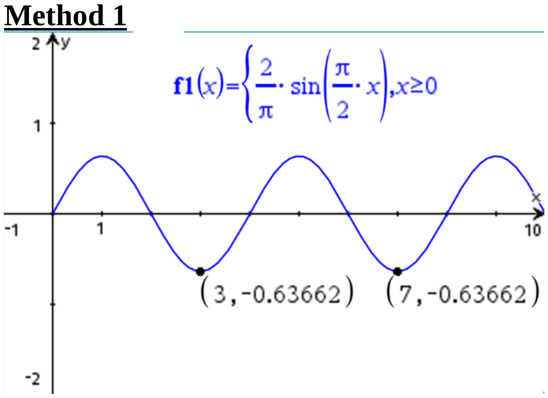
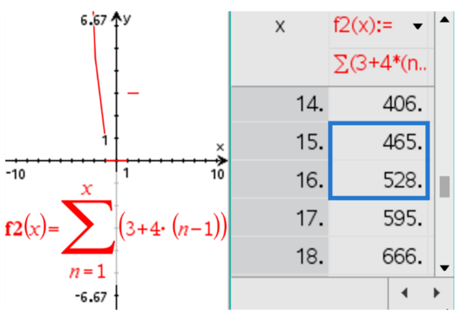
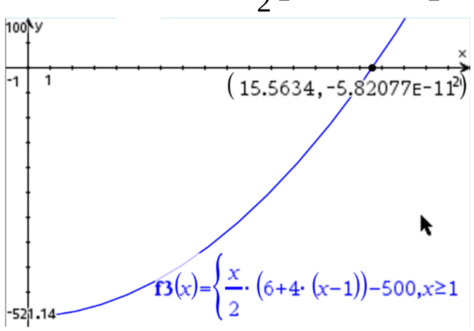
Year 6 HL MAA Preliminary Examination 2024 Paper 2 (Markscheme)

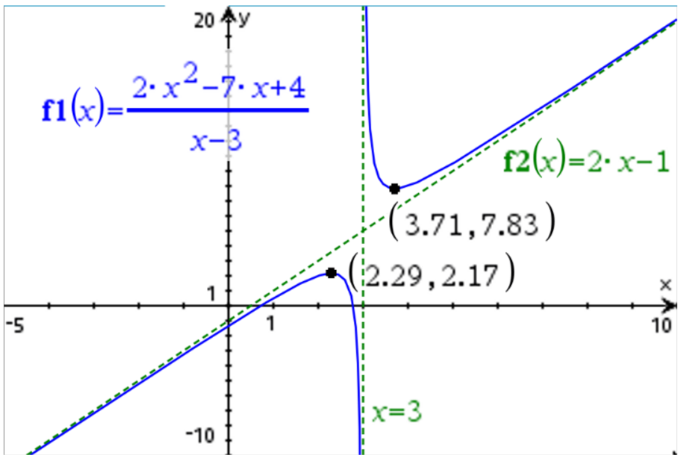
Section A

Qn	Suggested solution	Markscheme
1	***Normal distribution with conditional probability***	**[Marks: 6]**
(a)	$W \sim N\left(3.24,\ 0.375^2\right)$ $P\left(W > w\right) = 0.1 \Rightarrow P\left(W \le w\right) = 0.9$ By GDC (invNorm or nSolve), $w = 3.72$ (3 sf) invNorm(0.9,3.24,0.375) 3.72058 nSolve(normCdf(w,9.E999,3.24,0.375)=0.1,w▸ 3.72058	**(A1)** – cumulative prob **(M1)** – invNorm or nSolve **A1**
(b)	$P\left(W \ge 4 \mid W > w\right) = \dfrac{P\left(W \ge 4 \cap W > w\right)}{0.1} = \dfrac{P\left(W \ge 4\right)}{0.1}$ $= \dfrac{0.021348}{0.1}$ $= 0.213$ (3 sf) normCdf(4,9.E999,3.24,0.375) 0.021348 $\dfrac{0.021348187791259}{0.1}$ 0.213482	**M1 A1** – conditional prob **A1** – no follow through as $P\left(W > w\right) = 0.1$ is given
2	***Vectors***	**[Marks: 4]**
	$(\mathbf{a} - \mathbf{v}) \cdot \mathbf{b} = \left(\mathbf{a} - \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{\lvert\mathbf{b}\rvert^2}\right)\mathbf{b}\right) \cdot \mathbf{b}$ $= \mathbf{a} \cdot \mathbf{b} - \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{\lvert\mathbf{b}\rvert^2}\right)\mathbf{b} \cdot \mathbf{b}$ $= \mathbf{a} \cdot \mathbf{b} - \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{\lvert\mathbf{b}\rvert^2}\right)\lvert\mathbf{b}\rvert^2$ $= \mathbf{a} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$ $= 0$ Since $(\mathbf{a} - \mathbf{v}) \cdot \mathbf{b} = 0$, $(\mathbf{a} - \mathbf{v}) \perp \mathbf{b}$ $\lvert(\mathbf{a} - \mathbf{v}) \times \mathbf{b}\rvert = \lvert\mathbf{a} - \mathbf{v}\rvert\lvert\mathbf{b}\rvert \sin\dfrac{\pi}{2}$ $= \lvert\mathbf{a} - \mathbf{v}\rvert\lvert\mathbf{b}\rvert$	**M1** – distributive **M1** – $\mathbf{b} \cdot \mathbf{b} = \lvert\mathbf{b}\rvert^2$ **A1** **R1** – dot prod 0, perpendicular **AG**

Qn	Suggested solution	Markscheme
3	***Counting Techniques***	**[Marks: 5]**
(a)	(choose female P/VP **and** choose male P/VP **and** choose other 4 EXCO members) $2\left({}^{4}C_{1}\times{}^{4}C_{1}\right)\times{}^{28}C_{4}$ $=655200$	**M1 M1** – combination & multiplication principle **A1** (Accept 655000)
(b)	(arrange P/VP in middle two seats **and** arrange other 4 EXCO members in remaining seats) $(2!)(4!)=48$	**M1** – arrangement **A1**
4	***Sum and Product of Roots***	**[Marks: 5]**
(a)	$P(x)=(x^3+3)(x^4-6x^3+x-10)$ $A=-\sum\text{roots}=-(0+6)=-6$	**(M1)A1**
(b)	$\prod\text{roots}=(-1)^7G=-(-30)=30$	**M1A1**
(c)	$\prod\text{roots}=-(G-1)=31$	**A1**
5	***Arithmetic sequence & series (with horizontal scaling***	**[Marks: 8]**
(a)	$k=\dfrac{2}{\pi}$	**A1**
(b)	**<u>Method 1</u>**  From graph, $x_1=3$ and $x_2=7$ $a=7-3=4$ **<u>Method 2</u>** Period of $y=\dfrac{2}{\pi}\sin(x)$ is 2π	**M1** **A1** – consecutive min. pts. **A1** **M1** – use period of graphs

Qn	Suggested solution	Markscheme
	Period of $y=f\left(x\right)=2\pi\times\dfrac{2}{\pi}=4$ $\therefore a=4$ **<u>Method 3</u>** $x_{n+1}-x_n=2\times$difference in x values when $f(x)$=0 $\sin\left(\dfrac{\pi}{2}x\right)=0\Rightarrow\dfrac{\pi}{2}x=\pi,2\pi,\ldots$ $x_{n+1}-x_n=2\times\left(4-2\right)=4$	**M1** – scaling of period **A1** **M1** – use of x-intercepts **M1** **A1**
(c)	$\sum\limits_{n=1}^{N}x_n=\sum\limits_{n=1}^{N}\left[3+4\left(n-1\right)\right]=\dfrac{N}{2}\left[6+4\left(N-1\right)\right]$ **<u>Method 1</u>** By GDC Graph $y=\sum\limits_{n=1}^{x}\left[3+4\left(n-1\right)\right]$ and Table, $\sum\limits_{n=1}^{15}x_n=465<500$ $\sum\limits_{n=1}^{16}x_n=528>500$ Least $N=16$  **<u>Method 2</u>** By GDC Graph $y=\dfrac{x}{2}\left[6+4\left(x-1\right)\right]-500$  $N>15.563$ Least $N=16$	**M1 A1** – AP with first term 3 and common diff. 4 Follow through from part (a) **R1** **A1** **M1** **A1**

Qn	Suggested solution	Markscheme
6	***Domain, Area between Curves and Volume of Solid of revolution***	**[Marks: 6]**
(a)	For $f(x)$ to be defined, $x^2-7\geq 0$ $(x+\sqrt{7})(x-\sqrt{7})\geq 0$ $x\leq -\sqrt{7}$ or $x\geq \sqrt{7}$ Smallest $a=\sqrt{7}$	**M1** Allow $x\geq \pm\sqrt{7}$ **A1**
(b)	$y=x+\sqrt{x^2-7}$ $(y-x)^2=x^2-7$ $x=\dfrac{y^2+7}{2y}$ Vol. of solid $=\pi\int_{\sqrt{7}}^{7}\left(\dfrac{y^2+7}{2y}\right)^2 \mathrm{d}y$ $=142$ units^3 (3 sf) $\pi\cdot\int_{\sqrt{7}}^{7}\left(\dfrac{y^2+7}{2\cdot y}\right)^2 \mathrm{d}y$ 141.874	**M1** – Squaring **M1** – making x the subject **AG** **M1** – with square **A1** – follow through only for incorrect positive value of a from part (a)
7	***Area of sector + Roots of a complex number***	**[Marks: 11]**
(a)	$\theta=\dfrac{2\pi}{5}$	**A1**
(b)	$\dfrac{1}{2}r^2\left(\dfrac{2\pi}{5}\right)=5\pi$ $r^2=25$ Since $r>0$, $r=5$	**M1** **(A1)** **A1**
(c)i)	$z^5=\dfrac{k}{\sqrt{2}}(1+\mathrm{i})$ $=k\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{4}+2n\pi\right)}$ where $n=-2,-1,0,1,2$ $z=k^{\frac{1}{5}}\mathrm{e}^{\mathrm{i}\left(\frac{8n+1}{20}\right)\pi}$ $=k^{\frac{1}{5}}\mathrm{e}^{-\mathrm{i}\frac{3\pi}{4}},\ k^{\frac{1}{5}}\mathrm{e}^{-\mathrm{i}\frac{7\pi}{20}},\ k^{\frac{1}{5}}\mathrm{e}^{\mathrm{i}\frac{\pi}{20}},\ k^{\frac{1}{5}}\mathrm{e}^{\mathrm{i}\frac{9\pi}{20}},\ k^{\frac{1}{5}}\mathrm{e}^{\mathrm{i}\frac{17\pi}{20}}$	**M1 A1** – for $k\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ **M1** **A2, 1** (3 correct)**, 0**
(c)ii)	$k^{\frac{1}{5}}=r=5$ $k=5^5=3125$	**M1** **A1**

Qn	Suggested solution	Markscheme
8	***Sketch rational function (oblique asymptote) + Stationary points***	**[Marks: 10]**
(a)	$y=\dfrac{2x^2-7x+4}{x-3}$ $\quad=2x-1+\dfrac{1}{x-3}$ 	**(M1)** – finding oblique asymptote **A1** – shape (asymptotic behaviour), 2 branches **A1** – $y=2x-1$ (O.A.) **A1** – $x=3$ (V.A.) **A1** – Min. $(2.29, 2.17)$ **A1** – Max. $(3.71, 7.83)$
(b)	<u>**Method 1:**</u> $y=\dfrac{2x^2-7x+4}{x-3}=2x-1+\dfrac{1}{x-3}$ $\dfrac{dy}{dx}=2-\dfrac{1}{(x-3)^2}$ o.e. At stationary points, $\dfrac{dy}{dx}=0$: $\dfrac{1}{(x-3)^2}=2$ $x=3\pm\dfrac{1}{\sqrt{2}}$ x-coordinate of mid-point $=\dfrac{1}{2}\left[\left(3+\dfrac{1}{\sqrt{2}}\right)+\left(3-\dfrac{1}{\sqrt{2}}\right)\right]$ $=3$ Therefore, mid-point of the turning points of the graph lie on its vertical asymptote $x=3$ <u>**Method 2:**</u>	**A1** **M1** **A1** **M1** **AG** **A1**

Qn	Suggested solution	Markscheme
	$y=\dfrac{2x^2-7x+4}{x-3}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2x^2-12x+17}{(x-3)^2}$	**M1**
	At stationary points, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$: $\dfrac{2x^2-12x+17}{(x-3)^2}=0$ $2x^2-12x+17=0$	**A1**
	Sum of roots $=-\dfrac{-12}{2}=6$	**M1**
	x-coordinate of mid-point$=\dfrac{6}{2}=3$ Therefore, mid-point of the turning points of the graph lie on its vertical asymptote $x=3$	**AG**

Section B

Qn	Suggested solution	Markscheme
9	***Binomial Distribution + Discrete Random Variables***	**[Marks: 19]**

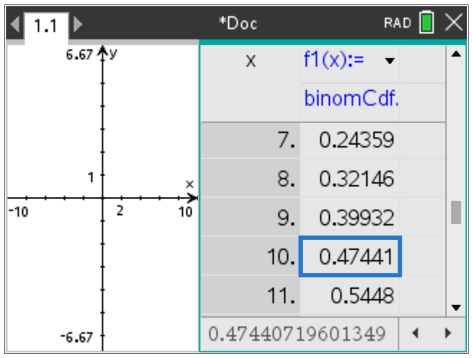
(a)

(i)

$$X \sim B\left(n, \frac{1}{4}\right) \Longrightarrow E(X) = \frac{n}{4}$$

M1 – evidence of 'np'
A1

(ii)



M1 – valid attempt, e.g., use of "binomCdf" or "binomInvN" <u>with correct parameters</u>

$n = 10 \Longrightarrow P(X \geq 3) < 0.5$
$n = 11 \Longrightarrow P(X \geq 3) > 0.5$

R1 – some comparison of probabilities must be seen

Therefore, $n = 10$.

A1

(b)

	0	**1**	**1**	**2**
0	0	1	1	2
1	1	2	2	3
1	1	2	2	3
2	2	3	3	4

t	0	1	2	3	4
$P(T = t)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

A1 – p_1
A1 – p_2
A1 – p_3

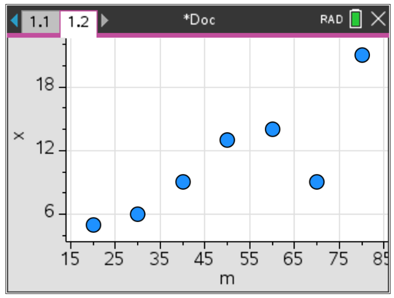
(c)

$S =$	**0**	**1**	**1**	**2**
0	0	1	1	2
1	1	2	2	3
1	1	2	2	3
2	2	3	3	4

M1 – sample space

$$P(T \geq 2S) = \frac{4 + 3 + 3 + 1}{16} = \frac{11}{16}$$

A1

<table>
<tr><th>Qn</th><th>Suggested solution</th><th>Markscheme</th></tr>
<tr><td>(d)</td><td>


$\Longrightarrow m = 70$</td><td>A1 – scatter plot must be seen

OR

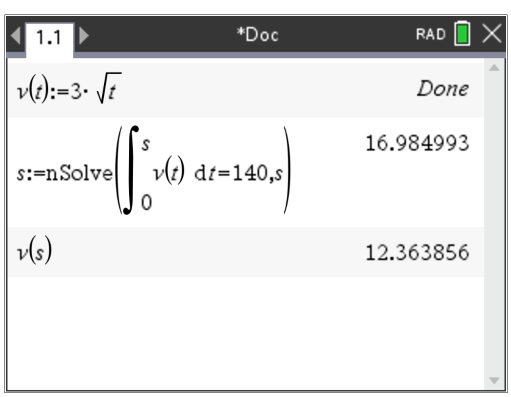
R1 – logical argument must be seen

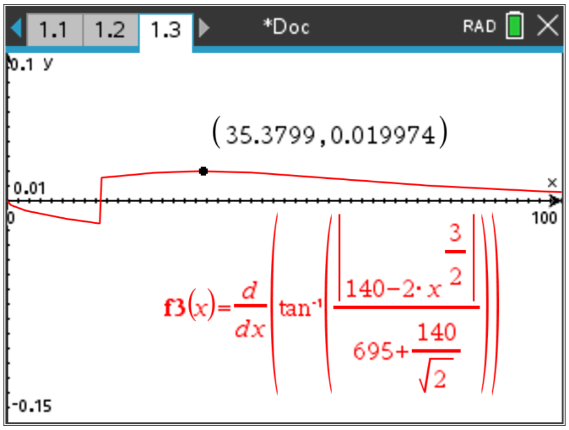
A1</td></tr>
<tr><td>(e)</td><td>$\overline{m} = \dfrac{20 + 30 + \cdots + 80}{7} = 50$
$\overline{x} = \dfrac{5 + 6 + 9 + 13 + 14 + x + 21}{7} = \dfrac{x + 68}{7}$

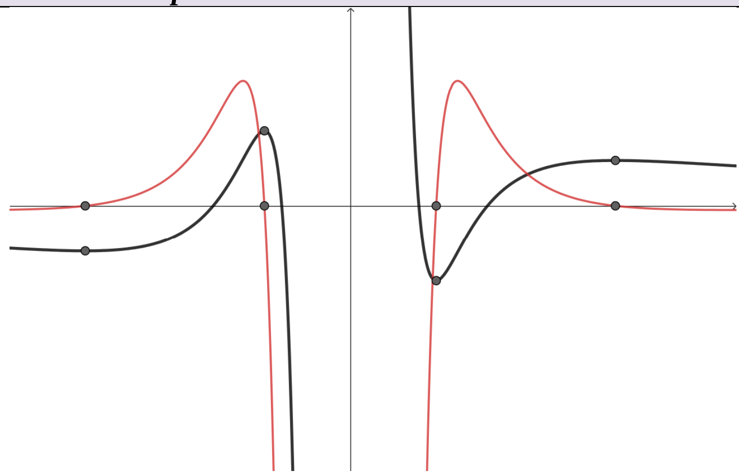
$\Longrightarrow 0.2834 \times 50 - 1.741 = \dfrac{x + 68}{7} \Longrightarrow x = 19.003 \approx 19$</td><td>A1
A1

M1
A1 – 19 (integer)</td></tr>
<tr><td>(f)</td><td>$r = 0.989$

Note: Allow FT2 for $r = 0.991$ ($x = 18$).</td><td>A2</td></tr>
<tr><td>(g)</td><td>The results are consistent (high r) and $\frac{x}{m} \approx 0.25$</td><td>A1</td></tr>
</table>

Qn	Suggested solution	Markscheme
10	***3D Geometry + Applications of Trigonometry***	**[Marks: 19]**
(a)	Let h be the height of the pyramid. Consider a triangle with height h, hypotenuse s and base $\frac{\sqrt{2}}{2}s$. Thus, $$s^2 = h^2 + \frac{1}{2}s^2 \Longrightarrow h = \frac{1}{\sqrt{2}}s$$	**(M1)** – correct triangle considered **A1** – correct use of Pythagoras'
(b)	Let θ be the angle between a triangular face and the base. Consider a triangle with height h, base $\frac{1}{2}s$ and hypotenuse connecting the midpoint of the base edge of the pyramid and its vertex. Thus, $$\tan\theta = \frac{\frac{1}{\sqrt{2}}s}{\frac{1}{2}s} = \sqrt{2} \Longrightarrow \theta \approx 0.955 \text{ rad or } 54.7^\circ$$	**(M1)** – correct triangle considered **M1A1**
(c)	Consider a triangle with height h, base $d + \frac{s}{2}$. Here the hypotenuse is from M to the vertex. $$h = 140 \Longrightarrow \frac{s}{2} = \frac{1}{\sqrt{2}}h = \frac{140}{\sqrt{2}} \approx 98.995 \text{ m}$$ Consider a triangle with height h, base $d + \frac{s}{2}$ and angle of elevation 10°. $$\Longrightarrow \tan 10^\circ = \frac{140}{d + \frac{140}{\sqrt{2}}}$$ $$\Longrightarrow d = 694.98 \approx 695.$$	**(M1)** – correct base **(A1)** **M1** – use of correct trigo ratio, eg. tan **A1** – valid intermediate step/s to find d **AG**
(d)	The drone's vertical distance from the ground is given by $$s(t) = \int v(t)\,\mathrm{d}t = 2t^{3/2} + C$$ And since $s(0) = 140$, we get $s(t) = 2t^{3/2}$. $s(t) = 0 \Longrightarrow t = 16.985$  Therefore, $v(16.985) = 12.4$ m/s.	**(M1)A1** – condone missing $+C$ **(A1)** – do not award if $C = 0$ was not considered **(M1)** **A1** – 16.985 **A1**

Qn	Suggested solution	Markscheme
(e)	Let α be the angle of elevation or depression. (Note: elevation when $s(t) < 140$ and depression when $s(t) > 140$.) Thus, $$\tan\alpha = \frac{\lvert 140 - 2t^{3/2}\rvert}{695 + \frac{140}{\sqrt{2}}} \Rightarrow \alpha = \arctan\frac{\lvert 140 - 2t^{3/2}\rvert}{695 + \frac{140}{\sqrt{2}}}$$	**M1A1**
	 	**R1 – considering α'**
	Therefore, the angle of depression is increasing the greatest when $t = 35.4$ seconds.	**A1**

<table>
<tr><th>Qn</th><th>Suggested solution</th><th>Markscheme</th></tr>
<tr><td>11</td><td>Derivative Graphs</td><td>[Marks: 17]</td></tr>
<tr><td>(a)</td><td></td><td>A1 – zeros at $\pm4, \pm\frac{5}{4}$

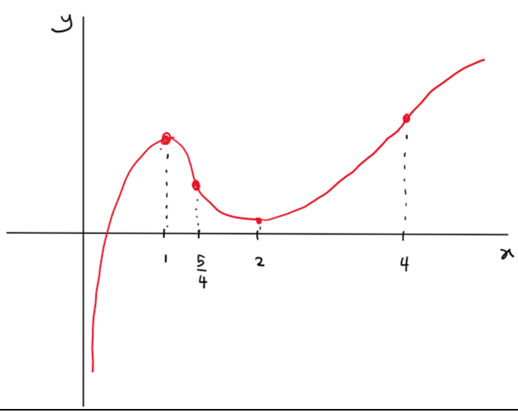
A1 – left branch + correct shape (asymptotic to $y = 0$)

A1 – right branch + correct shape (asymptotic to $y = 0$)</td></tr>
<tr><td>(b)</td><td>@ $x = \pm2 \implies$ minimum
@ $x = \pm1 \implies$ maximum

Award A1A0A0A0 if none are correct but all four x-coordinates are given.</td><td>A1A1
A1A1</td></tr>
<tr><td>(c)</td><td>4, as $y = f'(x)$ has 4 turning points.

Note: $f'' = 0$ is not enough reasoning.</td><td>A1R1</td></tr>
<tr><td>(d)</td><td>(i)
increasing $\implies (0,1] \cup [2,\infty)$

(ii)
concave down $\implies g'$ is decreasing $\implies \left(0,\frac{5}{4}\right) \cup (4,\infty)$</td><td>A1
A1 – condone the exclusion of 1 and/or 2

(M1) – considering the behaviour of g'
A1A1 – strictly exclusive of the boundaries</td></tr>
<tr><td>(e)</td><td>$y = g(x)$
</td><td>A1 – correct shape (eg concavity) from 0 to $\frac{5}{4}$.

A1 – correct shape from $\frac{5}{4}$ to 4

A1 – correct shape from 4 onwards</td></tr>
</table>