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**CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2022
PRELIMINARY EXAMINATION**

**ADDITIONAL MATHEMATICS
Paper 1**

**4049/01
23 August 2022**

Candidates answer on the Question Paper.
No Additional Materials are required.

2 hours 15 min

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the answer spaces provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Marks													

Table of Penalties		Question Number		Parent's/Guardian's Signature	90
Presentation	−1				
Accuracy / Units	−1				

1. ALGEBRA*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 A trapezium of area $(12\sqrt{7}+1)\text{cm}^2$ has parallel sides of length $(2\sqrt{7}-1)\text{cm}$ and $(\sqrt{7}+6)\text{cm}$. Without using a calculator, obtain an expression for the height, h , of the trapezium in the form $a\sqrt{7}+b$, where a and b are integers. [4]

- 2 Integrate $\sin^2 x + \frac{1}{x}$ with respect to x . [3]

- 3 Solve the simultaneous equations.

$$2^{x+1} + 5^y = 89$$

$$2^{x-1} - 5^{y-1} = 11$$

[4]

4 Express $\frac{7x^2 - 23x + 6}{(x-2)(x^2-4)}$ in partial fractions.

[5]

- 5 The gradient function of a curve is $\frac{4}{(1-2x)^2} + p$. It is given that the gradient of the curve at the point $(1, -3)$ is 7.

(a) Find the value of p . [2]

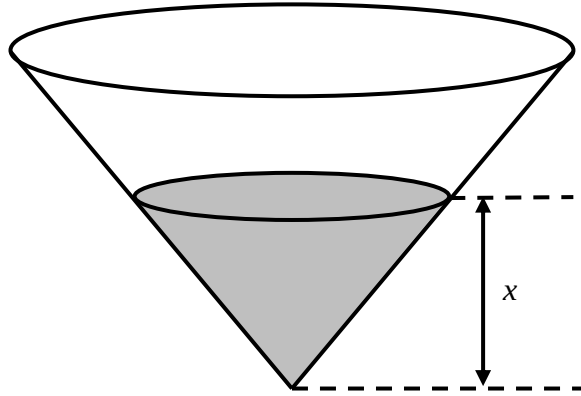
(b) Find the coordinates of the point where the curve meets the y -axis. [4]

- 6 (i) Express $\log_2 \sqrt{\frac{4^{3n-2}}{16^n}}$ in the form $an + b$, where a and b are integers. [3]

- (ii) Solve the equation $\log_2(x+4) = \log_4(7x+4) + \frac{1}{2}$. [4]

- 7 A container is in the shape of an inverted right circular cone where its base radius is equal to its height. Alcohol in the container evaporates at a constant rate of $10 \text{ cm}^3/\text{s}$.

The volume of alcohol, V , in the container is $\frac{1}{3}\pi x^3$ when the height of the alcohol is $x \text{ cm}$.



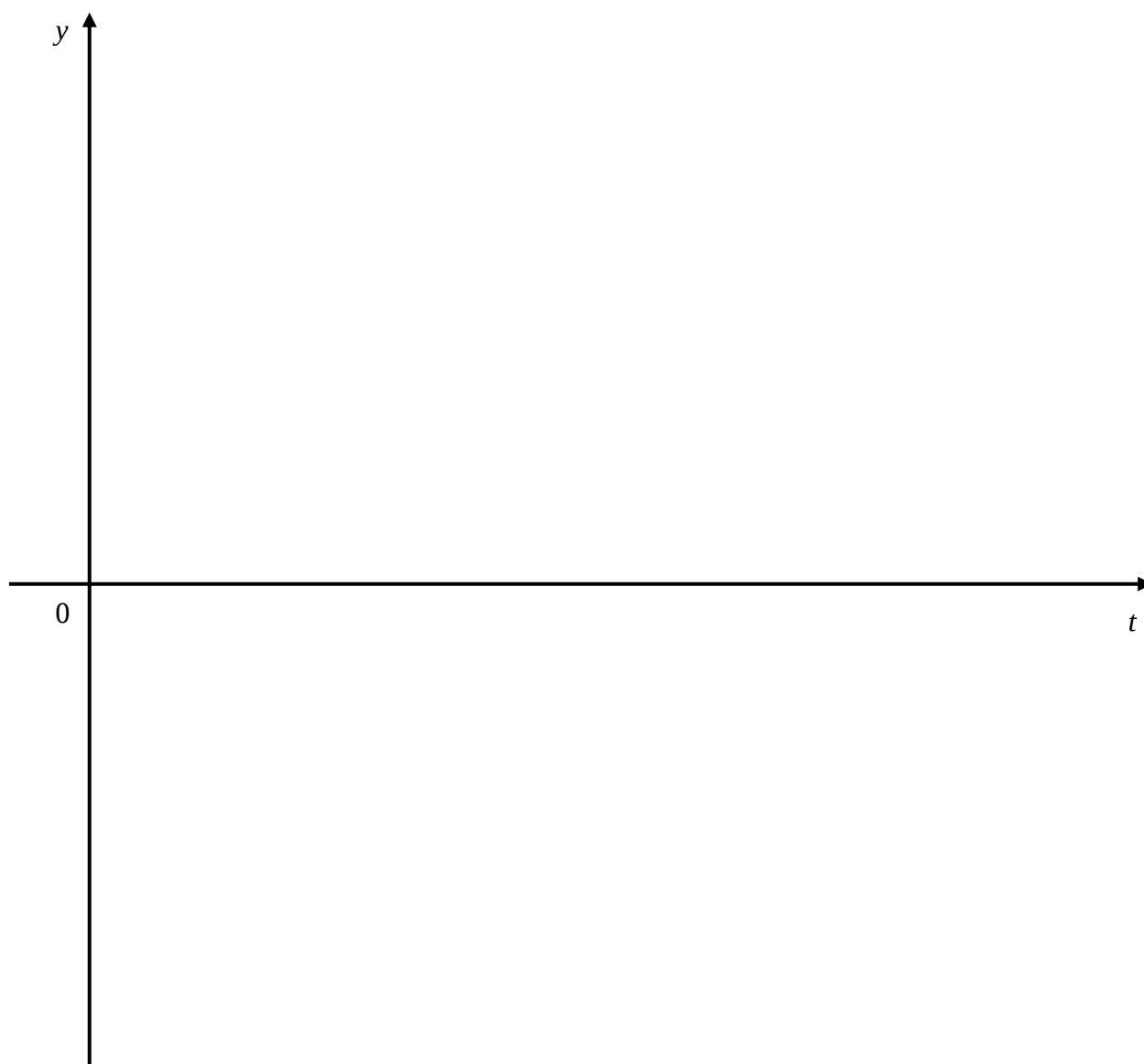
- (i) Calculate the rate of change of the height of alcohol when the height of alcohol is 5 cm, leaving your answer in exact form. [3]

- (ii) Calculate the rate of change of the exposed surface area of the alcohol when the depth of alcohol is 5 cm. [3]

- 8** The vertical displacement of the waves, y m, at a port can be modelled by the equation $y = -3\cos kt + 8.5$, where k is a constant and t is the number of hours after midnight. The time between two consecutive high tides is 6 hours.

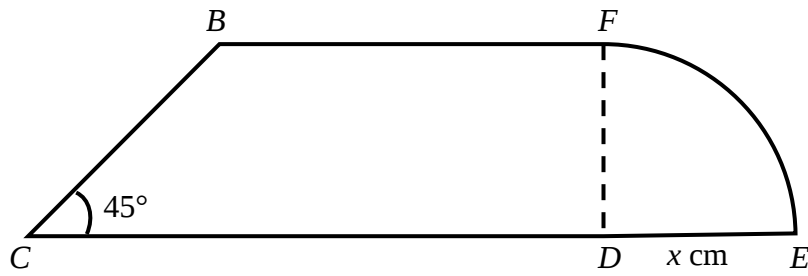
(i) Show that the value of k is $\frac{\pi}{3}$. [1]

(ii) Sketch the graph of $y = -3\cos\frac{\pi}{3}t + 8.5$ for $0 \leq t \leq 24$. [3]



- (iii) If a ship can only enter the port safely when the vertical displacement of the waves is at least 7 m, find the total length of time in hours in a day when the ship can enter the port. [4]

- 9 A piece of wire of length 460 cm is bent to form a figure consisting of a trapezium $BCDF$ and a quadrant DEF where angle $BCD = 45^\circ$ and $DE = x$ cm.



- (i) Show that the area of the figure is given by $A = 230x - \frac{\sqrt{2}+1}{2}x^2$. [5]

- (ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

- 10 (a)** A dolphin jumps out of the water during a dolphin show. The height, y m, of the dolphin is given by $y = -4.5t^2 + 9t$, where t is the time in seconds after the dolphin jumps out of the water.

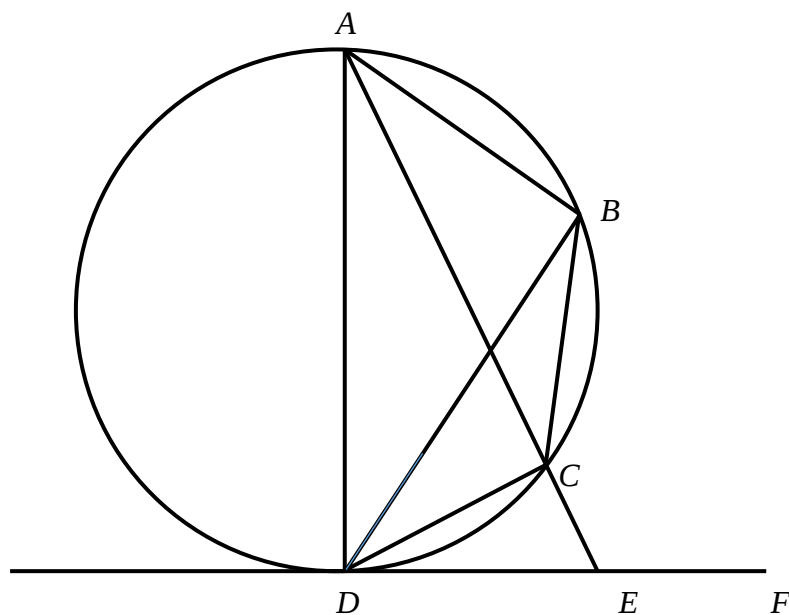
(i) Express $y = -4.5t^2 + 9t$ in the form $y = a(t - h)^2 + k$. [2]

(ii) How long did it take the dolphin to reach the maximum height and what is the maximum height? [2]

(iii) Find the range of values of t where the height of the dolphin is at least 3.375 m. [2]

- (b) The function f is defined by $f(x) = 2x^3 + ax - 3$, where a is a constant. $f(x)$ has a remainder of $-2p$ when divided by $x+1$ and a remainder of $3p+7$ when divided by $x-2$. Find the value of a and of p . [4]

11



In the figure above, DEF is a tangent to the circle at D . The line EA bisects angle DAB and intersects the circle at C .

- (i) Prove that $\angle CDB = \angle CDE$. [3]

(ii) Prove that $AD \times DE = AE \times CD$.

[3]

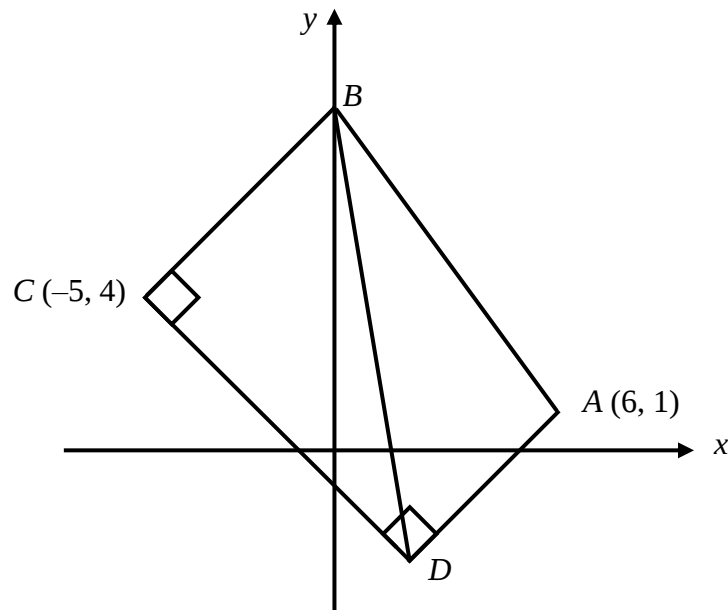
(iii) Prove that $BC = CD$.

[2]

12 (a) Prove that $\sin^3 x \sec^2 x + \sin x = \tan x \sec x$. [4]

(b) Find all the values of x between 0 and 5 for which $2 \cos^2 x - 6 \sin x \cos x = 1$. [5]

- 13 $ABCD$ is a trapezium such that the coordinates of A and C are $(6, 1)$ and $(-5, 4)$ respectively. Angle ADC and angle BCD are right angles and the gradient of AB is $-\frac{4}{3}$.



- (i) Show that the coordinates of B are $(0, 9)$.

[2]

(ii) Show that the coordinates of D are $(2, -3)$.

[4]

(iii) Find the area of triangle ABD .

[2]

(iv) Find angle ABC .

[3]

END OF PAPER

Answer Key

1. $13 - 3\sqrt{7}$

2. $\frac{1}{2}x - \frac{\sin 2x}{4} + \ln x + c$

3. $x = 5, y = 2$

4. $\frac{2}{x-2} - \frac{3}{(x-2)^2} + \frac{5}{x+2}$

5 (a) $p = 3$

(b) $(0, -2)$

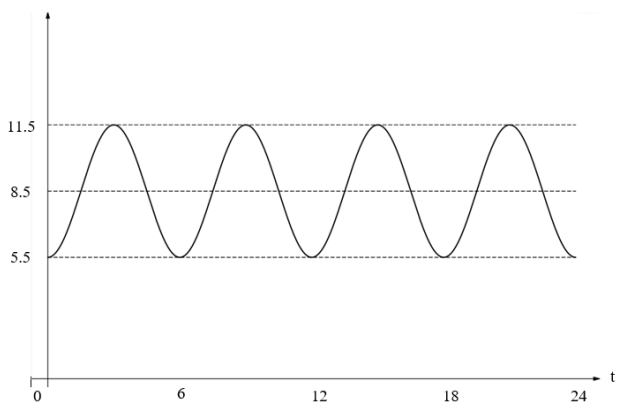
6 (i) $n - 2$

(ii) $x = 2$ or 4

7 (i) $-\frac{2}{5\pi} \text{ cm/s}$

(ii) $-4 \text{ cm}^2/\text{s}$

8(ii)



(iii) 16 hours

9 (ii) 11000, maximum

10 (ai) $-4.5(t-1)^2 + 4.5$

(ii) 1 seconds, 4.5m

(iii) $\frac{1}{2} \leq t \leq 1\frac{1}{2}$

(b) $a = 3, p = 4$

12 (b) 0.161, 1.73, 3.30, 4.87

13 (iii) 28 units^2

(iv) 81.9°