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CATHOLIC HIGH SCHOOL

Preliminary Examination

Secondary 4 (O-Level Programme)

Mathematics

4052/01

Paper 1

25 August 2025

2 hours 15 minutes

Additional Materials: Booklets A and B

Candidates answer in the space provided on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in **degrees to one decimal place**.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is **90**.

For Examiner's Use
90

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 Simplify $\frac{5}{2-x} + \frac{4x-8}{(x-2)^2}$.

$$\begin{aligned} & \frac{5}{2-x} + \frac{4x-8}{(x-2)^2} \\ &= -\frac{5}{x-2} + \frac{4(x-2)}{(x-2)^2} \quad \text{or} \quad = \frac{5}{2-x} - \frac{4}{2-x} \\ &= -\frac{1}{x-2} \quad \text{or} \quad = \frac{1}{2-x} \end{aligned}$$

Answer [3]

2 Rearrange the formula to make p the subject.

$$pr = \frac{6p^2 - 1}{p}$$

$$\begin{aligned} pr &= \frac{6p^2 - 1}{p} \\ p^2(6-r) &= 1 \\ p &= \pm \sqrt{\frac{1}{6-r}} \end{aligned}$$

Answer [2]

3 Factorise the following expressions completely.

(a) $5m + 2n - mn - 10$
 $5m + 2n - mn - 10$
 $= 5(m - 2) + n(2 - m)$
 $= (5 - n)(m - 2)$

or

$$= (n - 5)(2 - m)$$

Answer [2]

(b) $x^2 + 4xy - 9 + 4y^2$

$$\begin{aligned} & x^2 + 4xy + 4y^2 - 9 \\ &= (x + 2y)^2 - 9 \\ &= (x + 2y + 3)(x + 2y - 3) \end{aligned}$$

Answer [2]

- 4 The product of two numbers, x and y , is 6300 and their highest common factor is 6. Given $6 < x < y$, find the value of x and the value of y .

$$6300 = 2^2 \times 3^2 \times 5^2 \times 7$$

$$x = 2 \times 3 \times 7 = 42$$

$$y = 2 \times 3 \times 5^2 = 150$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [2]

- 5 Simplify $\left(\frac{49x^4}{y^{16}}\right)^{-\frac{3}{2}}$, leaving your answer in positive indices.

$$\left(\frac{49x^4}{y^{16}}\right)^{-\frac{3}{2}} = \frac{y^{24}}{343x^6}$$

Answer $\dots\dots\dots$ [2]

- 6 (a) Express $-x^2 - 8x - 19$ in the form of $-(x + p)^2 + q$.

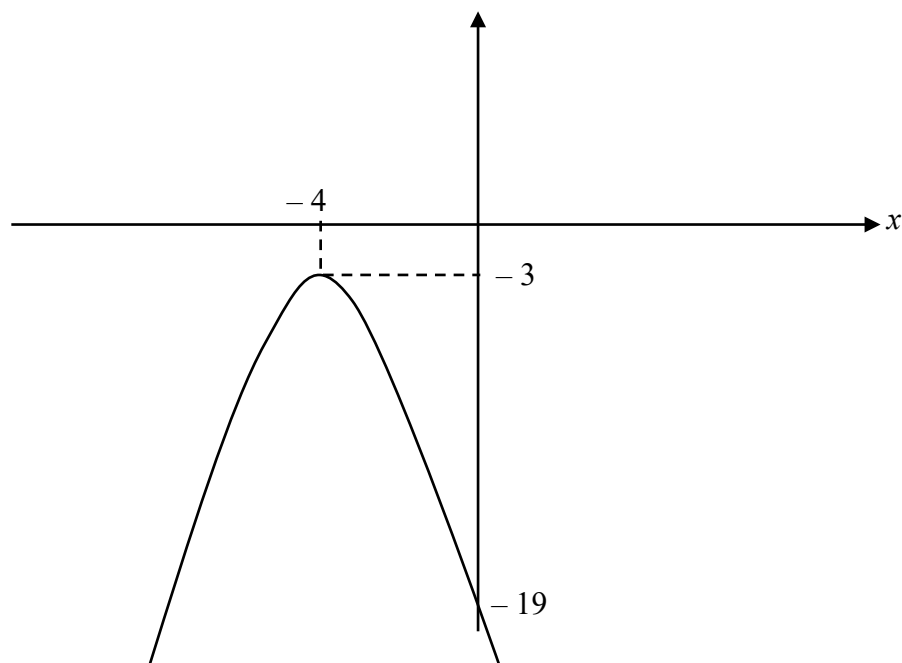
$$\begin{aligned} & -x^2 - 8x - 19 \\ & = -(x + 4)^2 - 3 \end{aligned}$$

Answer [2]

- (b) Hence, on the axes below, sketch the graph of $y = -x^2 - 8x - 19$.

Answer

[2]



7 Andy places an order for some polo shirts that come in three sizes S , M and L .

The ratio of the number of polo shirts $S : M = 3 : 8$.

The ratio of the number of polo shirts $S : L = 5 : 2$.

There are 306 more polo shirts in size M than L .

Find the total number of polo shirts ordered.

$$S : M \qquad S : L$$

$$3 : 8 \qquad 5 : 2$$

$$15 : 40 \qquad 15 : 6$$

$$S : M : L$$

$$15 : 40 : 6$$

$$\text{Total} = \left(\frac{306}{40-6} \right) (61)$$

$$= 549$$

Answer [2]

8 Solve $2^p \times 6 = 96$.

$$2^p \times 6 = 96$$

$$2^p = 16$$

$$p = 4$$

Answer $p =$ [2]

9 Solve the inequality $x - 6 \leq 3x - 17 < 32 - 4x$.

Show your solution on the number line below.

$$x - 6 \leq 3x - 17 < 32 - 4x$$

$$3x - 17 \geq x - 6$$

$$3x - 17 < 32 - 4x$$

$$x \geq 5\frac{1}{2}$$

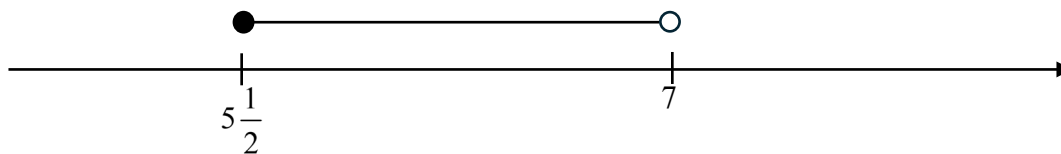
$$x < 7$$

$$5\frac{1}{2} \leq x < 7$$

Answer [2]

Answer

[1]



10 The cash price of a refrigerator is \$2499.

The hire-purchase price of the refrigerator is \$2999.

The hire-purchase price is a deposit of 25% of the cash price plus 24 equal monthly payments.

Calculate one monthly payment.

$$\text{Remaining amount} = 2999 - (2499)(0.25) = 2374.25$$

$$\text{Monthly Instalment} = \frac{2374.25}{24} = 98.93$$

Answer \$ [2]

- 11** Michael deposited some money into bank A which pays a simple interest rate of 3% per annum for 18 months.

If he had deposited the same amount of money with bank B, which pays a simple interest rate of 2.5% per annum for 24 months, he would receive \$500 more in interest.

- (a)** Find the amount of money he deposited.

$$\frac{P(2.5)(2)}{100} - \frac{P(3)(1.5)}{100} = 500$$
$$P = 100000$$

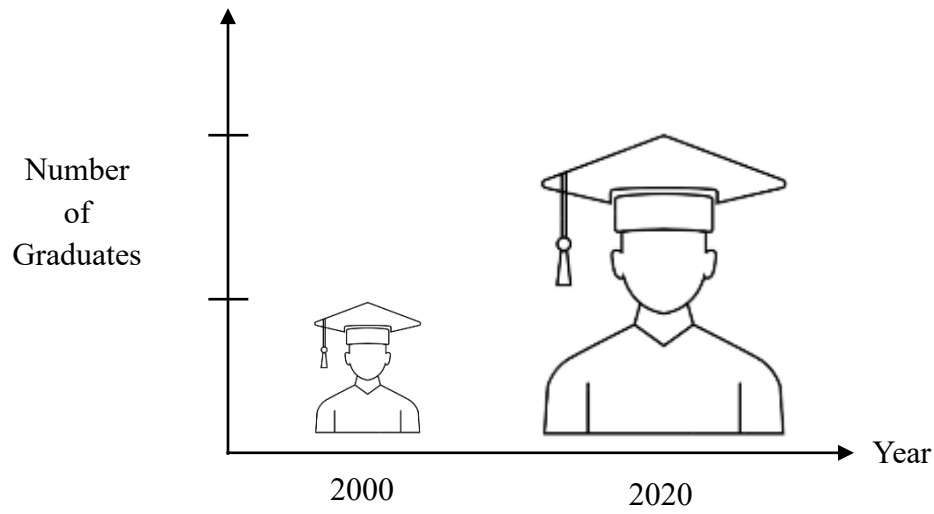
Answer \$ [2]

- (b)** Find the amount of interest Michael would receive if he were to deposit his money into bank C which pays a compound interest of 1.9% per annum compounded every 6 months for 24 months.

$$A = 100000 \left(1 + \frac{1.9}{200} \right)^4$$
$$= 103854.49$$
$$I = 3854.49$$

Answer \$ [2]

- 12 The number of university graduates for the year 2000 and year 2020 are represented by the chart below. Statistics has shown that the number of university graduates in year 2020 is doubled the number of university graduates in year 2000.



Explain how the above chart may be misleading and suggest a possible way to improve the chart.

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.....

.....

.....[2]

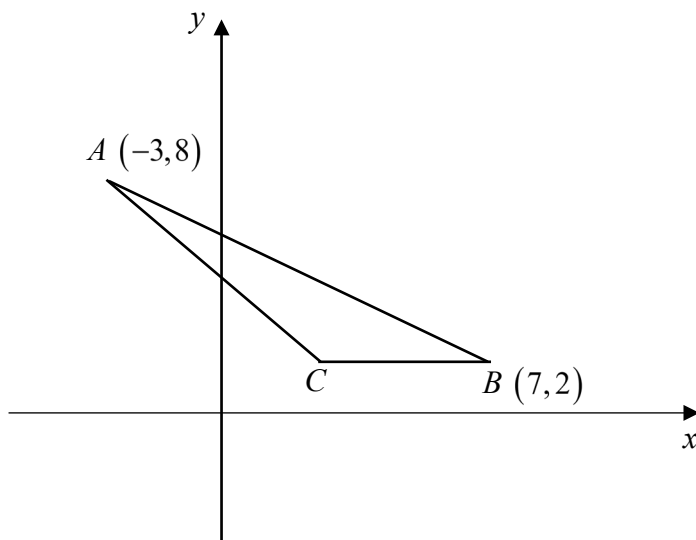
The **area** of the picture for year 2020 is more than twice the **area** of the picture for year 2000, which might mislead people into thinking that the number of graduates in 2020 is more than twice that of year 2000.

1. Replace with bar graphs of equal width and the height of bar of year 2020 twice the height of bar of year 2000.
2. Replace year 2020 picture with two identical 2000 pictures, and stack the two pictures on top of each other.

- 13 The vertices of A and B are $(-3,8)$ and $(7,2)$.

The line BC is parallel to the x – axis.

The area of triangle ABC is 12 square units.



- (a) Find the coordinates of C .

$$\frac{1}{2}(b)(6) = 12$$

$$b = 4$$

$$C(3,2)$$

Answer [2]

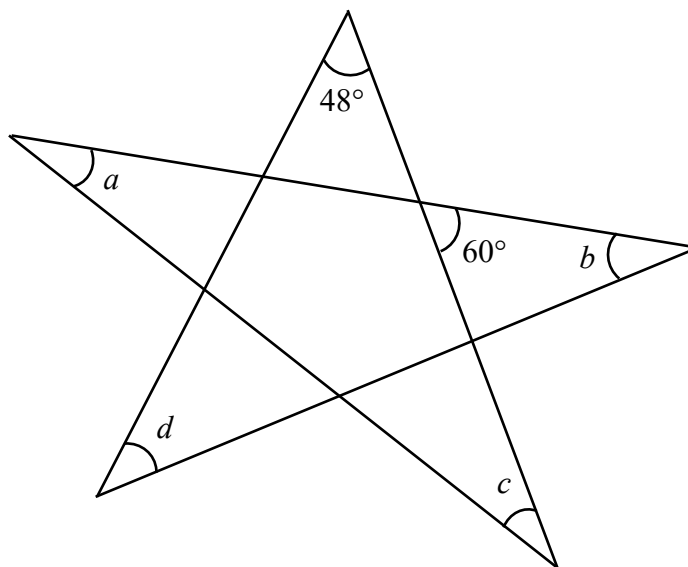
- (b) Find the exact value of $\cos \hat{ACB}$.

$$AC = \sqrt{(6^2) + 6^2} = 6\sqrt{2}$$

$$\cos \hat{ACB} = -\frac{6}{6\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Answer [2]

14 Find $\angle a + \angle b + \angle c + \angle d$.



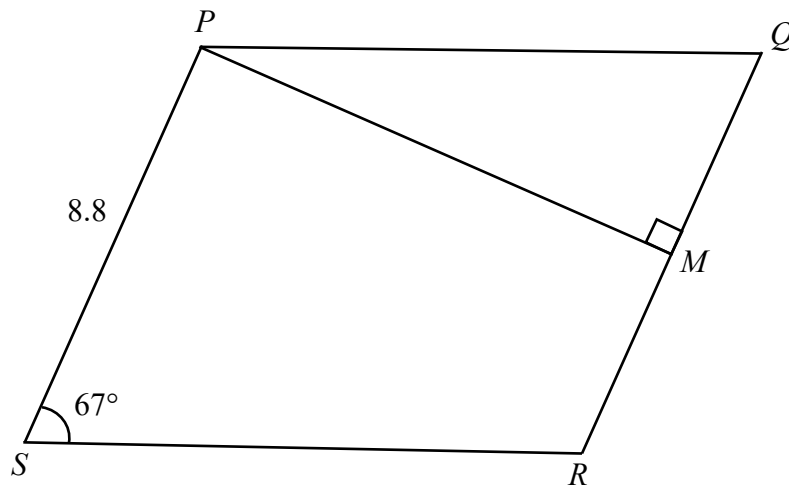
$$\angle a + \angle c = 60^\circ \text{ (exterior angle)}$$

$$\begin{aligned} \angle b + \angle d &= 180 - 60 - 48 \\ &= 72^\circ \text{ (exterior angle)} \end{aligned}$$

$$\begin{aligned} \angle a + \angle b + \angle c + \angle d &= 60 + 72 \\ &= 132^\circ \end{aligned}$$

Answer $^\circ$ [2]

- 15** $PQRS$ is a parallelogram. PM is perpendicular to QR and M is the midpoint of QR .
 $PS = 8.8\text{cm}$ and angle $PSR = 67^\circ$.
 Find the area of $PQRS$.



$$\tan 67 = \frac{PM}{4.4}$$

$$PM = 4.4 \tan 67$$

$$\begin{aligned} \text{Area} &= (4.4 \tan 67) 8.8 \\ &= 91.2 \end{aligned}$$

Answercm² [3]

16 The first four terms of a sequence are 1, 5, 9 and 13.

(a) Find an expression, in terms of n , for the n th term of the sequence.

$$4n - 3$$

Answer [1]

(b) Find an expression, in terms of n , for the $(n + 1)$ th term of the sequence.

$$4(n + 1) - 3$$

$$= 4n + 1$$

Answer [1]

(c) Explain why the difference between the squares of two consecutive terms is always a multiple of 8.

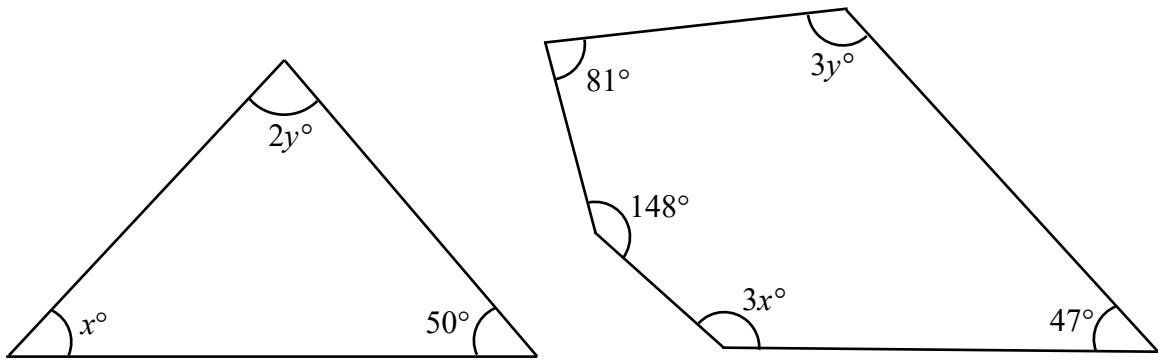
$$(4n + 1)^2 - (4n - 3)^2$$

$$= 16n^2 + 8n + 1 - (16n^2 - 24n + 9)$$

$$= 32n - 8$$

$$= 8(4n - 1)$$

$(4n - 1)$ is an integer and since 8 is a factor, the difference is always multiple of 8.



- (a) Using the information shown in the diagrams, write down and simplify two equations in x and y .

Answer

[3]

$$x + 2y = 130$$

$$3x + 3y = 540 - 81 - 148 - 47$$

$$x + y = 88$$

- (b) Solve these equations to find the value of x and of y .

$$x + 2y = 130 \quad -(1)$$

$$x + y = 88 \quad -(2)$$

$$(1) - (2), y = 42$$

$$x = 46$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [2]

- 18 (a) In a company of 120 workers, the ratio of female worker to male workers is 2 : 3.

	Female	Male
Full time	p	q
Part time	9	r

20% of the workers work part time.

Find the value of p , of q and of r .

$$\begin{aligned} r &= (120)(0.2) - 9 \\ &= 15 \end{aligned}$$

$$\begin{aligned} p &= (120)\left(\frac{2}{5}\right) - 9 \\ &= 39 \end{aligned}$$

$$\begin{aligned} q &= 120 - 9 - 39 - 15 \\ &= 57 \end{aligned}$$

Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$

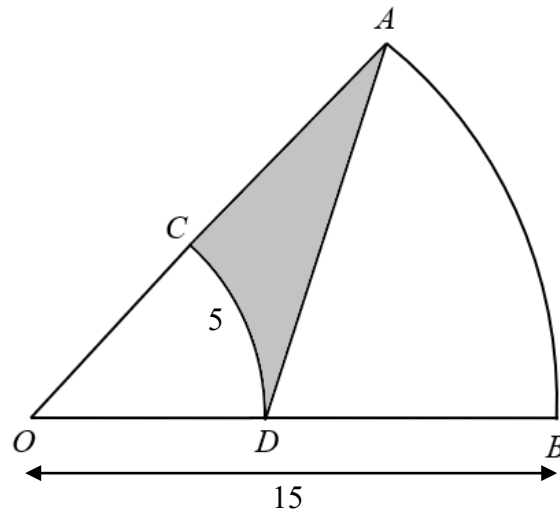
$r = \dots\dots\dots$ [2]

- (b) The annual salary of a particular male worker is \$85000 in 2023. His salary in 2023 was 12% higher than in 2021. Find his annual salary in 2021.

$$\begin{aligned} &\frac{85000}{1.12} \\ &= 75892.86 \end{aligned}$$

Answer \$..... [1]

- 19 The diagram shows two sectors OAB and OCD with centre at O . OB is 15 cm and CD has an arc length of 5 cm. The ratio of area of sector OCD : area of $ABDC$ is 16 : 65.



- (a) Find angle COD in radian.

Mtd 1:

$$\begin{aligned}\frac{A_{OCD}}{A_{OAB}} &= \frac{16}{81} \\ \frac{OD}{OB} &= \sqrt{\frac{16}{81}} \\ \frac{OD}{15} &= \frac{4}{9} \\ OD &= 6\frac{2}{3} \\ COD &= \frac{5}{6\frac{2}{3}} \\ &= \frac{3}{4} \text{ rad}\end{aligned}$$

Mtd 2:

$$\begin{aligned}\frac{A_{OCD}}{A_{OAB}} &= \frac{16}{81} \\ \frac{CD}{AB} &= \sqrt{\frac{16}{81}} \\ \frac{5}{AB} &= \frac{4}{9} \\ AB &= 11.25 \\ COD &= \frac{11.25}{15} \\ &= \frac{3}{4} \text{ rad}\end{aligned}$$

Mtd 3:

Let $\angle COD = \theta$ and $OD = r$.

$$r = \frac{5}{\theta}$$

$$\frac{\frac{1}{2} \left(\frac{5}{\theta} \right)^2 \theta}{\frac{1}{2} (15)^2 \theta - \frac{1}{2} \left(\frac{5}{\theta} \right)^2 \theta} = \frac{16}{65}$$

$$\frac{\frac{25}{2\theta^2}}{\frac{225}{2} - \frac{25}{2\theta^2}} = \frac{16}{65}$$

$$\frac{25}{225\theta^2 - 25} = \frac{16}{65}$$

$$\theta^2 = \frac{9}{16}$$

$$\theta = \frac{3}{4} \text{ rad}$$

Mtd 4:

Let $\angle COD = \theta$ and $OD = r$.

$$r = \frac{5}{\theta}$$

$$\text{Area of sector } OAB = \frac{1}{2} (15)^2 \theta = \frac{225}{2} \theta$$

$$\begin{aligned} \text{Area of sector } OAB &= \frac{16}{81} \times \frac{225}{2} \theta \\ &= \frac{200}{9} \theta \end{aligned}$$

$$\frac{1}{2} \left(\frac{5}{\theta} \right)^2 \theta = \frac{200}{9} \theta$$

$$\theta^2 = \frac{9}{16}$$

$$\theta = \frac{3}{4} \text{ rad}$$

Answerrad [3]

- (b) Find the area of the shaded region.

Mtd 1:

$$\begin{aligned} A &= \frac{1}{2}(15)\left(6\frac{2}{3}\right)\sin\frac{3}{4} - \frac{1}{2}\left(6\frac{2}{3}\right)^2\left(\frac{3}{4}\right) \\ &= 34.082 - 16.667 \\ &= 17.4 \end{aligned}$$

Mtd 2:

Let h = perpendicular height of A to OB

$$\begin{aligned} h &= 15 \sin \frac{3}{4} \\ &= 10.22 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}\left(6\frac{2}{3}\right)(10.22) - \frac{1}{2}\left(6\frac{2}{3}\right)^2\left(\frac{3}{4}\right) \\ &= 17.4 \end{aligned}$$

Answercm² [3]

- 20 In this table, p is inversely proportional to q^2 .

p	0.08	0.5	b
q	a	4	2

Calculate the value of a and of b .

$$0.5 = \frac{k}{4^2}$$

$$k = 8$$

$$a = 10$$

$$b = 2$$

Answer $a =$

$b =$ [3]

- 21 (a)** A river with an actual length of 1 km is represented on a map with a length of 8 cm.
Find the map scale in the form 1 : n .

8 : 100,000

1 : 12500

Answer [1]

- (b)** A forest is represented by an area of 200 cm² on the map. Calculate the actual area of forest in square metres, giving your answer in standard form.

1 : 125

1 : 15625

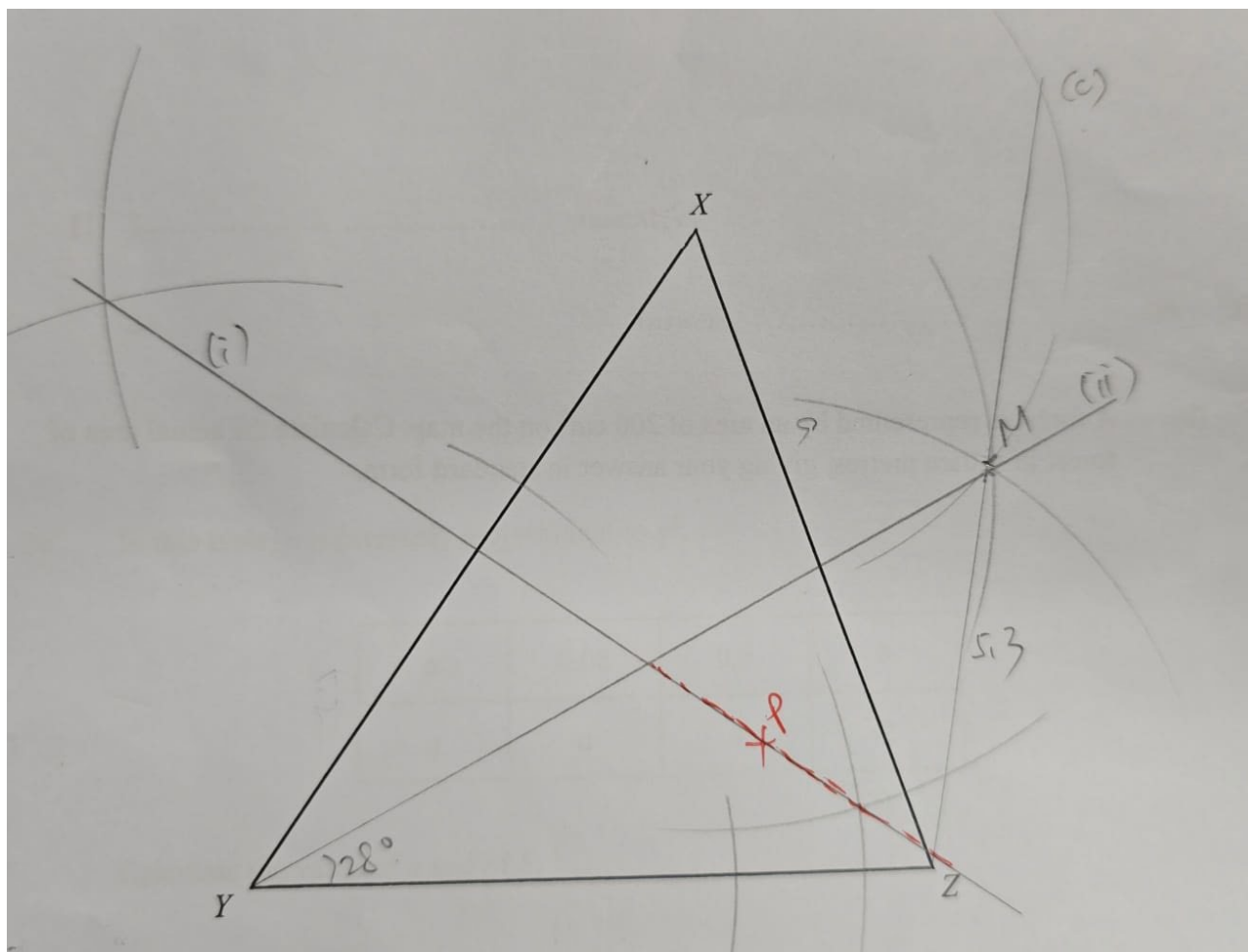
200 : 3125000

3.125×10^6

Answerm² [2]

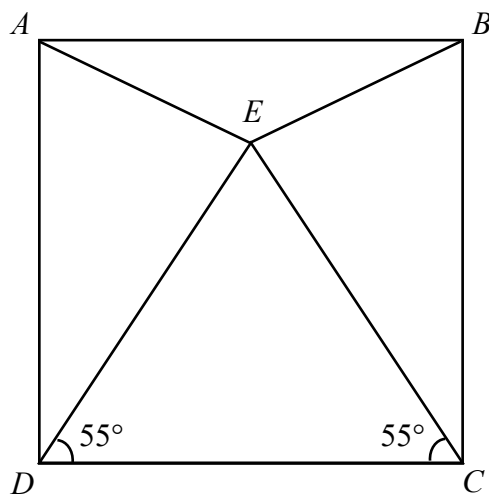
22 A triangle XYZ is shown below.

Answer



- (a) Construct
- (i) the perpendicular bisector of XY , [1]
 - (ii) the bisector of angle XYZ . [1]
- (b) Mark clearly a possible point which is inside the triangle, equidistant from X and Y , and nearer to YZ than XY . Label this point P . [1]
- (c) The point M is such that angle $YZM = 100^\circ$ and $XM = 5$ cm. Find the two possible positions of M and label them M_1 and M_2 . [2]

- 23 $ABCD$ is a square and angle $EDC = \text{angle } ECD = 55^\circ$.



Prove that triangle AEB is an isosceles triangle.

[4]

Mtd 1:

Since $\angle EDC = \angle ECD = 55^\circ$, $\triangle EDC$ is an isosceles triangle, $ED = EC$

$\angle ADE = \angle BCE = 90^\circ - 55^\circ = 35^\circ$

$AD = BC$ (sides of sq)

$\triangle ADE \equiv \triangle BCE$ (SAS)

$\therefore AE = BE$, $\triangle AEB$ is an isosceles triangle

Mtd 2:

Let P be the foot of the perpendicular from E to DC .

Since $\angle EDC = \angle ECD = 55^\circ$,

$\Rightarrow \triangle EDC$ is an isosceles triangle,

$\Rightarrow EP \perp DC$,

$\Rightarrow DP = PC$

Let EP extended to cut AB at Q .

Since $ABCD$ is a square and EP extended to Q on AB ,

$$\left. \begin{array}{l} \angle AQE = \angle BQE = 90^\circ \\ AQ = BQ \end{array} \right\}$$

EQ is a shared side for $\triangle QAE$ and $\triangle QBE$,

$\triangle QAE \equiv \triangle QBE$ (SAS)

$\therefore \angle QAE = \angle QBE$

$\Rightarrow \triangle AEB$ is an isosceles triangle.

Mtd 3:

$$\left. \begin{array}{l} \text{Since } \angle EDC = \angle ECD = 55^\circ, \\ \Rightarrow \triangle EDC \text{ is an isosceles triangle} \\ \Rightarrow E \text{ lies on the perpendicular bisector of } DC. \\ \text{Since } ABCD \text{ is a square,} \\ \Rightarrow E \text{ lies on the perpendicular bisector of } AB. \end{array} \right\}$$

Let Q = foot of the perpendicular bisector of AB from E .

$$\left. \begin{array}{l} \Rightarrow AQ = BQ \text{ and } \angle AQE = \angle BQE \\ EQ \text{ is a shared side for } \triangle QAE \text{ and } \triangle QBE, \end{array} \right\}$$

$\triangle QAE \equiv \triangle QBE$ (SAS)

$\therefore \angle QAE = \angle QBE$

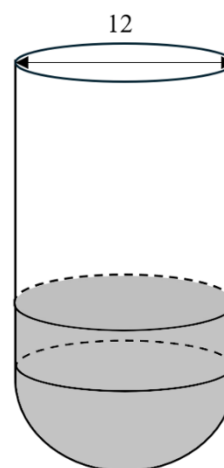
$\Rightarrow \triangle AEB$ is an isosceles triangle.

- 24 (a) A container is made from a hemisphere and cylinder.

The container has a diameter of 12 cm.

The container is filled with water such that the total surface area of water in contact with the container is 120π .

Find the volume of water in the container, leaving your answer in terms of π .



$$2\pi(6^2) + 2\pi(6)(h) = 120\pi$$

$$72\pi + 12\pi h = 120\pi$$

$$h = 4$$

$$\text{Vol} = \frac{2}{3}\pi(6)^3 + \pi(6^2)(4)$$

$$= 144\pi + 144\pi$$

$$= 288\pi$$

Answercm³ [4]

- (b) All the water in the container was poured into a cone with diameter 24 cm and a slant height of 20 cm. Calculate the depth, d , of the empty space.

Mtd 1:

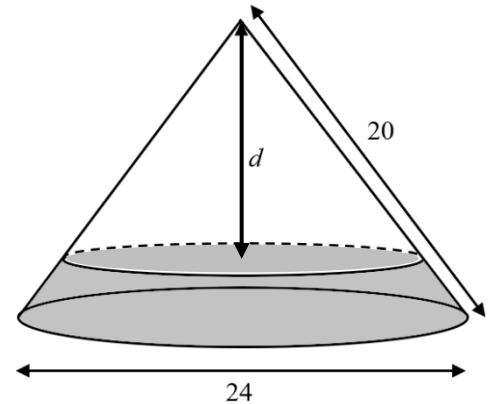
$$h = \sqrt{20^2 - 12^2} = 16$$

$$\begin{aligned} V_{air} &= \frac{1}{3} \pi (12^2)(16) - 288\pi \\ &= 768\pi - 288\pi \\ &= 480\pi \end{aligned}$$

$$\frac{V_{air}}{V_{cone}} = \left(\frac{d_{air}}{d_{cone}} \right)^3$$

$$\frac{480\pi}{768\pi} = \left(\frac{d_{air}}{16} \right)^3$$

$$\begin{aligned} d_{air} &= \sqrt[3]{\frac{480\pi}{768\pi}} (16) \\ &= 13.7 \end{aligned}$$



Mtd 2:

$$h = \sqrt{20^2 - 12^2} = 16$$

$$\frac{r}{12} = \frac{d}{16}$$

$$r = \frac{3}{4}d$$

$$\frac{1}{3} \pi (12^2)(16) - \frac{1}{3} \pi (r^2)(d) = 288\pi$$

$$768\pi - \frac{1}{3} \pi \left(\frac{3}{4}d \right)^2 (d) = 288\pi$$

$$\frac{9}{16} \pi d^3 = 480\pi$$

$$d^3 = 2560$$

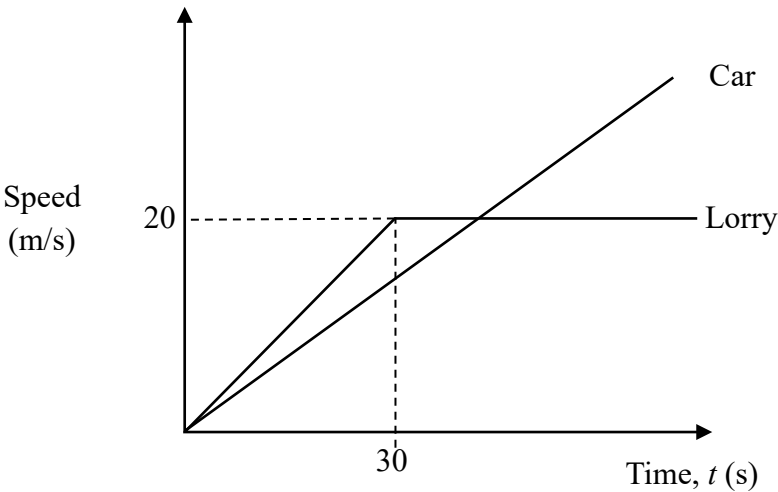
$$d_{air} = 13.7$$

Answer $d = \dots\dots\dots$ cm [4]

25 The diagram shows the speed-time graphs of a car and lorry traveling along the same road in the same direction. Both vehicles begin their journey from the same starting point.

The lorry accelerates for 30 seconds and then travels at a constant speed of 20 m/s.

The car accelerates at a uniform rate of 0.5 m/s^2 from rest.

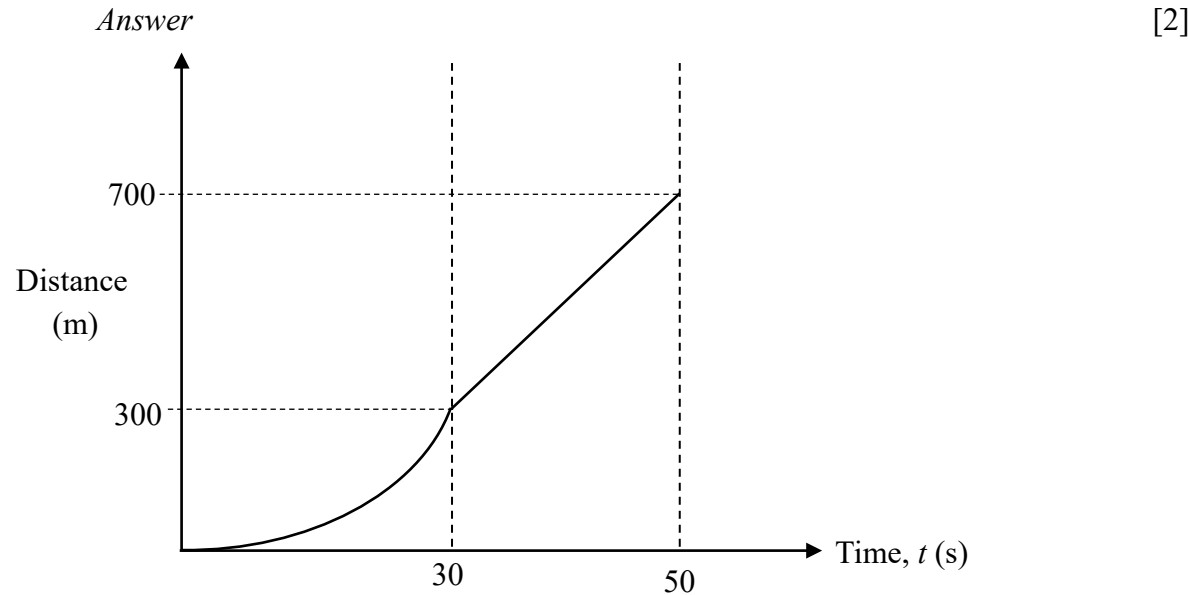


(a) Find the speed of lorry at 12 seconds.

$$\begin{aligned} \text{Speed} &= \frac{20}{30}(12) \\ &= 8 \text{ m/s} \end{aligned}$$

Answerm/s [2]

(b) Draw the distance-time graph of the lorry for $0 \leq t \leq 50$ s.



- (c) Find the time taken for the car to meet the lorry.

Mtd 1:

$$V \text{ of car} = \frac{1}{2}t$$

$$\text{Dist traveled by car} = \frac{1}{2}\left(\frac{1}{2}t\right)(t)$$

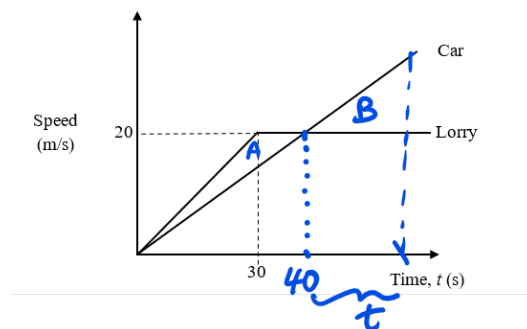
$$\text{Dist traveled by lorry} = 300 + 20(t - 30)$$

$$\frac{1}{2}\left(\frac{1}{2}t\right)(t) = 20(t) - 300$$

$$\frac{1}{4}t^2 - 20t + 300 = 0$$

$$t = 60 \quad \text{or} \quad t = 20(\text{rej})$$

Mtd 2:



$$V \text{ of car} = \frac{1}{2}t$$

$$\text{When } v = 20, \frac{1}{2}t = 20 \Rightarrow t = 40$$

$$\text{Area of region A} = \frac{1}{2}(40 - 30)(20) = 100$$

$$\text{Area of region B} = \frac{1}{2}\left(\frac{1}{2}t\right)t$$

$$\frac{1}{2}\left(\frac{1}{2}t\right)(t) = 100$$

$$t^2 = 400$$

$$t = 20$$

$$\text{Time taken } 40 + 20 = 60$$

Answers [4]