



Term 2 W4 Revision: Discrete Random Variables, Binomial Distribution

Questions

1 JPJC BT2 9758/2021/02/Q9

A biased four-sided die has the numbers ‘1’ to ‘4’ labelled on its faces. When the die is thrown, the number shown on the die is the score, X , which follows the probability distribution:

$$P(X = x) = \begin{cases} kx & , x = 1, 2 \\ \frac{1}{x} - \frac{1}{4}k & , x = 3, 4 \end{cases}$$

where k is a non-zero positive constant.

- (i) Show that the value of k is $\frac{1}{6}$. [2]

In a game, Evan tosses two such dice. The probability distribution of Y , which is the sum of the scores shown on the dice, is given in the following table:

y	2	3	4	5	6	7	8
$P(Y = y)$	a	$\frac{1}{9}$	$\frac{5}{24}$	b	$\frac{43}{192}$	$\frac{35}{288}$	$\frac{25}{576}$

- (ii) Find the values of a and b . [3]
- (iii) If Y is an odd number, Evan wins \$ Y . If Y is an even number, Evan loses \$ Y . Let W be Evan’s winnings after one game. Find the expectation and variance of W . [3]

2 SAJC BT2 8865/2021/10

A glass vial is an important primary packaging component for pharmaceutical drugs. Vaccines, and most injectable drugs, need to be packaged in these sterile glass vials. On average, it was found that 5% of glass vials shipped by a certain pharmaceutical company are defective. The glass vials are packed in boxes. Each box consists of 50 glass vials.

- (i) State, in context, two assumptions needed for the number of defective vials in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of defective vials in a box of shipment follows a binomial distribution.

- (ii) Find the most likely number of defective glass vials in a box. [2]

A box of glass vials is rejected if there are more than 5 defective glass vials.

- (iii) Find the probability that a box is rejected. [1]

- (iv) 2 boxes of glass vials are randomly chosen. Find the probability that 1 box is rejected. [2]
- (v) n boxes of glass vials are to be chosen. Find the least value of n such that the probability of at least 20 boxes that are not rejected is greater than 0.95. [3]

A sample of 50 boxes of glass vials is taken. The sample is rated as 'substandard' if 4 or more boxes are rejected. If none of the boxes are rejected, the sample is rated as 'excellent'. Otherwise, the sample is rated as 'fair'.

- (vi) Find the probability that 2 boxes are rejected if the sample is rated as 'fair'. [3]