Topic: Trigonometry

Question 1 [2008 EOY, Q5]	marks
Solve for $0^\circ \le x < 360^\circ$ when $2\cos(2x-40^\circ)+1=0$. **Solution:** $2\cos(2x-40^\circ)+1=0 \Rightarrow \cos(2x-40^\circ)=-\frac{1}{2}$ basic angle is 60° and $2x-40^\circ$ is in the 2nd and 3rd quadrants $-40^\circ \le 2x-40^\circ < 680^\circ$ so $2x-40^\circ = 120^\circ,\ 240^\circ,\ 480^\circ,\ 600^\circ$ Hence $x=80^\circ,\ 140^\circ,\ 260^\circ,\ 320^\circ$	[5]

Question 2 [2008 EOY, Q8]	marks
Prove the identity $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}\equiv 2\operatorname{cosec} x$. **Solution:** LHS $=\frac{\sin^2 x+(1-\cos x)^2}{\sin x(1-\cos x)}=\frac{\sin^2 x+1-2\cos x+\cos^2 x}{\sin x(1-\cos x)}$ $=\frac{2-2\cos x}{\sin x(1-\cos x)}$ $=\frac{2(1-\cos x)}{\sin x(1-\cos x)}$ $=\frac{2}{\sin x}$ $=2\operatorname{cosec} x$ = RHS	[3]

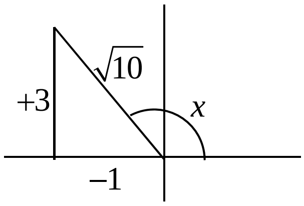
Question 3 [2008 EOY, Q16] — marks

Given that $\tan x = -3$ and $\sin x$ is positive, without using the calculator, find the exact values of

(i) $\cos x + \sin x$, **[2]**

Solution:

x is in 2nd quadrant so



$$\cos x + \sin x = -\frac{1}{\sqrt{10}} + \frac{3}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

(ii) $\sec x - \operatorname{cosec} x$. **[2]**

Solution:

$$\sec x - \operatorname{cosec} x = \frac{1}{\cos x} - \frac{1}{\sin x} = -\sqrt{10} - \frac{\sqrt{10}}{3} = -\frac{4\sqrt{10}}{3}$$

Question 4 [2009 EOY, Q3] — marks

Prove the identity $\dfrac{2\cot\theta + \operatorname{cosec}^2\theta}{(\sin\theta + \cos\theta)^2} \equiv \operatorname{cosec}^2\theta$. **[4]**

Solution:

$$\text{LHS} = \frac{\dfrac{2\cos\theta}{\sin\theta} + \dfrac{1}{\sin^2\theta}}{1 + 2\sin\theta\cos\theta}$$

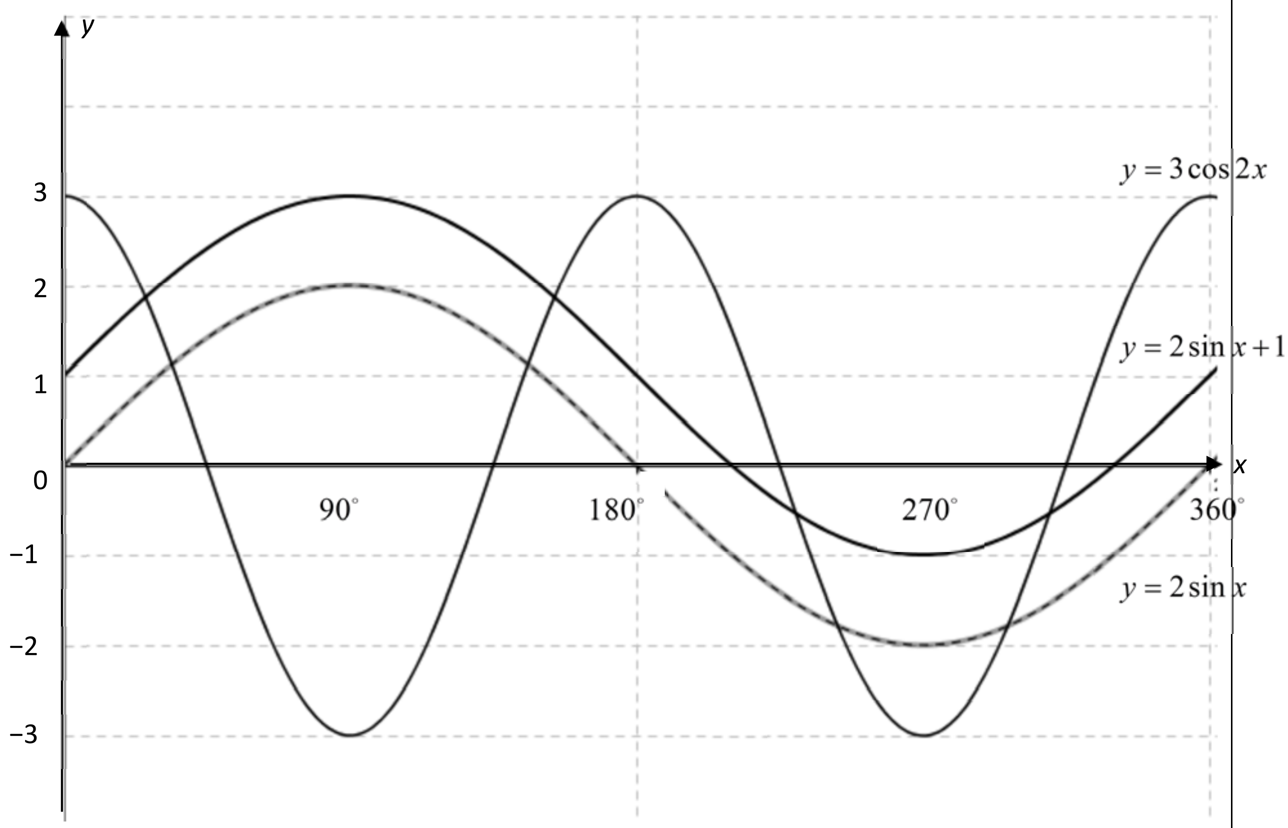
$$= \frac{\dfrac{2\cos\theta\sin\theta + 1}{\sin^2\theta}}{1 + 2\sin\theta\cos\theta}$$

$$= \frac{1}{\sin^2\theta}$$

$$= \operatorname{cosec}^2\theta = \text{RHS (proven)}$$

Question 5 [2009 EOY, Q5]	marks
(a) Given that $\cos A = -\frac{2}{3}$ and $180^\circ < A < 360^\circ$. Without solving for angle *A*, find the value of (i) $\cot A$, **Solution:** $\cot A = \frac{1}{\tan A}$ $= \frac{1}{\frac{\sqrt{5}}{2}}$ $= \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$	[2]
(ii) $\text{cosec } (180^\circ + A)$. **Solution:** $\text{cosec } (180^\circ + A) = \frac{1}{\sin(180^\circ + A)}$ $= \frac{1}{\frac{\sqrt{5}}{3}}$ $= \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$	[2]
(b) Solve $\tan(2x - 65^o) = -1$ for $0^\circ \le x \le 360^\circ$. **Solution:** $BA = 45^\circ$ $65^\circ \le 2x - 65^\circ \le 655^\circ$ $2x - 65^\circ = 135^\circ, 315^\circ, 495^\circ, -45^\circ$ $x = 10^\circ, 100^\circ, 190^\circ, 280^\circ$	[4]

Question 6 [2009 EOY, Q12]	marks
(i) Show that ($2\sin x - 1$) is a factor of the expression $6\sin^3 x + \sin^2 x - 4\sin x + 1$. **<u>Solution:</u>** $f(\sin x) = 6\sin^3 x + \sin^2 x - 4\sin x + 1$ $f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 0$ $\therefore$ By Factor Theorem, $(2\sin x - 1)$ is a factor.	[2]
(ii) Factorise the expression $6\sin^3 x + \sin^2 x - 4\sin x + 1$ completely. **<u>Solution:</u>** $6\sin^3 x + \sin^2 x - 4\sin x + 1 = (2\sin x - 1)(A\sin^2 x + B\sin x + C)$ By comparing coefficients, *A* = 3, *B* = 2, *C* = −1 $(2\sin x - 1)(3\sin^2 x + 2\sin x - 1) = (2\sin x - 1)(3\sin x - 1)(\sin x + 1)$ **OR** $f\left(\frac{1}{3}\right) = 0, \quad f(1) = 0$ $\therefore f(\sin x) = 6(\sin x - \frac{1}{2})(\sin x - \frac{1}{3})(\sin x + 1)$	[3]
(iii) Hence find all values of *x* between 0° and 360° inclusive for which $6\sin^3 x + \sin^2 x - 4\sin x + 1 = 0$. **<u>Solution:</u>** $(2\sin x - 1)(3\sin x - 1)(\sin x + 1) = 0$ $\sin x = \frac{1}{2}$, $BA = 30^\circ$, $x = 30^\circ, 150^\circ$ $\sin x = \frac{1}{3}$, $BA = 19.47^\circ$, $x = 19.5^\circ, 160.5^\circ$ $s\,in\, x = -1$, $BA = 90^\circ$, $x = 270^\circ$ *x* = 19.5°, 30°, 160.5°, 150°, 270°.	[4]

Question 7 [2009 EOY, Q13]	marks
(i) Sketch, on the same diagram, the graphs of $y = 2\sin x + 1$ and $y = 3\cos 2x$ for the domain $0^\circ \le x \le 360^\circ$. **Solution:** 	
(ii) Hence, deduce the smallest possible positive value of *m* for which the equation $2\sin x + 1 = 3\cos 2x + m$ has 3 solutions. **Solution:** *m* = 2	[1]

Question 8 [2010 EOY, Q10]	marks
(i) Solve, for $0° \le x \le 180°$, the equation $2\cos 2x = 3\cos 2x \sin^2 2x$.	[4]

Solution:

$$\cos 2x(3\sin^2 2x - 2) = 0 \Rightarrow \cos 2x = 0 \text{ or } \sin 2x = \pm\sqrt{\frac{2}{3}}$$

For $\cos 2x = 0$, $x = 45°, 135°$

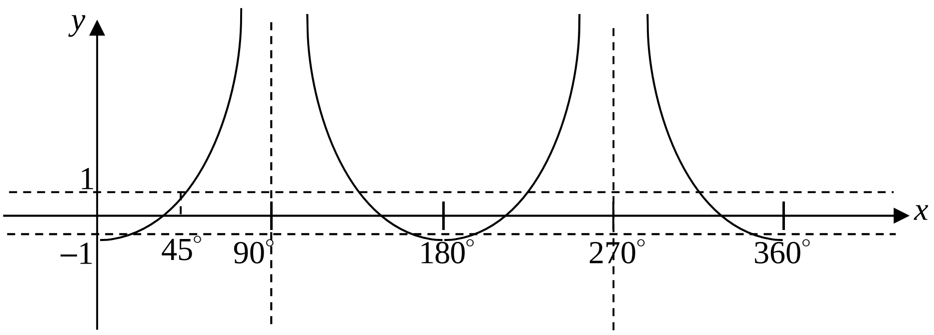
For $\sin 2x = \pm\sqrt{\frac{2}{3}}$, basic angle is $54.74°$ hence

$2x = 54.74°, 125.26°, 234.74°, 305.26°$

$\Rightarrow x = 27.4°, 62.6°, 117.4°, 152.6°$

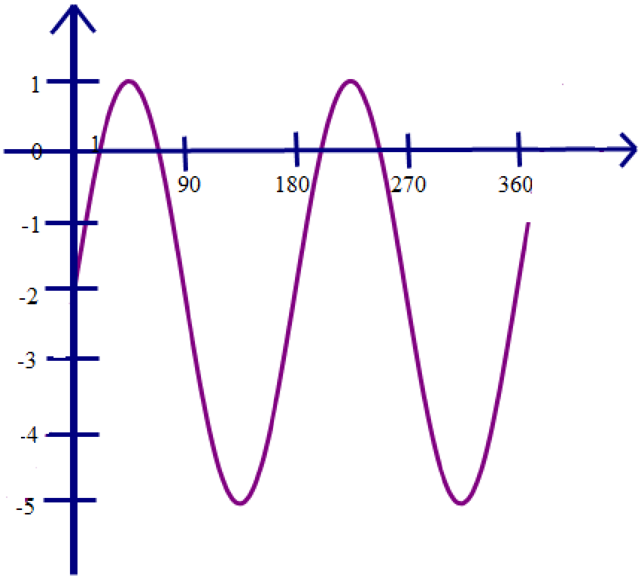
(ii)	
Sketch the graph of $y = 2\lvert\tan x\rvert - 1$ for $0° \le x \le 360°$.	[3]

Solution:



Question 9 [2010 EOY, Q11]	marks
Prove the identity $\dfrac{1-\operatorname{cosec} x}{\tan x - \sec x} \equiv \cot x$. **Solution:** $\dfrac{1-\operatorname{cosec} x}{\tan x - \sec x} \equiv \left(1-\dfrac{1}{\sin x}\right) \div \left(\dfrac{\sin x}{\cos x} - \dfrac{1}{\cos x}\right)$ $\equiv \dfrac{\sin x - 1}{\sin x} \times \dfrac{\cos x}{\sin x - 1}$ $\equiv \dfrac{\cos x}{\sin x}$ $\equiv \cot x$	[3]

Question 10 [2010 EOY, Q16]	marks
Simplify $\dfrac{\sin(-135°)}{\sec 240°} + \dfrac{\tan 200° \tan 300°}{\cot(-290°)}$, leaving your answer in exact form. **Solution:** $\dfrac{\sin(-135°)}{\sec 240°} + \dfrac{\tan 200° \tan 300°}{\cot(-290°)}$ $= -\sin 135° \cos 240° - \tan 200° \tan 300° \tan 290°$ $= -\sin 45°(-\cos 60°) - \tan 20°(-\tan 60°)(-\tan 70°)$ $= \dfrac{1}{\sqrt{2}}\left(\dfrac{1}{2}\right) - \sqrt{3} \tan 20°(\tan 70°)$ $= \dfrac{1}{2\sqrt{2}} - \sqrt{3}$ or $\dfrac{1-2\sqrt{6}}{2\sqrt{2}}$	[3]

Question 11 [2011 EOY, Q2]	marks
Sketch the graph of $y = 3\sin 2x - 2$ for $0^\circ < x < 360^\circ$, **<u>Solution:</u>** Correct Amplitude i.e. 3 Correct no of cycles (2 cycles) Graph shifted down by 2 units 	[3]

Question 12 [2011 EOY, Q9]	marks
Solve the following equations for angles between 0° and 360°. (i) $\cos x = 4\sin^2 x\cos x$, **<u>Solution:</u>** $\cos x\left(4\sin^2 x - 1\right) = 0$ $\cos x = 0 \;\; or \;\; \sin x = \pm\frac{1}{2}$ $\cos x = 0 \Rightarrow 90^\circ, 270^\circ$ $\sin x = \pm\frac{1}{2} \Rightarrow 30^\circ, 150^\circ, 210^\circ, 330^\circ$ (Without $\cos x = 0$, 2 marks deducted. Basic $\angle$ cannot be $0^\circ, 90^\circ, 180^\circ$.)	[5]

(ii)

$\tan\left(2y-85^\circ\right)=-5$. [4]

Solution:

$\tan\left(2y-85^0\right)=-5$

$B.A.=78.69^0$

$2y-85^0$

$=-78.69^0, 101.31^0, 281.31^0, 461.31^0$

$y=3.2^0, 93.2^0, 183.2^0, 273.2^0$

Question 13 [2011 EOY, Q12] marks

(i)

Given that $\dfrac{2\cos^2 x}{1+3\sin^2 x}=\dfrac{8}{13}$, where $90^\circ < x < 180^\circ$. Without using a calculator, find the value of $\dfrac{2\cos x}{1+3\sin x}$. [4]

Solution:

$$\frac{2\cos^2 x}{1+3\sin^2 x}=\frac{8}{13}$$

$$26\cos^2 x=8+24\sin^2 x$$

$$26\left(1-\sin^2 x\right)=8+24\sin^2 x$$

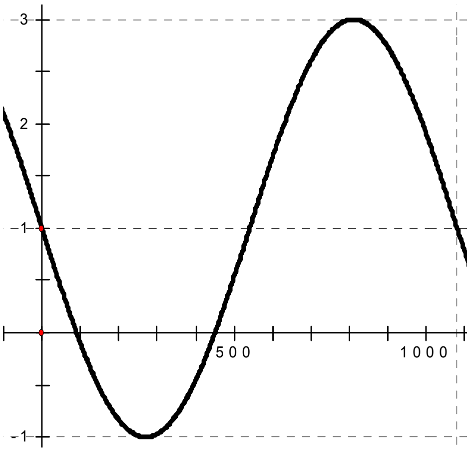
$$\sin^2 x=\frac{18}{50}=\frac{9}{25}$$

$$\sin x=\frac{3}{5}$$

$$\cos x=-\frac{4}{5}$$

$$\frac{2\cos x}{1+3\sin x}=\frac{2\left(-\frac{4}{5}\right)}{1+3\left(\frac{3}{5}\right)}=-\frac{4}{7}$$

(ii) Prove the identity $\dfrac{\cot\theta}{1+\cos\text{ec}\,\theta}-\dfrac{\cot\theta}{1-\cos\text{ec}\,\theta}\equiv 2\sec\theta$. **Solution:** $LHS=\dfrac{\cot\theta}{1+\cos ec\theta}-\dfrac{\cot\theta}{1-\cos ec\theta}$ $=\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta+1}{\sin\theta}}-\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta-1}{\sin\theta}}$ $=\dfrac{\cos\theta}{\sin\theta+1}-\dfrac{\cos\theta}{\sin\theta-1}$ $=\dfrac{\cos\theta\sin\theta-\cos\theta-\cos\theta\sin\theta-\cos\theta}{\sin^2\theta-1}$ $=\dfrac{-2\cos\theta}{-\cos^2\theta}$ $=2\sec\theta$ $=RHS$	[3]
Question 14 [2012 EOY, Q6]	marks
The function f is defined by $\mathrm{f}:x\mapsto 1-2\sin\left(\dfrac{x}{3}\right)$. (i) State the amplitude and the period of f. **Solution:** Amplitude = 2 Period = 1080^0	[2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le 1080^\circ$. **Solution:** 	[4]
Question 15 [2012 EOY, Q7]	marks
(i) Solve the equation $\tan\left(\frac{x}{2} - 50^\circ\right) + 1 = 0$ for angles between 0° and 360°. **Solution:** $\tan\left(\frac{x}{2} - 50^\circ\right) + 1 = 0$ Domain: $0^\circ < x < 360^\circ$ $\Rightarrow -50^\circ < \frac{x}{2} - 50^\circ < 130^\circ$ basic $\angle = 45^\circ$ $\frac{x}{2} - 50^\circ = -45^\circ$ $\therefore x = 10^\circ$	[4]

(ii) Given that $\dfrac{\cos x}{\tan x-1}+\dfrac{\sin x}{\cot x-1}=k\left(\sin x+\cos x\right)$ for all values of x, find the value of k. **<u>Solution:</u>** $=\dfrac{\cos^2 x}{\cos x\left(\tan x-1\right)}+\dfrac{\sin^2 x}{\sin x\left(\cot x-1\right)}$ $=\dfrac{\cos^2 x}{\sin x-\cos x}+\dfrac{\sin^2 x}{\cos x-\sin x}$ $=\dfrac{\cos^2 x-\sin^2 x}{\sin x-\cos x}$ $=\dfrac{\left(\cos x-\sin x\right)\left(\cos x+\sin x\right)}{\sin x-\cos x}$ $=-\left(\sin x+\cos x\right)$ $\therefore k=-1$	[3]
Question 16 [2013 EOY, Q2]	marks
Angles A and B are in different quadrants such that $\tan A=\dfrac{3}{4}$ and $\sin B=-\dfrac{4}{5}$. Find, without using calculator, the possible values of ($2\sin A-\tan B$) in the exact form. **<u>Solution:</u>** Given that $\tan A=\dfrac{3}{4}$ $\Rightarrow$ A is in the 1st & 3rd Q $\sin B=-\dfrac{4}{5}$ $\Rightarrow$ B is in the 3rd & 4th Q When A is in the 1^{st} Q, $\sin A=\dfrac{3}{5}$, When A is in the 3^{rd} Q, $\sin A=-\dfrac{3}{5}$. When B is in the 3^{rd} Q, $\tan B=\dfrac{4}{3}$, When B is in the 4^{th} Q, $\tan B=-\dfrac{4}{3}$. When A and B are in different quadrants : Case (i) A is in the 1^{st} Q and B is in the 3^{rd} Q, $2\sin A-\tan B=2(\dfrac{3}{5})-\dfrac{4}{3}=\dfrac{18-20}{15}=-\dfrac{2}{15}$ Case (ii) A is in the 1^{st} Q and B is in the 4^{th} Q,	[6]

$$2\sin A - \tan B = 2(\frac{3}{5}) - (-\frac{4}{3}) = \frac{18+20}{15} = 2\frac{8}{15}$$ **A1**

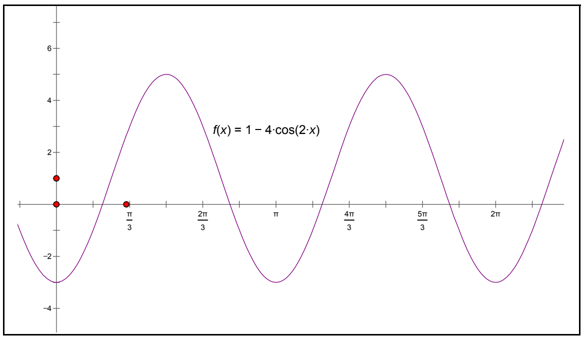
Case (iii) A is in the 3rd Q and B is in the 4th Q,

$$2\sin A - \tan B = 2(-\frac{3}{5}) - (-\frac{4}{3}) = \frac{-18+20}{15} = \frac{2}{15}$$ **A1**

Question 17 [2013 EOY, Q7] marks

Sketch the graph $y = 1 - 4\cos 2x$ for $0° \le x \le 360°$. [3]

Solution:



Question 18 [2013 EOY, Q8] marks

Solve the equations for $0° \le x \le 180°$.

(i)

$\sin 3x = \cos 2x$, [3]

Solution:

$$\sin 3x = \sin(90° - 2x)$$

$$\Rightarrow 3x = 90° - 2x,\ 180° - (90° - 2x),$$

$$360° + (90° - 2x),\ 720° + (90° - 2x)$$

$$\Rightarrow x = 18°,\ 90°,\ 90°,\ 162°$$

Hence $x = 18°,\ 90°,\ 162°$

(ii)

$\tan(2x - 40^\circ) = -2.718$. [3]

Solution:

(b) Let

$\tan(2x - 40°) = 2.718 \Rightarrow x = 69.800°$ (B.A.)

Hence $2x - 40° = 180° - 69.80°,$

$360° - 69.80°$

$\Rightarrow \quad x \simeq 75.1°, \quad 165.1°$

Question 19 [2013 EOY, Q9]	marks

Prove the identities.

(i)

$\sin^4 x + \sin^2 x \cos^2 x + \cos^2 x \equiv 1$, [4]

Solution:

$$\text{LHS} = \sin^2 x(\sin^2 x + \cos^2 x) + \cos^2 x$$
$$\equiv \sin^2 x + \cos^2 x \equiv 1 = \text{RHS}$$

Alternatively

$$\text{LHS} = (\sin^2 x)^2 + (1 - \cos^2 x)\cos^2 x + \cos^2 x$$
$$\equiv (1 - \cos^2 x)^2 + (1 - \cos^2 x)\cos^2 x + \cos^2 x$$
$$\equiv 1 - 2\cos^2 x + \cos^4 x + \cos^2 x$$
$$- \cos^4 x + \cos^2 x$$
$$\equiv 1 = \text{RHS}$$

(ii)

$$\frac{2-\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x+2\cot x} \equiv \frac{\sin x-\cos x}{\sin x+\cos x}.$$

[4]

Solution:

(b)

$$\text{LHS}=\frac{2-(1+\cot^2 x)}{1+\cot^2 x+2\cot x}$$

$$\equiv \frac{1-\cot^2 x}{(1+\cot x)^2}$$

$$\equiv \frac{1-\cot x}{1+\cot x}$$

$$\equiv \frac{1-\frac{\cos x}{\sin x}}{1+\frac{\cos x}{\sin x}} \equiv \frac{\sin x-\cos x}{\sin x+\cos x}=\text{RHS}$$

Alternatively

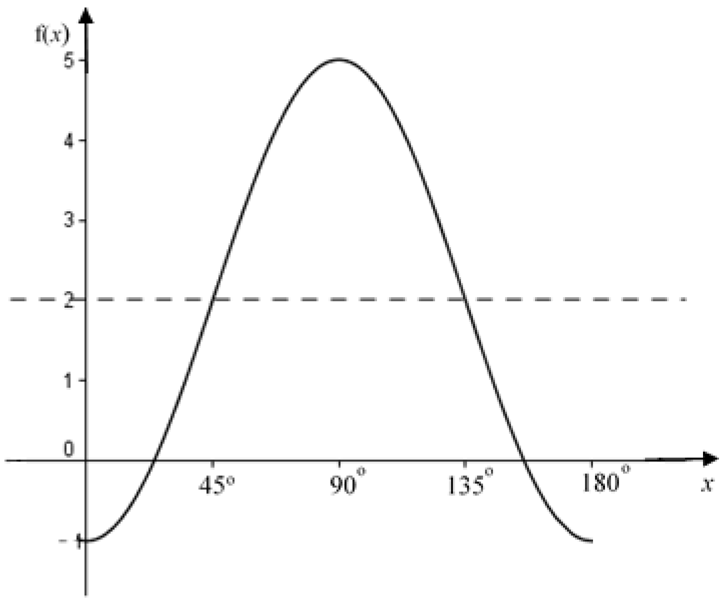
$$\text{LHS}=\frac{2-\frac{1}{\sin^2 x}}{\frac{1}{\sin^2 x}+\frac{2\cos x}{\sin x}} \equiv \frac{\frac{2\sin^2 x-1}{\sin^2 x}}{\frac{1+2\sin x\cos x}{\sin^2 x}}$$

$$\equiv \frac{\sin^2 x-\cos^2 x}{\sin^2 x+2\sin x\cos x+\cos^2 x}$$

$$\equiv \frac{(\sin x-\cos x)(\sin x+\cos x)}{(\sin x+\cos x)^2} \equiv \frac{\sin x-\cos x}{\sin x+\cos x}$$

$$=\text{RHS}$$

Question 20 [2014 EOY, Q9]	marks
(i) Solve the trigonometric equation $6-4\cos^2\left(\frac{x}{2}+30°\right)=9\sin\left(\frac{x}{2}+30°\right)$ for $0° \le x \le 720°$. **<u>Solution:</u>** $0° \le x \le 720°$ $\Rightarrow 30° \le \frac{x}{2}+30° \le 390°$ $6-4\cos^2\left(\frac{x}{2}+30°\right)=9\sin\left(\frac{x}{2}+30°\right)$ $6-4+4\sin^2\left(\frac{x}{2}+30°\right)=9\sin\left(\frac{x}{2}+30°\right)$ $4\sin^2\left(\frac{x}{2}+30°\right)-9\sin\left(\frac{x}{2}+30°\right)+2=0$ $\left[4\sin\left(\frac{x}{2}+30°\right)-1\right]\left[\sin\left(\frac{x}{2}+30°\right)-2\right]=0$ $\sin\left(\frac{x}{2}+30°\right)=\frac{1}{4}$ or 2 (rejected) $\left.\begin{array}{r}\text{B.A.} = 14.48°\\ \frac{x}{2}+30° = 165.52° \text{ or } 374.48°\end{array}\right\}$ $x = 271.0°$ or $689.0°$	[6]

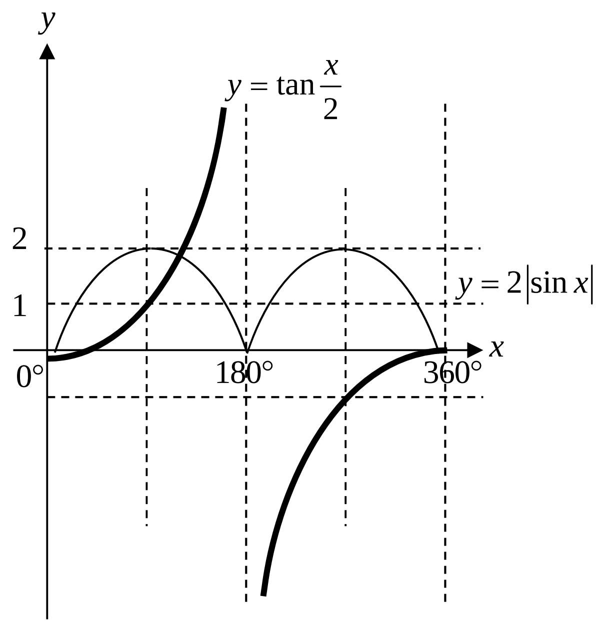
(ii) Prove that $2\sin^2 x-\dfrac{\tan x-\cot x}{\tan x+\cot x}=1$. **Solution:** $\text{LHS}=2\sin^2 x-\dfrac{\tan x-\cot x}{\tan x+\cot x}$ $=2\sin^2 x-\dfrac{\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$ $=2\sin^2 x-\dfrac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}$ $=2\sin^2 x-(\sin^2 x-\cos^2 x)$ $=\sin^2 x+\cos^2 x$ $=1=\text{RHS}$	[3]
Question 21 [2014 EOY, Q12]	marks
The function f is given by $\mathrm{f}:x\mapsto 2-3\cos 2x$ for the domain $0°\le x\le 180°$. Sketch the graph $y=\mathrm{f}(x)$. Hence, state the range of the function for its given domain. **Solution:**  Hence, the range of the function is $-1\le \mathrm{f}(x)\le 5$.	[4]

Question 22 [2014 EOY, Q15]	marks
Given that θ is an angle that lies in the 4th quadrant, simplify the expression $\dfrac{1}{\cos\theta\sqrt{1+\tan^2\theta}}+\dfrac{2\cot\theta}{\sqrt{\dfrac{1}{\sin^2\theta}-1}}$. **Solution:** $\dfrac{1}{\cos\theta\sqrt{1+\tan^2\theta}}+\dfrac{2\cot\theta}{\sqrt{\dfrac{1}{\sin^2\theta}-1}}$ $=\dfrac{1}{\cos\theta\sqrt{\sec^2\theta}}+\dfrac{2\cot\theta}{\sqrt{\cot^2\theta}}$ $=\dfrac{1}{\cos\theta\lvert\sec\theta\rvert}+\dfrac{2\cot\theta}{\lvert\cot\theta\rvert}$ $\because$ θ lies in the 4th quadrant, $\sec\theta>0$, $\cot\theta<0$, $\therefore \lvert\sec\theta\rvert=\sec\theta$, $\lvert\cot\theta\rvert=-\cot\theta$ Hence, the expression $=\dfrac{1}{\cos\theta\sec\theta}+\dfrac{2\cot\theta}{-\cot\theta}$ $=1-2=-1$	[3]
Question 23 [2015 EOY, Q13]	marks
Given that $\sec x+\tan^2 x=0$, show that $\cos x=-\dfrac{2}{1+\sqrt{5}}$. **Solution:** $\sec x+\tan^2 x=0 \Rightarrow \sec^2 x+\sec x-1=0$ Hence $\sec x=\dfrac{-1\pm\sqrt{5}}{2} \Rightarrow \cos x=\dfrac{2}{-1\pm\sqrt{5}}$ But $-1\le\cos x\le 1$ hence $\cos x=-\dfrac{2}{1+\sqrt{5}}$	[4]

(i) Prove that $\dfrac{\operatorname{cosec} x - \sec x}{\operatorname{cosec} x + 1} + \dfrac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x - 1} = 2\sec^2 x + 2\sec x \tan^2 x$. **<u>Solution:</u>** $\dfrac{\operatorname{cosec} x - \sec x}{\operatorname{cosec} x + 1} + \dfrac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x - 1}$ $= \dfrac{2\operatorname{cosec}^2 x + 2\sec x}{\operatorname{cosec}^2 x - 1}$ $= \dfrac{2\operatorname{cosec}^2 x + 2\sec x}{\cot^2 x}$ $= 2\operatorname{cosec}^2 x \tan^2 x + 2\sec x \tan^2 x$ $= 2\sec^2 x + 2\sec x \tan^2 x$	[3]
(ii) Hence, solve for $0° < x < 360°$ the equation $\dfrac{\operatorname{cosec} x - \sec x}{1 + \operatorname{cosec} x} = \dfrac{\operatorname{cosec} x + \sec x}{1 - \operatorname{cosec} x}$. **<u>Solution:</u>** $\dfrac{\operatorname{cosec} x - \sec x}{1 + \operatorname{cosec} x} = \dfrac{\operatorname{cosec} x + \sec x}{1 - \operatorname{cosec} x}$ $\Rightarrow \dfrac{\operatorname{cosec} x - \sec x}{\operatorname{cosec} x + 1} + \dfrac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x - 1} = 0$ Hence $2\sec^2 x + 2\sec x \tan^2 x = 0$ $\Rightarrow \sec x(\sec x + \tan^2 x) = 0$ Since $\sec x \neq 0$, $\sec x + \tan^2 x = 0 \Rightarrow \cos x = -\dfrac{2}{1+\sqrt{5}}$ Hence $x = 180° - 51.8°$ (1 d.p.) or $180° + 51.8°$ (1 d.p.) $= 128.2°$ (1 d.p.) or $231.8°$ (1 d.p.)	[4]

Question 24 [2015 EOY, Q14]	marks
Sketch the graph of $y = \tan\frac{x}{2}$ for $0^\circ \le x \le 360^\circ$, showing clearly any asymptotes and intercepts with the axes. On the same axes, sketch the graph of $y = 2\lvert\sin x\rvert$. Hence state the number of solutions between 0° and 360° inclusive when $\tan\frac{x}{2} = 2\lvert\sin x\rvert$.	[4]

Solution:



$\tan\frac{x}{2} = 2\lvert\sin x\rvert$

By including the graph of $y = 2\lvert\sin x\rvert$ it can be seen that there are three solutions between 0° and 360° inclusive.