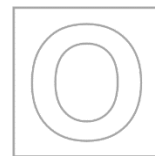




中正中学 义顺

CHUNG CHENG HIGH SCHOOL (YISHUN)



2024 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE
NAME

CLASS

INDEX
NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

22 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need of clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [] at the end of each question or part question.

For Examiner's Use		
Presentation Deduction	- 1 / - 2	
TOTAL	90	

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 (i) Given that the line $y = 2x - k$ meets the curve $y = \frac{k}{4}x^2 - 2kx + 1$, find the range of values of k . [4]

- (ii) A student claimed that when $k = -\frac{1}{2}$, the curve does not intersect the line. By showing your workings clearly, explain whether the statement is valid. [2]

- 2 A triangle has a base of $\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{32}}{2} + \frac{16}{\sqrt{48}}\right)$ m and a height of h m. Given that the area of the triangle is $(2\sqrt{2} - \sqrt{3})$ m², find, without using calculator, the value of h in the form $\frac{a\sqrt{6} + b}{5}$ where a and b are integers. [5]

3 (i) Show that $3x^2 - 5x + 21$ is always positive for all real values of x . [3]

(ii) The curve $y = ax^n$, where a and n are constants, passes through $(2, 48)$, $(3, 108)$ and $(k, 192)$. Find the values of a , n and k where $k > 0$. [3]

- 4 **(a)** Given that $f(x) = \frac{2x+3}{\sqrt{4x-3}}$, find $f'(x)$. [3]

- (b)** Find the set of values of x for $y = \ln\left(\frac{3+8x}{8x-5}\right)$ to be a decreasing function. [3]

- 5 **(a)** Explain why $\frac{2x^3 - 4x^2 + 13x - 10}{(x^2 + 4)(x - 2)}$ is considered an improper fraction. [1]

- (b)** Express $\frac{2x^3 - 4x^2 + 13x - 10}{(x^2 + 4)(x - 2)}$ in the form of $A + \frac{Bx + C}{(x^2 + 4)(x - 2)}$ and hence, express $\frac{2x^3 - 4x^2 + 13x - 10}{(x^2 + 4)(x - 2)}$ in partial fractions. [6]

- 6** **(a)** Given that $6x^2 - 9x + 18 = A(2x + 1)(x - 1) + B(x - 2) + C$ for all values of x , find the values of A , B and C . [3]

- (b)** Given that $f(x) = 2x^3 - 11x^2 + 3x + 36$, show that $2x + 3$ is a factor of $f(x)$ and hence solve the equation $f(x) = 0$. [5]

- 7 Without using a calculator, evaluate $\frac{\log_5 9 + 2\log_5 6 - 4\log_5 3}{\log_{25} 4}$. [4]

- 8 Simplify $2^{3x-4} \times 9^{x-3} = 6^{2(x-2)}$ in the form $\left(\frac{m \times n}{36}\right)^x = 9$, where m and n are integers.

Hence solve for x , giving your answer in the simplest form $\frac{\ln a}{\ln b}$, where a and b are integers.

[5]

- 9 The function f is defined, for $0^\circ \leq x \leq 180^\circ$, by $f(x) = c + a \cos bx$, where a , b and c are positive integers. Given that the amplitude of f is 4 and the period of f is 120° ,

(i) state the value of a and of b , [2]

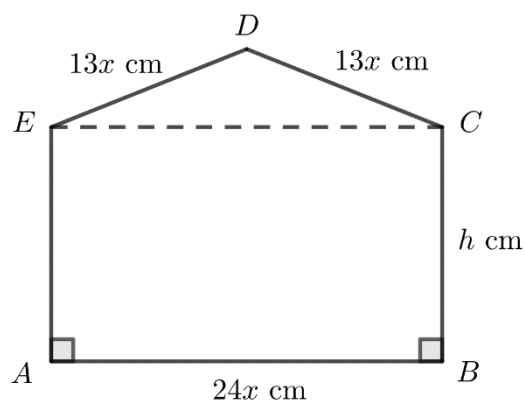
Given that the minimum value of f is -5 ,

(ii) find the value of c , [1]

(iii) sketch the graph of $y = f(x)$ for $0^\circ \leq x \leq 180^\circ$. [3]

(iv) A line $y = k$ is drawn on the graph of $y = f(x)$. State the range of values of k that will give 3 intersections. [1]

- 10 The diagram shows an outline of a board, $ABCDE$, consisting of a rectangle $ABCE$ of height h cm and width $24x$ cm and an isosceles triangle CDE in which $CD = DE = 13x$ cm. The perimeter of the board is 480 cm.



- (a) Show that the area of the board, $A \text{ cm}^2$, is given by $A = 5760x - 540x^2$. [3]

- (b) Calculate the stationary value of A and determine whether it is a maximum or a minimum. [5]

- 11 (a)** If $a \cos^2 \theta + b \sin^2 \theta = c$, show that $\tan^2 \theta = \frac{a-c}{c-b}$. [4]

(b) (i) Given that $\sqrt{(2 \cos^2 x - \sin x)} = 2 \sin x$, show that $6 \sin^2 x + \sin x - 2 = 0$. [2]

(ii) Hence, solve the equation $\sqrt{(2 \cos^2 x - \sin x)} = 2 \sin x$ for $0 \leq \theta \leq 2\pi$. [4]

12 The positive x - and y -axes are tangents to a circle C with equation $(x-a)^2 + (y-b)^2 = r^2$.

(a) What condition(s) must apply to the constants a , b and r ? [1]

(b) It is given that $N(1, 2)$ is a point on the circle and it is further from the centre of the circle C as compared to the origin. Show that the coordinates of the centre M are $(5, 5)$. [4]

- (c) Find the equation of the tangent to C at the point N . [3]

- (d) The tangent intersects the axes at the points P and Q . The point R has coordinates $(k^2, -k)$, where $a > 2$. Find the area of the quadrilateral $MPQR$ in terms of k . [4]

13 (a) Show that $\sin 2x \cot x = 2 \cos^2 x$.

[1]

(b) Hence, without using a calculator, find the value of the constant k for which

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \operatorname{cosec} 6x \tan 3x \, dx = \frac{\sqrt{3}}{k}.$$

[5]