HOUGANG SECONDARY SCHOOL
PRELIMINARY EXAMINATION / 2022
SECONDARY FOUR (EXPRESS)

CANDIDATE NAME:		**CLASS:**	

CENTRE NUMBER:	**S**					**INDEX NUMBER:**					

ADDITIONAL MATHEMATICS 4049 / 02

Paper 2

Tuesday 23 August 2022
2 hours 15 minutes

Candidates answer on the Question Paper

Instructions to students:

- Write your name, centre number, index number and class in the spaces at the top of this page.
- Write in dark blue or black pen on spaces provided.
- You may use a HB pencil for any diagrams or graphs.
- Do not use staples, paper clips, glue or correction fluid.
- Answer **all** the questions in this paper.
- Give non-exact numerical answers correct to 3 significant figures, or one decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
- The use of an approved scientific calculator is expected, where appropriate.
- You are reminded of the need for clear presentation in your answers.

Information for pupils

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total mark for this paper is 90.

Calculator Model: ________________________

/ 90

This question paper consists of 17 printed pages (including this cover page).

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + ... + \binom{n}{r} a^{n-r} b^r + ... + b^n$$

where n is appositive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)..........(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Sketch the graph of $y = e^x + 2$. [2]

(b) Solve the equation $3 - \mathrm{e}^{-x} = 2\mathrm{e}^x$. [4]

2 The cubic polynomial f(x) is such that the coefficient of x^3 is -1 and the roots of f(x) = 0 are 1, k and k^2. It is given that f(x) has a remainder of -7 when divided by $x - 2$.

(i) Show that $k^3 - 2k^2 - 2k - 3 = 0$. [3]

(ii) Hence find a value for k and explain that there are no other real values of k which satisfy this equation. [6]

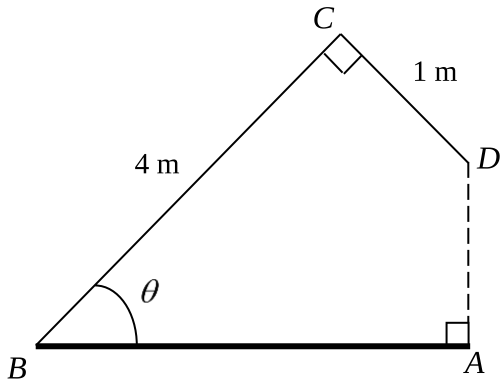
3 **(i)** Given that $y = (x+2)\sqrt{x-1}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{kx}{2\sqrt{x-1}}$ where k is constant. [4]

Hence

(ii) find the rate of change of x when $x = 2$, given that y is changing at a constant rate of 2 units per second, [2]

(iii) evaluate $\displaystyle\int_2^5 \frac{x}{\sqrt{x-1}}\,\mathrm{d}x$. [3]

4



The diagram above shows the side view of a bus stop shelter *BCD* such that *BC* = 4 m, *CD* = 1 m, angle $BCD = 90°$ and angle $CBA = \theta$. *AB* is a concrete pavement under the shelter such that *DA* is perpendicular to *AB*.

(i) Show that $AB = 4\cos\theta + \sin\theta$. [2]

(ii) Express *AB* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^o < \alpha < 90^o$. [3]

(iii) State the maximum value of AB and find the corresponding value of θ when AB is maximum. [2]

(iv) Find the value of θ when $AB = 3$ m. [2]

5 **(a)** A curve has the equation $y = 2x^2 - 6x + c$, where c is a constant.
Find the value of c for which the line $y + 2x = 8$ is a tangent to the curve. [3]

(b) Represent the solution set of $3(x^2 - 5) > x - 1$ on the number line. [3]

(c) Find the greatest value of integer p for which $-2x^2 + x - p$ has real roots for all real values of x. [3]

6 **(i)** Expand and simplify $\left(\frac{1}{2}-2x\right)^5$ in ascending powers of x, up to the first 4 terms. [2]

(ii) Hence find the value of a if the coefficient of x^2 in the expansion of

$\left(1+ax+3x^2\right)\left(\frac{1}{2}-2x\right)^5$ is $\frac{13}{2}$. [4]

(iii) Using the answer from part **(i)**, evaluate $\left(0.47\right)^5$ correct to 5 decimal places. [3]

7 The table shows experimental values of two variables, x and y.

x	0.5	1.0	1.5	2.0
y	15.9	19.1	23.4	30.2

It is known that x and y are related by the equation $y = 10 + Ab^x$.

(i) On **Pg 11**, draw the graph of $\lg(y-10)$ against x. [2]

(ii) Use your graph in **(i)** to estimate the value of A and of b. [4]

(ii) By drawing a suitable line on your graph, solve the equation $Ab^x = 10^{2x}$. [3]

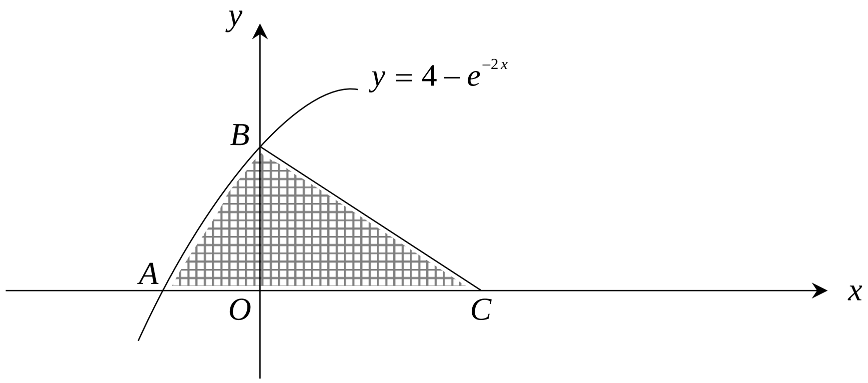
8 A particle P moves in a straight line so that, t seconds after passing through a fixed point O, its velocity v m/s, is given by $v = 3t^2 + kt + 18$, where k is a constant. When $t = 1$, the acceleration of the particle is -9 m/s^2.

(i) Show that $k = -15$. [2]

(ii) Find the values of t for which particle P is instantaneously at rest. [2]

(iii) Find the total distance travelled by P in the first 3 seconds after passing through point O. [4]

9



The diagram shows part of the curve $y = 4 - e^{-2x}$ which crosses the axes at A and at B.

(i) Find the coordinates of A and of B. [2]

The normal to the curve at B meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region. [5]

10 A circle, C, has equation $x^2 + y^2 - 6x + 4y - 12 = 0$.

(i) Find the radius and the coordinates of the centre of C. [2]

The equation of the normal to the circle at the point A is $3y = m - 4x$.

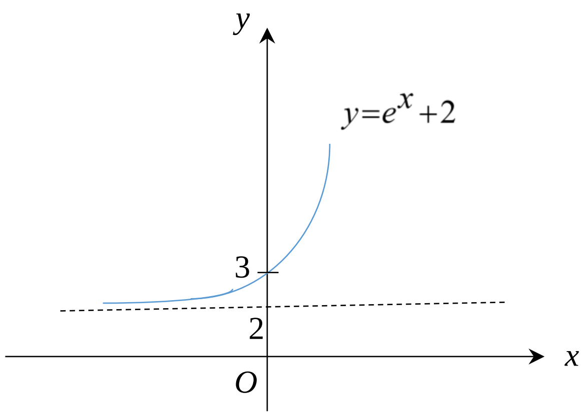
(ii) Find the value of the constant m. [2]

The tangent to the circle at A cuts at the positive y-axis.

(iii) Use your answer in **(ii)** to show that A is (0, 2). [4]

(iv) B is a point on the circle. Given that the equation of tangent to the circle at B is parallel to the equation of the tangent to the circle at A, find the equation of tangent to the circle at B. [3]

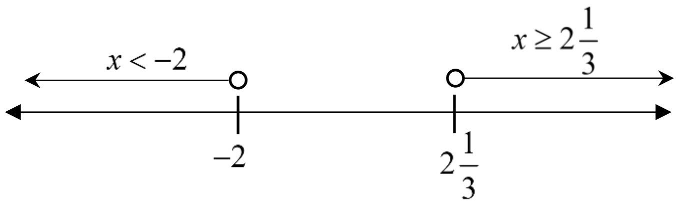
End of Paper

Marking Scheme for A Maths Paper 2			
Qn	**Working**	**Marks**	**Remarks**
1(a)	 	B1 B1	Shape of the curve y-intercept and horizontal asymptote
(b)	$3-e^{-x}=2e^{x}$		
	$3-\frac{1}{e^{x}}=2e^{x}$		
	$3e^{x}-1=2e^{2x}$		
	$2e^{2x}-3e^{x}+1=0$	M1	
	Let $y=e^{x}$		
	$2y^{2}-3y+1=0$		
	$(2y-1)(y-1)=0$	M1	
	$y=\frac{1}{2}$ or $y=1$		
	$e^{x}=\frac{1}{2}$ or $e^{x}=1$	M1	
	$x=\ln\frac{1}{2}$ or $x=\ln 1$		
	$x=-0.693$ or $x=0$	A1	

2(i)	Since roots of f(x) = 0 are 1, k and k^2 $\mathrm{f}(x) = -(x-1)(x-k)(x-k^2)$ $f(2) = -7$ $-(2-1)(2-k)(2-k^2) = -7$ $(2-k)(2-k^2) = 7$ $4-2k^2-2k+k^3 = 7$ $k^3+2k^2+2k-3=0$ (shown)	M1 M1 A1	
(ii)	Let $\mathrm{g}(k) = k^3-2k^2-2k-3$ $\mathrm{g}(3) = 3^3-2(3)^2-2(3)-3$ $= 0$ Since g(3) = 0, k – 3 is a factor of g(k). $k^3-2k^2-2k-3=0$ $(k-3)(k^2+k+1)=0$ $k^2+k+1=0$ k = 3 or $b^2-4ac = 1^2-4(1)(1)$ $= -3 < 0$ (no real roots) Therefore $k^3-2k^2-2k-3=0$ has only 1 real root.	M1 M1 M1, M1 M1 B1	

3(i)	$y=(x+2)\sqrt{x-1}$ $\dfrac{dy}{dx}=(x+2)[\dfrac{1}{2}(x-1)^{-\frac{1}{2}}]+\sqrt{x-1}(1)$ $=(x-1)^{-\frac{1}{2}}[\dfrac{1}{2}x+1+x-1]$ $=(x-1)^{-\frac{1}{2}}(\dfrac{3}{2}x)$ $=\dfrac{3x}{2\sqrt{x-1}}$	M1 M1 M1 A1	
(ii)	By Chain Rule, $\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$ $2=\dfrac{3x}{2\sqrt{x-1}}\times\dfrac{dx}{dt}$ $\dfrac{dx}{dt}=2\div\dfrac{3(2)}{2\sqrt{2-1}}$ $=\dfrac{2}{3}$ units/s	 M1 A1	
(iii)	$\int_2^5\dfrac{x}{\sqrt{x-1}}\,dx=\dfrac{2}{3}\int_2^5\dfrac{3x}{2\sqrt{x-1}}\,dx$ $=\dfrac{2}{3}[(x+2)\sqrt{x-1}]_2^5$ $=\dfrac{2}{3}[7\sqrt{4}-4\sqrt{1}]$ $=\dfrac{2}{3}(10)$ $=6\dfrac{2}{3}$	M1 M1 A1	

4(i)	Let E be the point on AB such that BE is perpendicular to CE. F is the point on CE such that CF is perpendicular to FD. $\left.\begin{array}{l}\cos\theta = \dfrac{BE}{4} \\ BE = 4\cos\theta \\ \\ \sin\theta = \dfrac{FD}{1} \\ FD = \sin\theta\end{array}\right\}$ $AB = BE + FD$ $= 4\cos\theta + \sin\theta$ (shown)	M1 for either one correct A1	
(ii)	$AB = \sqrt{4^2+1^2}\cos\left(\theta - \tan^{-1}\dfrac{1}{4}\right)$ $= \sqrt{17}\cos(\theta - 14.036°)$ $= \sqrt{17}\cos(\theta - 14.0°)$	M1 for finding R, M1 for finding α A1	
(iii)	Max $AB = \sqrt{17}$ m For AB to be maximum, the value of $\cos(\theta - 14.036°)$ must be 1. $\cos(\theta - 14.036°) = 1$ $\theta - 14.036° = 0°$ $\theta = 14.036°$ $= 14.0°$ (1 d.p.)	B1 B1	
(iv)	$3 = \sqrt{17}\cos(\theta - 14.036°)$ $\cos(\theta - 14.036°) = \dfrac{3}{\sqrt{17}}$ $\theta - 14.036° = 43.313°$ $\theta = 43.313° + 14.036°$ $= 57.349°$ $= 57.3°$ (1 d.p.)		

5(a)	$y = 2x^2 - 6x + c$ --------- (1) $y + 2x = 8$ --------------- (2) Equating (1) & (2): $2x^2 - 6x + c = 8 - 2x$ $2x^2 - 4x + c - 8 = 0$ The line is a tangent to the curve, $b^2 - 4ac = 0$ $(-4)^2 - 4(2)(c-8) = 0$ $16 - 8c + 64 = 0$ $c = 10$	M1 M1 A1	
(b)	$3(x^2 - 5) > x - 1$ $3x^2 - 15 - x + 1 > 0$ $3x^2 - x - 14 > 0$ $(3x-7)(x+2) > 0$ $x < -2$ or $x > 2\frac{1}{3}$ 	M1 A1 A1	
(c)	$-2x^2 + x - p$ $a = -2,\ b = 1,\ c = -p$ For real roots, $b^2 - 4ac \geq 0$ $1 - 4(-2)(-p) \geq 0$ $1 - 8p \geq 0$ $p \leq \frac{1}{8}$ Greatest value of integer $p = 0$	M1 M1 A1	

6(i)	$\left(\frac{1}{2}-2x\right)^5$ $=\left(\frac{1}{2}\right)^5+\binom{5}{1}\left(\frac{1}{2}\right)^4(-2x)^1+\binom{5}{2}\left(\frac{1}{2}\right)^3(-2x)^2+\binom{5}{3}\left(\frac{1}{2}\right)^2(-2x)^3+\cdots$ $=\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+\cdots$	M1 A1	
(ii)	$\left(1+ax+3x^2\right)\left(\frac{1}{2}-2x\right)^5$ $=\left(1+ax+3x^2\right)\left(\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+...\right)$ Considering x^2, $5x^2-\frac{5}{8}ax^2+\frac{3}{32}x^2=\frac{13}{2}x^2$ $5-\frac{5}{8}a+\frac{3}{32}=\frac{13}{2}$ $a=-2\frac{1}{4}$	M1 M1 M1 A1	
(iii)	$(0.47)^5=\left(\frac{1}{2}-2x\right)^5$ $\frac{1}{2}-2x=0.47$ $x=0.015$ $(0.47)^5=\frac{1}{32}-\frac{5(0.015)}{8}+5(0.015)^2-20(0.015)^3+...$ ≈ 0.02293	M1 M1 A1	

<table>
<tr><td>7(i)</td><td>$y = 10 + Ab^x$
$y - 10 = Ab^x$
Take log to the base 10 on both sides,
$\lg(y-10) = \lg b(x) + \lg A$
Plot $\lg(y-10)$ against x to obtain a straight line.
<table><tr><td>x</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td></tr><tr><td>y</td><td>15.9</td><td>19.1</td><td>23.4</td><td>30.2</td></tr><tr><td>$y-10$</td><td>5.9</td><td>9.1</td><td>13.4</td><td>20.2</td></tr><tr><td>$\lg(y-10)$</td><td>0.77</td><td>0.96</td><td>1.13</td><td>1.31</td></tr></table></td><td>M1, G1</td><td>1 mark for drawing the straight line</td></tr>
<tr><td>(ii)</td><td>$\lg A =$ vertical intercept of the graph
$= 0.60$ (Accept ± 0.02)
$A = 10^{0.60}$
≈ 3.98

$\lg b =$ Gradient of the graph
$= \dfrac{1.31 - 0.60}{2 - 0}$
$= 0.355$ (Accept ± 0.02)
$b = 10^{0.355}$
≈ 2.26</td><td>M1

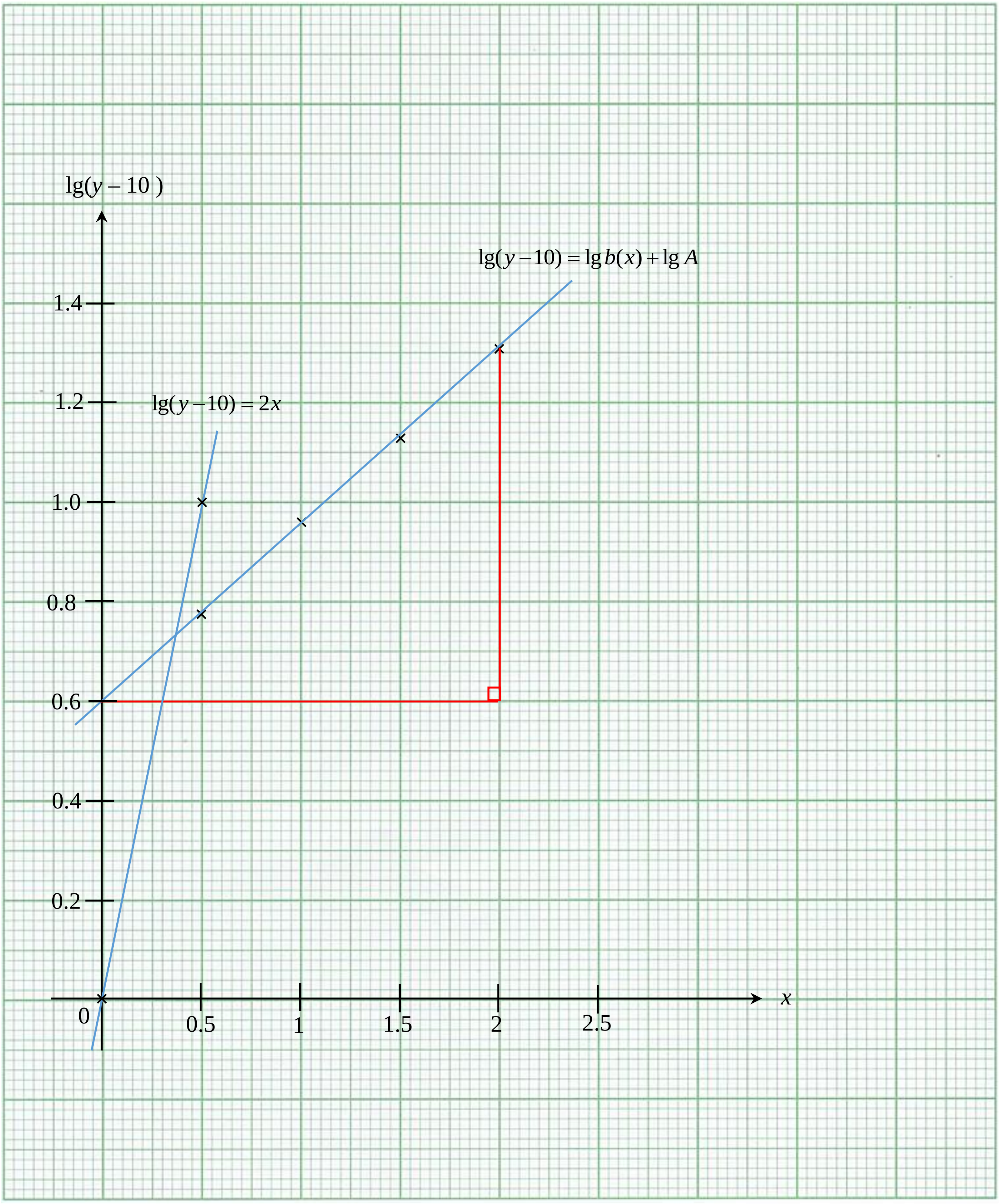
A1

M1

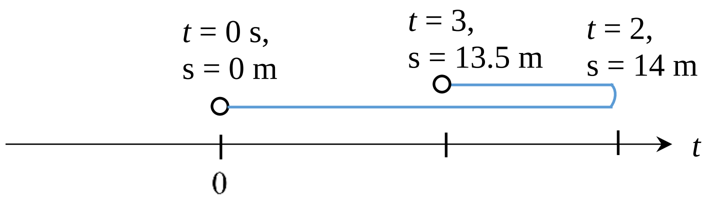
A1</td><td></td></tr>
<tr><td>(iii)</td><td>$Ab^x = 10^{2x}$
$\lg A + x\lg b = 2x$
$\therefore$ draw $\lg(y-10) = 2x$

From the graph, $x \approx 0.375$ (Accept ± 0.01)</td><td>M1
M1

A1</td><td></td></tr>
</table>



$\lg(y-10)$
$\lg(y-10)=\lg b(x)+\lg A$
$\lg(y-10)=2x$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
x

8(i)	$v = 3t^2 + kt + 18$ $a = \dfrac{dv}{dt}$ $= 6t + k$ When $t = 1$, $6(1) + k = -9$ $k = -15$ (shown)	M1 A1	
(ii)	$3t^2 - 15t + 18 = 0$ $t^2 - 5t + 6 = 0$ $(t-2)(t-3) = 0$ $t = 2$ or $t = 3$	M1 A1 for both answers	
(iii)	$s = \int v \, dt$ $= \int \left(3t^2 - 15t + 18\right) dt$ $= t^3 - \dfrac{15}{2}t^2 + 18t + C$ When $t = 0, s = 0, C = 0$ $\therefore s = t^3 - \dfrac{15}{2}t^2 + 18t$ when $t = 2$ s, $s = 14$ m when $t = 3$ s, $s = 13.5$ m  Distance travelled in first 3 s = 14 + (14 – 13.5) = 14.5 m	M1 M1 M1 A1	

9(i)	when $y = 0$, $4 - e^{-2x} = 0$ $e^{-2x} = 4$ $-2x = \ln 4$ $x = -\ln 2$ $\therefore A(-\ln 2, 0)$ when $x = 0$, $y = 4 - e^{0}$ $= 3$ $\therefore B(0,3)$	B1 B1	
(ii)	$y = 4 - e^{-2x}$ $\frac{dy}{dx} = -(-2)e^{-2x}$ $= 2e^{-2x}$ when $x = 0$, $\frac{dy}{dx} = 2e^{0}$ $= 2$ Gradient of $BC = -\frac{1}{2}$ Let the coordinates of C be (x, y). $\frac{3-0}{0-c} = -\frac{1}{2}$ $c = 6$ $\therefore C(6,0)$	M1 M1 M1 A1	
(iii)	Area of shaded region $= \int_{-\ln 2}^{0} (4 - e^{-2x})\ dx + \frac{1}{2}(6)(3)$ $= \left[4x + \frac{1}{2}e^{-2x}\right]_{-\ln 2}^{0} + 9$ $= \left(0 + \frac{1}{2}\right) - \left(-4\ln 2 + \frac{1}{2}e^{2\ln 2}\right) + 9$ $= 7\frac{1}{2} + 4\ln 2$ $\approx 10.3 \text{ units}^2$	M1, M1 M1 M1 A1	

10(i)	$(x-3)^2+(y+2)^2=12+9+4$ $(x-3)^2+(y+2)^2=25$ $C(3,-2)$ Radius $=\sqrt{25}$ $=5$ units	B1 B1	
(ii)	Normal of a circle passes through centre of a circle. $3(-2)=m-4(3)$ $m=6$	M1 A1	
(iii)	$3y=6-4x$ $y=-\frac{4}{3}x+2$ ---------------- (1) $(x-3)^2+(y+2)^2=25$ ---------------- (2) $(x-3)^2+(-\frac{4}{3}x+2+2)^2=25$ $(x-3)^2+(4-\frac{4}{3}x)^2=25$ $x^2-6x+9+(\frac{16}{9}x^2-\frac{32}{3}x+16)=25$ $\frac{25}{9}x^2-\frac{50}{3}x=0$ $25x^2-150x=0$ $25x(x-6)=0$ $x=6$ (rejected) or $x=0$ when $x=0$, $y=-\frac{4}{3}(0)+2$ $y=2$ $\therefore A(0,2)$ (shown)	M1 M1 A1 A1	
(iv)	$(3,-2)$ is midpoint of AB and the required coordinates B be $B(e,f)$ $(\frac{0+e}{2},\frac{2+f}{2})=(3,-2)$ $B(6,-6)$ Equation of tangent at B: $y+6=\frac{3}{4}(x-6)$ $y=\frac{3}{4}x-\frac{21}{2}$	M1 M1 A1	