TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2022
Secondary 4

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **Wednesday 24 Aug 2022**

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1).......(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Answer **all** questions.

1 Solve the simultaneous equations [4]

$$3x + 4y = 54,$$
$$\frac{4}{x} + \frac{3}{y} - 1 = 0.$$

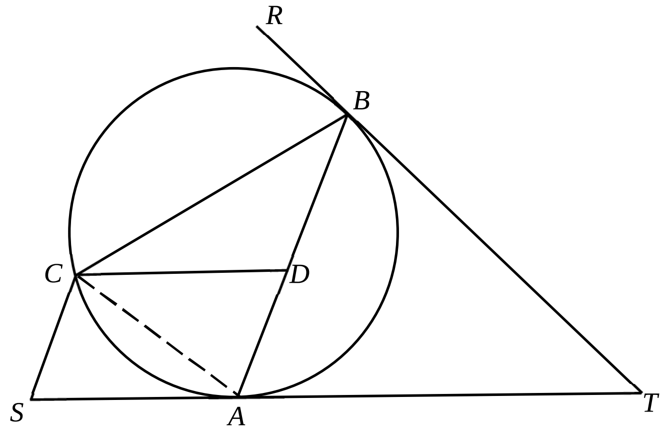
2(a) Express $-\sqrt{3}\sin\theta + 3\cos\theta$ in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [3]

(b) Find the minimum value of $y = \dfrac{1}{-\sqrt{3}\sin\theta + 3\cos\theta - 1}$ and the corresponding value of θ when it occurs. [3]

3 **(a)** Solve the equation $e^{2x}(e^{2x}-3)=28$. [3]

(b) Solve the equation $\log_4 x-\log_{16} x=(\log_4 x)(\log_{16} x)$. [5]

4



The diagram shows a circle passing through the points A, B and C. The straight lines SAT and RBT are tangents to the circle at A and B respectively. AB and BC are chords of the circle. $ADCS$ is a parallelogram.

(a) Prove that triangle ADC is similar to triangle ACB. [2]

(b) Prove that $AC^2 = CS \times AB$. [2]

(c) Explain why $\angle CBA + \angle CAD + \angle ABT = 180°$. [2]

5 **(a)** The equation of a curve is $y = 6 + 2x - 4x^2$. Find the range of values of x for $y > 0$. [3]

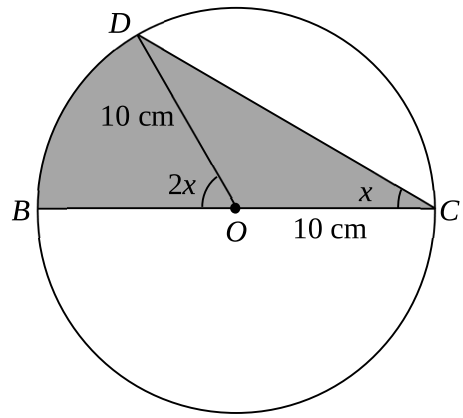
(b) Given that $(m-8)x^2 + 6x + m$ is never negative for all values of x, find the smallest integer value of m. [5]

6 **(a)** Write down, in descending powers of x, the first three terms in the expansion of $\left(x^2+\dfrac{a}{x}\right)^9$, where a is an integer. [2]

(b) The coefficient of x^3 in the expansion of $\left(x^2+\dfrac{a}{x}\right)^9$ is -126, find the value of a. [3]

(c) Hence, find the term independent of x in the expansion of $\left(1-\dfrac{2}{x^3}\right)\left(x^2+\dfrac{a}{x}\right)^9$. [3]

7 [The area of a sector is $\frac{1}{2}r^2\theta$, where θ is the angle, in radians, subtended by the arc at the centre of the circle of radius r.]



In the diagram, O is the centre of the circle with diameter BC and radius 10 cm.
A is the shaded area enclosed by the chord CD, the diameter BC and the minor arc BD.
Angle $BCD = x$ radians.

(a) Show that $A = 100x + 50\sin 2x$. [3]

(b) If x is increasing at a rate of 0.2 radian/second, find the rate of change of A when $x = \frac{\pi}{6}$. [3]

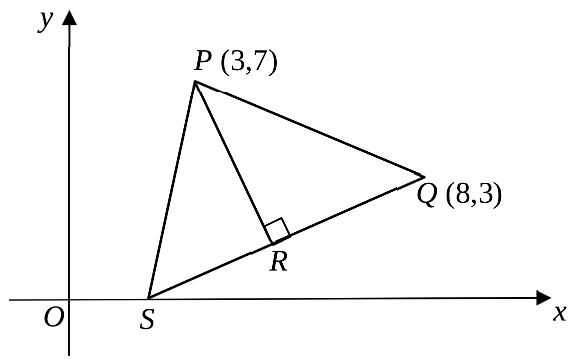
(c) Find the value of x when the rate of change of A is 50 times the rate of increase of x. [3]

8 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(x^2+1\right)e^{2x-1}\right]=2e^{2x-1}\left(x^2+x+1\right)$. [3]

(b) Hence, show that $\int -2e^{2x-1}\left(x^2+x\right)\,\mathrm{d}x=-x^2e^{2x-1}+c$. [4]

(c) Find the value of $\int_{\frac{1}{2}}^{1} -2e^{2x-1}\left(x^2+x\right)\,\mathrm{d}x$ and explain what the result possibly implies about the curve. [2]

9 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a triangle *PQR* with vertices *P*(3, 7), *Q*(8, 3) and *S*, which lies on the *x*-axis. The equation of line *QS* is $2y = x - 2$. *R* is a point on *QS* such that *PR* is perpendicular to *QS*.

(a) Find the equation of *PR*. [3]

(b) Find the coordinates of *S* and *R*. [4]

It is given that the point $T\ (14, 6)$ is such that S, Q and T are collinear points.

(c) What is the significance of point Q in relation to the line ST? Explain why. [3]

V is another point on the y-axis such that VP is parallel to the x-axis.

(d) Find the area of enclosed by $OVPS$. [3]

10 The function of a graph, f, is defined for $x > a$ and is such that $\mathrm{f}''(x) = \dfrac{8}{(4x+1)^2}$.

The graph $y = \mathrm{f}(x)$ passes through the origin and at $x = 0$, the gradient of the graph is -2.

(a) Find an expression for $\mathrm{f}'(x)$. [3]

(b) Find an expression for $\mathrm{f}(x)$. [3]

(c) Hence, explain why $a = -\frac{1}{4}$. [2]

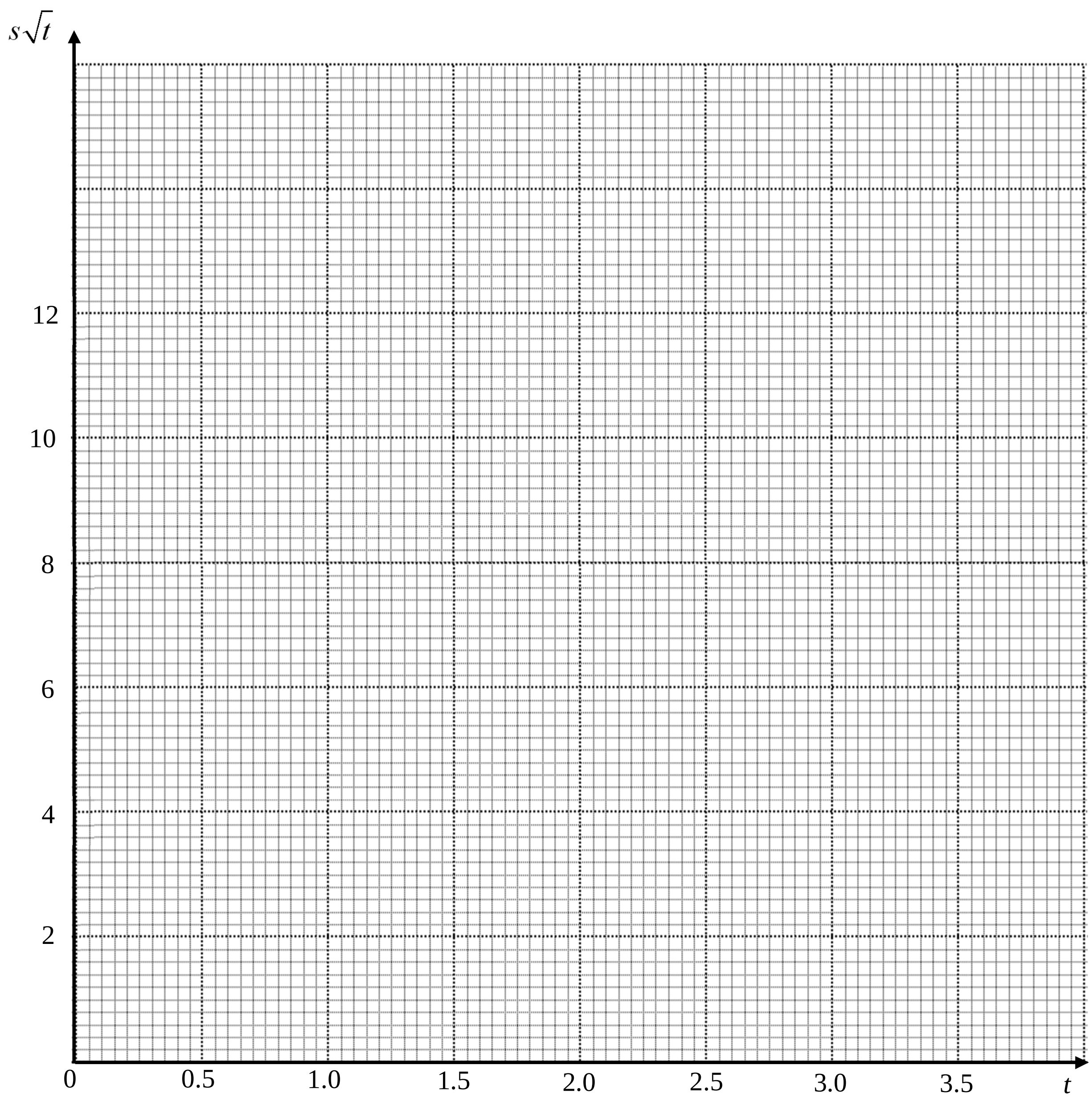
(d) Sketch the graph of $y = \mathrm{f}(x)$. [2]

11 An experiment is conducted to find the distance, s metres, travelled by an object over time, t seconds. The table below shows the corresponding values of t and s.

t	1.0	1.5	2.0	2.5	3.0	3.5
s	4.20	4.65	5.09	5.50	5.89	6.25

It is known that s and t are related by the equation $s = a\sqrt{t} + \dfrac{b}{\sqrt{t}}$, where a and b are constants.

(a) On the grid below, plot $s\sqrt{t}$ against t and draw a straight line. [2]



(b) Use your graph to estimate *a* and *b*. [4]

(c) By drawing a suitable straight line on your graph, find the value of *t* that satisfies the equation $2s\sqrt{t} + 2t - 10 = at + b$. [3]

End of Paper

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Answer Keys

1 $x=12, y=4\frac{1}{2}$ and $x=6, y=9$

2(a) $-\sqrt{3}\sin\theta+3\cos\theta=4\sqrt{3}\cos(\theta+\frac{\pi}{6})$

2(b) Minimum value of $y=-1$ and θ is $\frac{\pi}{3}$.

3(a) $x=\frac{1}{2}\ln 7 \approx 0.973$

3(b) $x=1$ or 4

4(a) $\angle ACD=\angle CAS$ (alternate angles, SA parallel to CD)

$\angle ABC=\angle CAS$ (tangent chord theorem)

$\therefore \angle ACD=\angle ABC$

$\angle CAB$ is common.

$\therefore$ Triangle ADC is similar to Triangle ACB. (AAA)

4(b) $\frac{AC}{AB}=\frac{AD}{AC}$

$AC^2=AB\times AD$

$=CS\times AB$ ($AD=SC$ as $SADC$ is a parallelogram)

4(c) Method 1:

$\angle BCA=\angle ABT$ (tangent chord theorem)

$\angle CBA+\angle CAD+\angle BCA=180°$ (Angles sum of triangle)

$\therefore \angle CBA+\angle CAD+\angle ABT=180°$

Method 2:

$\angle CAS=\angle CBA$ (tangent chord theorem)

$\angle ABT=\angle DAT$ (tangents from external point)

$\angle CAS+\angle CAD+\angle DAT=180°$ (adjacent angles on a straight line)

$\therefore \angle CBA+\angle CAD+\angle ABT=180°$

5(a) $-1<x<1\frac{1}{2}$

5(b) The smallest integer value of $m=9$.

6(a) $x^{18}+9ax^{15}+36a^2x^{12}+\ldots$

6(b) $a=-1$

6(c) 336

7(a) $\angle BOD=2\angle BCD$ ($\angle$ at the centre $=2$ $\angle$ at circumference)

$=2x$

Area of sector BOD

$=\frac{1}{2}(10)^2(2x)$

$=100x \text{ cm}^2$

$$\text{Area of } \Delta COD = \frac{1}{2}(10)(10)\sin\angle COD$$
$$= 50\sin(\pi - \angle BOD)$$
$$= 50\sin(\pi - 2x)$$
$$= 50\sin 2x \text{ cm}^2$$
$$A = \text{Area of sector } BOD + \text{Area of } \Delta COD$$
$$= 100x + 50\sin 2x$$

7(b) $30 \text{ cm}^2 \text{ s}^{-1}$

7(c) $x = \dfrac{\pi}{3}$

8(a) $2e^{2x-1}\left(x^2 + x + 1\right)$

8(b) $-x^2e^{2x-1} + c$

8(c) $1 - e$. The area between the graph and the x-axis in the interval $\frac{1}{2} < x < 1$ lies below x-axis.

9(a) $y = -2x + 13$

9(b) $R = \left(5\frac{3}{5}, 1\frac{4}{5}\right)$

9(c) Q is the midpoint of S and T.

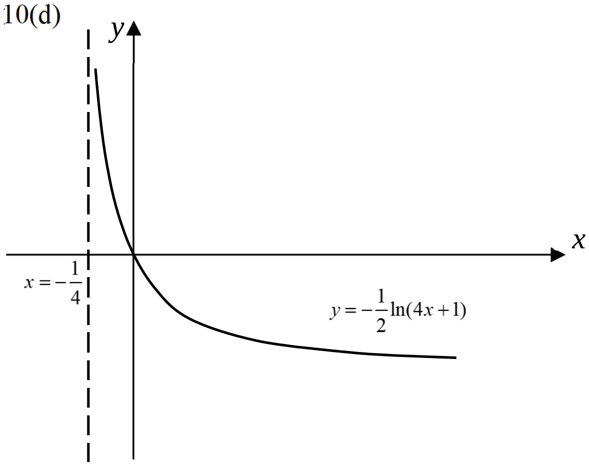
9(d) $17\frac{1}{2}$ sq units

10(a) $f'(x) = \dfrac{-2}{4x+1}$

10(b) $f(x) = -\dfrac{1}{2}\ln(4x+1)$

10(c) $a = -\dfrac{1}{4}$

10(d)



11(a)

t	1.0	1.5	2.0	2.5	3.0	3.5
$s\sqrt{t}$	4.20	5.70	7.20	8.70	10.20	11.69

11(b) a= 2.99714 (actual) [acceptable range: 2.80 to 3.19]
b = 1.204 (actual) [acceptable range: 1.10 to 1.30]

11(c) Draw $s\sqrt{t} = -2t + 10$.
t=1.75 [acceptable range: 1.70 to 1.80]