

Name: _____

Class: _____



JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2026

MATHEMATICS
Higher 2

9758/01

2 September 2026

Paper 1

3 hours

Additional materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given by [] at the end of each question or part question.

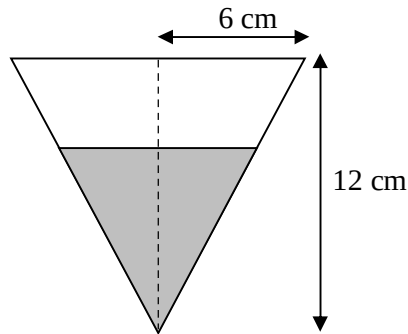
- 1 The curve C has equation $y = |1 + 2 \cos x|$, $0 \leq x \leq 2\pi$.

Sketch the graph of C , stating the exact coordinates of the turning point(s) and where the curve meets the axes. [3]

Hence find the set of values of k for which the equation $|1 + 2 \cos x| = k$ has at least 3 distinct roots in the interval $0 \leq x \leq 2\pi$. [2]

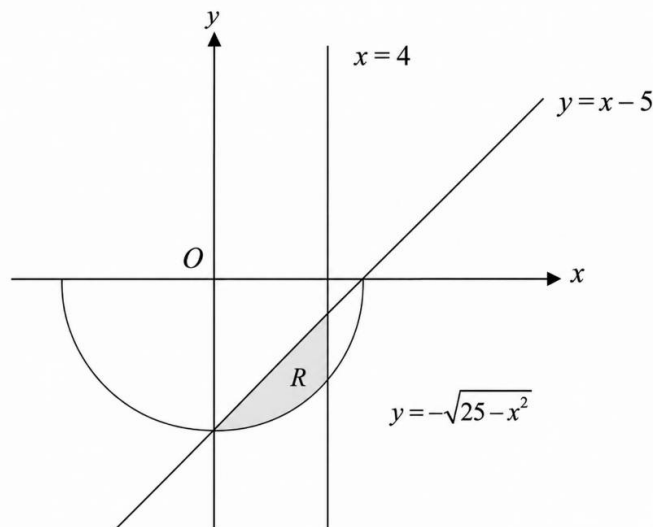
- 2 Water is draining from an inverted right circular conical container with its vertex at the bottom. The container has a height of 12 cm and a base radius of 6 cm. Suppose water drains at a constant rate of $2 \text{ cm}^3/\text{min}$, find the rate at which the water depth h is decreasing when $h = 4 \text{ cm}$. [4]

[It is given that the volume of a cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$.]



- 3 The diagram shows the region R which is bounded by the curve $y = -\sqrt{25 - x^2}$, the lines $y = x - 5$ and $x = 4$.

- (i) Calculate the area of R . [2]
 (ii) Find the volume of the solid of revolution obtained when R is rotated completely about the y -axis. [3]



- 4 A solid cylinder has radius r cm, height h cm and total surface area 1500 cm^2 . Find the maximum volume of the cylinder in the form $a\sqrt{\frac{10}{\pi}} \text{ cm}^3$, where a is a constant to be determined. [5]

- 5 (i) A cartesian equation of a plane is given by $ax + by + cz = d$, where a , b , c and d are constants.

(a) State the significance of the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in relation to the plane. [1]

(b) If $a^2 + b^2 + c^2 = 1$ and $d > 0$, state what d represents. [1]

- (ii) (a) The non-zero vectors $\mathbf{u}$ and $\mathbf{v}$ are such that $(\mathbf{u} - \mathbf{v}) \times (\mathbf{u} + \mathbf{v}) = \mathbf{0}$. Determine the geometric relationship between $\mathbf{u}$ and $\mathbf{v}$. [3]

(b) If $\mathbf{u}$ and $\mathbf{v}$ are unit vectors which are perpendicular, state the magnitude of $(\mathbf{u} - \mathbf{v}) \times (\mathbf{u} + \mathbf{v})$. [1]

- 6 It is given that $-2 + 3i$ is a root of the equation $z^3 + az^2 - 7z + b = 0$, where a and b are real constants.

(a) Find the values of a and b and the other two roots of the equation. [5]

(b) Represent all the three roots on an Argand diagram. [2]

(c) Hence find the area of triangle PQR , where P , Q and R are points that represent the roots. [1]

- 7 (a) A sequence $v_1, v_2, v_3, \dots$ is such that $20v_{n+1} = 17v_n + 14$, where $n > 0$ and $v_1 = 3$.

(i) Find the value of v_{15} , giving your answer to 3 decimal places. [1]

(ii) Given that as $n \rightarrow \infty$, $v_n \rightarrow L$, find the exact value of L . [2]

- (b) It is given that $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$.

Find in terms of n , the sum of the series

(i) $1^2 + 2^2 + 3^2 + \dots + 4n^2$, [1]

(ii) $(n+1)^2 + (n+2)^2 + \dots + 4n^2$. [3]

By using your answer to (b)(i), find in terms of n , the sum of the series

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2. \quad [2]$$

8 (a) Find

(i) $\int \cos^2 2x \, dx,$ [2]

(ii) $\int \frac{xe^{x^2}}{\sqrt{e^{x^2} + 2}} \, dx,$ [2]

(iii) $\int x \tan^{-1}(x^2) \, dx.$ [4]

(b) By using the substitution $x = 4 \cos^2 \theta$, find the exact value of $\int_2^3 \frac{1}{x\sqrt{4-x}} \, dx.$ [5]

9 The curve C is defined by the parametric equations

$$x = 3t^2 - 4t + \frac{1}{2}, \quad y = 3t^3 - \frac{9}{2}t^2 + 2t + \frac{49}{18}, \quad \text{for } 0 < t < 1, \, t \neq \frac{2}{3}.$$

(i) Find $\frac{dy}{dx}$ in terms of t . [2]

(ii) Find the coordinates of the stationary point Q . Hence write down the equation of the tangent to C at Q . [3]

(iii) The tangent at Q cuts C again at the point R . Find the equation of the normal to C at R . [4]

(iv) The normal at R cuts the y -axis at the point N . Find the area of the triangle QRN . [2]

10 A line L_1 has cartesian equation given by $x - 4 = -z, y = 1$. Another line L_2 is parallel to $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$

and passes through the point $A(4, 3, c)$, where c is a constant.

(a) Find the acute angle between L_1 and L_2 . [3]

(b) Determine the possible values of c if L_1 and L_2 are skew. [3]

(c) Show that the perpendicular distance from A to L_1 is $\sqrt{\frac{1}{2}(c^2 + 8)}$. [2]

A plane Π has equation given by $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 0$.

(d) Find the value of c if A is equidistant from L_1 and Π . [3]

- 11 (i) The graphs of $y = |f(x)|$ and $y = f'(x)$, where $f'(x) = \frac{d}{dx}[f(x)]$, are shown in Figure 1 and Figure 2 respectively.

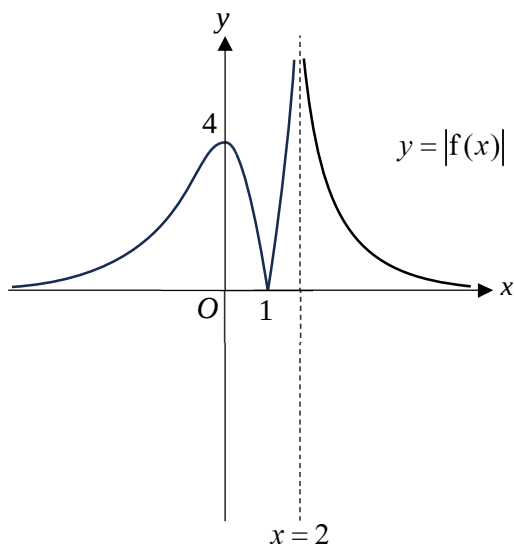


Figure 1

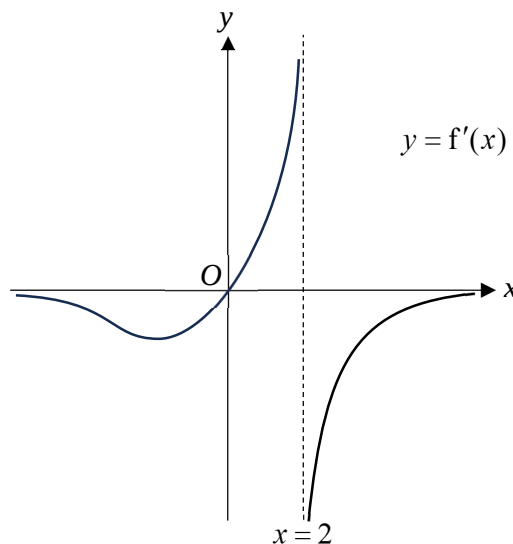


Figure 2

On separate diagrams, sketch the graphs of

(a) $y = f(x)$, [3]

(b) $y = \frac{1}{|f(x)|}$, [4]

showing clearly the coordinates of any axial intercepts, stationary points, and the equations of any asymptotes.

- (ii) Each of the following two sequences of transformations can be used to transform the graph of $y = g(x)$ to the graph of $y = g(3x+1)$:

Sequence A:

A translation of a unit(s) in the negative direction of the x -axis, followed by a scaling parallel to the x -axis by a scale factor of b .

Sequence B:

A scaling parallel to the x -axis by a scale factor of c , followed by a translation of d unit(s) in the negative direction of the x -axis.

Determine the values of a , b , c and d . [4]

- 12** The population growth of a certain species of fish in a farm is studied. The number of fish, t days after the study begins, is denoted by x (in thousands). It is found that the birth rate per day is twice of x and the death rate per day is proportional to x^2 .

(i) If there is no change in the population of fish when its population hits 8000, show that the

differential equation relating x and t is given by $\frac{dx}{dt} = 2x - \frac{x^2}{4}$. [2]

(ii) The owner of the farm decides to sell 1750 fish daily. Modify the differential equation in part **(i)** to model the new situation. Hence, show that the resulting differential equation

can be written as $\frac{dx}{dt} = -\frac{1}{4}[(x-4)^2 - a^2]$, where a is a constant to be determined. [2]

(iii) It is known that the initial population of the fish is 15 000. Solve the differential equation found in part **(ii)**, expressing x in terms of t . [5]

(iv) Hence sketch the part of the curve which is relevant in this context. Explain what happens to the number of fish in the farm in the long run. [3]

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