, 4), find the expression of the cubic polynomial, leaving it in the general form

$f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. [3]

x -1 = y

(x-1)(x+1)

y=f

8 Function f is defined as $f : x \to x^2 - 1,\ -2 \le x \le 3$.

(i) Find the range of f. [2]

(ii) Sketch the graph $y = |f(x)|$, labelling clearly the intercepts, the turning point and the end points. [3]

(iii) Does f^{-1} exist? Justify your answer. [2]

(iv) Another function $g : x \to x^2 - 1,\ k \le x \le 3$ has an inverse function.

(a) Write down the smallest possible value of *k*. [1]

(b) Assuming the value of *k* in **(a)**, find the value of $g^{-1}(2)$. [2]

9 **Round off all your answers to the nearest whole number.**

Mathematical modelling can be used to analyse data and predict trends.

Using the logistic growth model, $l(t)$ is the **cumulative** number of confirmed COVID-19 cases in Singapore, where *t* is the number of days after 15 February 2020.

The equation, $l(t) = \dfrac{30766.634}{1 + 6764e^{-0.119t}}$, is obtained based on the data collected from 15 February 2020 ($t = 0$) to 20 May 2020 ($t = 95$).

(i) Find the cumulative number of confirmed cases on 15 February 2020. [2]

Singapore imposed Circuit Breaker (stay-at-home order) on 7 April 2020 ($t = 52$).

(ii) Show that the cumulative number of confirmed cases is 2066 on 7 April 2020. [1]

(iii) Hence, find the number of confirmed cases on 7 April 2020 itself. [2]

(iv) According to this model, calculate the cumulative number of confirmed cases in the long run, i.e. as time tends to infinity. Comment on the number of the **daily** confirmed cases in the long run. [3]

End of Paper

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Name: Wan Run Hua　　Subject: Math　　Date: 8th July

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C.T Group/Class: 3I1　　Serial No: 27

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1. $4x^3-5x+3 \equiv (x-2)(4x^2+ax+11)+b$

By remainder theorem, let $x=2$

$\therefore 4(2)^3-5(2)+3=b$

$b=25$

$4x^3-5x+3=(x-2)(4x^2+ax+11)+25$

$= 4x^3+ax^2+11x-8x^2-2ax-22+25$

$=(4)x^3+(a-8)x^2+(11-2a)x+3$

Comparing coefficients,

$a-8=0$

$a=8$

$\therefore a=8,\ b=25$ 　　4

2. (i) $f(-2)=2(-2)^3+9(-2)^2+7(-2)-6$

$=0$

Since $f(-2)=0$, by factor theorem, $(x+2)$ is a linear factor of $f(x)$. 　　2

(ii) Using long division,

$$\begin{array}{r} 2x^2+5x-3 \\ x+2\ \overline{)\ 2x^3+9x^2+7x-6} \\ -)\ 2x^3+4x^2 \\ \hline 5x^2+7x \\ -)\ 5x^2+10x \\ \hline -3x-6 \\ -)\ -3x-6 \\ \hline 0 \end{array}$$

Quadratic factor of $f(x)$ is $2x^2+5x-3$ 　　2

(iii) When $f(x)=0$,

$(x+2)(2x^2+5x-3)=0$

$(x+2)(2x-1)(x+3)=0$

$x=\frac{1}{2}$ or -2 or -3 　　3

3. (i) $h(x) = \frac{3}{x-2}, \ x<2$

Let $h(x) = y$

$\therefore y = \frac{3}{x-2}, \ x<2$

$xy - 2y = 3$

$xy = 3 + 2y$

$x = \frac{2y+3}{y}$

$h^{-1}(y) = \frac{2y+3}{y}, \ y \neq 0$

$\therefore h^{-1}(x) = \frac{2x+3}{x}, \ x \neq 0$

Domain of $h^{-1}(x)$: $x \neq 0$

2

(ii) When $h^{-1}(x) = h(x)$, it also equals to x.

$\therefore h(x) = x$

$\frac{3}{x-2} = x, \ x<2$

$3 = x^2 - 2x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ (rejected as $x<2$) or -1.

$\therefore x = -1$

2

4. $f(x) = Q(x)(x-3) + 4$

$= Q'(x)(x+2) - 11$

$= Q''(x)(x^2 - x - 6) + R(x)$

(i) The remainder must be at least 1 degree lower than the divisor so why degree 1 (linear) and not degree 0 (constant)?

1*

(ii) Let $R(x)$ be $(ax+b)$

Since $x-3$ results in 4

and $x-2$ results in -11

$a = \frac{4-(-11)}{3-2}$

$= 15$

$b = 4 - 45$

$= -41$

What concept / logic? is this how you were taught on solving simultaneous equations?

0

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Name Wan Run Hao Subject Math Date 8th July

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$\therefore ax+b = 15x-41$

$R(x) = 15x-41$

5. (a) $x^5 = e^{9x+6}$

$\ln x^5 = 9x+6$

$5\ln x = 9x+6$

$\ln x = \frac{9x+6}{5}$

$\ln x = \frac{9}{5}x + \frac{6}{5}$

3

(b) $e^{2x} - 2e^{-2x} = 1$

$e^{2x} - \frac{1}{2e^{2x}} = 1$ Why '2' becomes '$\frac{1}{2}$'?

Let $e^{2x} = y$, $y>0$

$\therefore y - \frac{1}{2y} = 1$

$2y^2 - 1 = 2y$

$2y^2 - 2y - 1 = 0$

$y = \frac{2+\sqrt{12}}{4}$ or $\frac{2-\sqrt{12}}{4}$ (reject as $y>0$)

$e^{2x} = \frac{2+\sqrt{12}}{4}$

2

$\ln\left(\frac{2+\sqrt{12}}{4}\right) = 2x$

$x = \frac{1}{2}\ln\left(\frac{2+\sqrt{12}}{4}\right)$

$= 0.156$ (3sf.)

(c) $\log_6(1-2x) - 2\log_6 x = 1 - \log_6(2-5x)$, $x < \frac{1}{2}$

$\log_6(1-2x) - \log_6 x^2 + \log_6(2-5x) = 1$

$\log_6\left(\frac{(1-2x)(2-5x)}{x^2}\right) = 1$

$\frac{(2-5x-4x+10x^2)}{x^2} = 6^1$

$10x^2 - 9x + 2 = 6x^2$

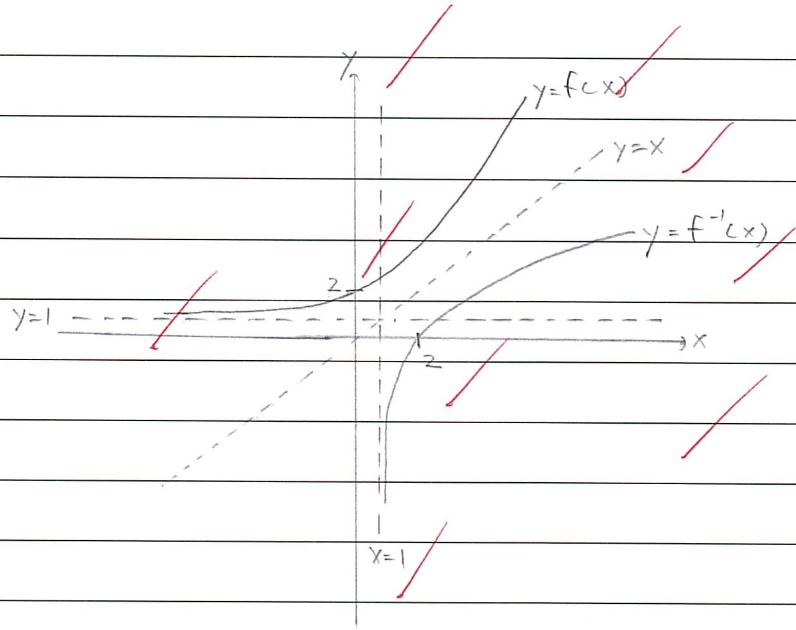
$4x^2 - 9x + 2 = 0$

5

$(4x-1)(x-2) = 0$

$x = 2$ (reject as $x < \frac{1}{2}$) or $\frac{1}{4}$

6.



Let $f(x) = e^{x+1}$

Let $y = f(x)$

7. (i) Cubic polynomial : $k[x(x-2)(x+1)]$ —— ①

$\therefore$ Infinite polynomials satisfy the conditions. as k can be any real number.

4

2

(ii) Let ① $= y$

$\therefore y = kx(x-2)(x+1)$ —— ②

Sub $(1, 4)$ into ②

$\therefore 4 = k \cdot (-1)(2)$

$= -2k$

$k = -2$

$\therefore y = -2x(x-2)(x+1)$

$= -2x(x^2 - x - 2)$

$= -2x^3 + 2x^2 + 4x + \cancel{0}$

$f(x) = -2x^3 + 2x^2 + 4x + \cancel{0}$

3

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Name ..Wan Run Hao.. Subject ..Math.. Date ..8th July..

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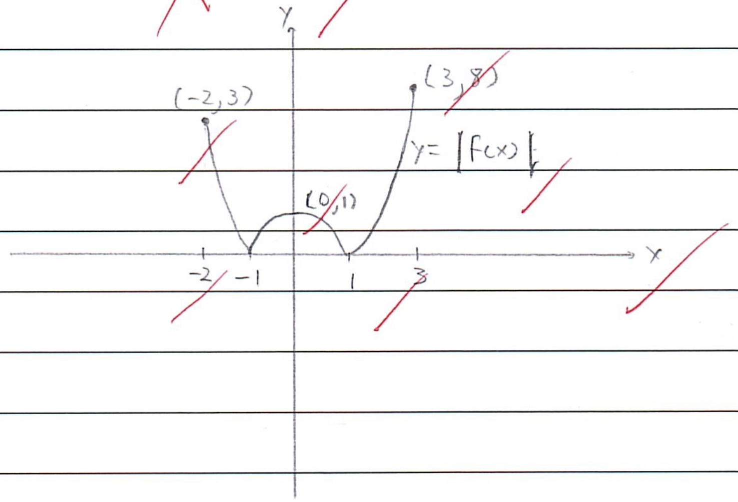
8. (i) Minimum point of f(x) : $(0,-1)$

$\therefore$ Range of f: $f(x) \geq -1$ 0

(ii)



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(iii) f^{-1} does not exist as f(x) is not a one to one function 2

(iv) (a) For g(x) to be a function, k must be larger or equal to the turning point.

Turning point of g(x) : $(0,-1)$

$k \geq 0$

$\therefore k = 0$ 1

(b) $g(x) = x^2 - 1$, $0 \leq x \leq 3$

let $g(x) = y$

$\therefore y = x^2 - 1$

$x^2 = y + 1$

$x = \sqrt{y+1}$, $y > -1$ ($-\sqrt{y+1}$ is rejected as the function is below y-axis)

$\therefore g^{-1}(y) = \sqrt{y+1}$, $y > -1$

$\therefore g^{-1}(x) = \sqrt{x+1}$, $x > -1$

$g^{-1}(2) = \sqrt{2+1}$

$= \sqrt{3}$

$= 1.73$ (3s.f.) 2

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9\. (i) On 15th Feb 2020, $t = 0$.

$$\therefore I(0) = \frac{30766.634}{1+6764e^{-0.119\times 0}}$$

$= 5$ (nearest whole number)

The cumulative number of cases on 15th Feb is 5. 2

(ii) Let $t = 52$

$$\therefore I(52) = \frac{30766.634}{1+6764e^{-0.119\times 52}}$$

$= 2066$ (nearest whole number) (shown) Statement? 1

(iii) Let 6th April be $t = 51$

$$\therefore I(51) = \frac{30766.634}{1+6764e^{-0.119\times 51}}$$

$= 1848$ (nearest whole number)

Number of cases on 7th April $= I(52) - I(51)$

$= 2066 - 1848$

$= 218$

$\therefore$ 7th April 2020 has 218 confirmed cases. 2

(iv) As $t \to \infty$, $e^{-0.119t}$ will move nearer to 0.

Thus, $1+6764e^{-0.119t}$ will close upon 1 as $t \to \infty$.

The cumulative number of cases in the long run will be 30767.

The number of daily cases will 0, when rounded to the nearest whole number. 3

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