



READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 80.

Compound interest

Mensuration

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f} \right)^2}$$

Trigonometry

Statistics

This question paper consists of 21 printed pages.

MATHS7015DS
© SAS 2020

Turn Over

- 4 Given that $A : B = \frac{1}{2} : \frac{1}{3}$ and $B : C = 1 : 2$, find the ratio of A to C in its simplest form.

$$\begin{aligned} A : B &= \frac{1}{2} : \frac{1}{3} \\ A : B : C &= \frac{1}{2} : \frac{1}{3} : 2 \\ \frac{1}{2} : \frac{1}{3} : 2 &= 3 : 2 : 12 \\ \frac{3}{3} : \frac{2}{3} : \frac{12}{3} &= 3 : 2 : 12 \end{aligned}$$

Answer 3 : 4 [1]

- 5 The numbers 60 and 63, written as products of their prime factors, are

$$60 = 2^2 \times 3 \times 5, \quad 63 = 3^2 \times 7.$$

Find

- (a) the smallest integer which is an exact multiple of both 60 and 63,

$$\begin{aligned} \text{LCM} &= 2^2 \times 3^2 \times 5 \times 7 \\ &= 1260 \end{aligned}$$

Answer 1260 [1]

- (b) the smallest positive integer value of n for which $63n$ is a multiple of 60.

$$\begin{aligned} n &= \frac{1260}{63} \\ &= 20 \end{aligned}$$

Answer 20 [1]

- 1 The kinetic energy (E) of an object depends on both its mass (m) and its speed (v). Their relationship is given by the formula $E = mv^2$ where the units of E , m and v are expressed in J, kg and m/s respectively. A rocket of mass 15 Mg, has an initial speed of 5200 m/s. Find, in standard form, the kinetic energy of the rocket.

$$\begin{aligned} [1 \text{ Mg} &= 10^6 \text{ g}] \\ E &= mv^2 \\ 15 \text{ Mg} &= 15 \times 10^6 \text{ kg} \\ &= 15 \times 10^3 \text{ kg} \\ &= 15 \times 10^3 \times 5200^2 \\ &= 4.056 \times 10^{11} \end{aligned}$$

Answer 4.056×10^{11} J [2]

- 2 A map is drawn to a scale of 1 : 50 000.

A farm has an area of 20 cm^2 on the map. Calculate, in km^2 , the actual area of the farm.

$$\begin{aligned} 1 \text{ cm} &= 0.5 \text{ km} \\ 1 \text{ cm}^2 &= 0.25 \text{ km}^2 \\ 20 \text{ cm}^2 &= 0.25 \times 20 \\ &= 5 \text{ km}^2 \end{aligned}$$

Answer 5 km^2 [2]

- 3 The height of a tree is expected to increase by $x\%$ of its value each year. The height is now 30 m. Calculate the value of x if the tree grows 6 m in 4 years.

$$\begin{aligned} 36 &= 30 \left(1 + \frac{x}{100} \right)^4 \\ \sqrt[4]{\frac{36}{30}} &= 1 + \frac{x}{100} \\ x &= \left(\sqrt[4]{\frac{36}{30}} - 1 \right) 100 \end{aligned}$$

Answer 4.66 [2]

- 8 Andrew wants to buy a computer which is being advertised for \$2 500. There is an option of paying a 10% deposit with 24 monthly instalments using a hire purchase scheme. The interest is calculated at 7.5% per annum. Calculate Andrew's monthly instalment. Give your answer correct to the nearest dollars.

$$\begin{aligned}
 \text{outstanding balance} &= 2500 \times 2500 \quad \text{monthly instalment} \\
 &= 2250 \\
 I &= 2250 \times 7.5 \times 2 \\
 &= 107.1875 \\
 &= \$108
 \end{aligned}$$

108

Answer \$..... [3]

- 9 (a) p is inversely proportional to the square of v . It is known that $p = 4$ for a particular value of v . What is the value of p when v is doubled?

$$\begin{aligned}
 p &= \frac{k}{v^2} = 4 \\
 p_1 &= \frac{k}{(2v)^2} \\
 &= \frac{k}{4v^2} \\
 &= 1
 \end{aligned}$$

Answer..... [1]

- (b) A car skids to a halt in an accident. Investigators measure the length, x m, of the skid to estimate the speed, y m/s, at which it had been travelling before the accident. The speed is proportional to the square root of the length of the skid. Given that the difference in the values of y when $x = 25$ and $x = 100$ is 20, find the equation connecting x and y .

$$\begin{aligned}
 y &= k\sqrt{x} \\
 \text{when } x &= 25, y = 5k \\
 x &= 100, y = 10k \\
 10k - 5k &= 20 \\
 5k &= 20 \\
 k &= 4 \\
 y &= 4\sqrt{x}
 \end{aligned}$$

Answer..... $y = 4\sqrt{x}$ [2]

- 6 Trees will be planted on one side of road A and on one side of road B . Road A is 0.48 km long and road B is 792 m long. A tree will be planted at the beginning and at the end of each road. The distance between each tree on road A and on road B is the same. Find the maximum possible distance between 2 consecutive trees.

$$\begin{array}{r}
 2 \overline{) 480, 792} \\
 \underline{240, 396} \\
 2 \overline{) 120, 198} \\
 \underline{60, 99} \\
 3 \overline{) 20, 133} \\
 14 \text{ CF} = 23 \times 3 \\
 = 24
 \end{array}$$

24

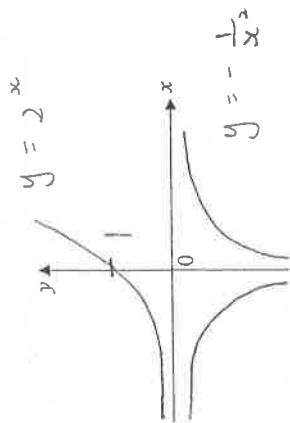
Answer..... m [2]

- 7 Express $\frac{x}{x^2 - 6x + 9} - \frac{3}{3 - x}$ as a single fraction in its simplest form.

$$\begin{aligned}
 \frac{x}{(x-3)^2} - \frac{3}{-(x-3)} \\
 = \frac{x}{(x-3)^2} + \frac{3}{(x-3)} \\
 = \frac{x + 3(x-3)}{(x-3)^2} \\
 = \frac{4x-9}{(x-3)^2}
 \end{aligned}$$

Answer..... $\frac{4x-9}{(x-3)^2}$ [3]

- 12 (a) Sketch $y = 2^x$ and $y = -\frac{1}{x^2}$ on the same axes below. [2]



- (b) Hence, explain why $-\frac{1}{x^2} = 2^x$ has no solutions. [1]

Answer

This is no solutions because there is no point of intersection between the graphs

- 13 (a) Solve the inequality $x + 5 \leq 3 + 2x < 21 - x$.

$$\begin{aligned} x + 5 &\leq 3 + 2x & \text{and} & & 3 + 2x < 21 - x \\ 2 &\leq x & & & 3x < 18 \\ & & & & x < 6 \end{aligned}$$

$$2 \leq x < 6$$

Answer $2 \leq x < 6$ [2]

- (b) Show your solution on the number line below. [1]

Answer



- 10 The ratio of the circumferences of the bases of two similar jugs is in the ratio 2 : 3.

- (a) Find the height of the smaller jug if the height of the larger jug is 36 cm.

$$\frac{h}{36} = \frac{2}{3}$$

$$h = \frac{2}{3} \times 36$$

$$= 24$$

24

Answer cm [1]

- (b) Both jugs are completely filled with fruit juice. The cost of the fruit juice in the smaller jug and the larger jug are \$4 and \$14 respectively. Which jug is better value for money? Justify your answer. [2]

Answer

$$\frac{V_1}{V_2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

8 units $\rightarrow$ 4

27 units $\rightarrow$ \$13.50

$\therefore$ small jug is better value for money.

- 11 Given that $y = \frac{4x+3}{x-5}$, express x in terms of y .

$$y(x-5) = 4x+3$$

$$xy - 5y = 4x + 3$$

$$xy - 4x = 5y + 3$$

$$x(y-4) = 5y+3$$

$$x = \frac{5y+3}{y-4}$$

Answer $x = \frac{5y+3}{y-4}$ [3]

- 15 (a) Simplify and give your answer in positive index,

(i) $\frac{1}{\sqrt{a^3}} \times a^3$

$a^{-\frac{3}{2}} \times a^3$

$= a^{\frac{3}{2}}$

$a^{\frac{3}{2}}$

Answer [1]

(ii) $\left(\frac{1}{x^4}\right)^{-2}$

$(x^4)^2$

$= x^8$

x^8

Answer [1]

- (b) Solve the equation $5^{x+2} = 25^0$

$5^{x+2} = (5^2)^0$

$x+2 = 0$

$x = -2$

Answer [2]

-2

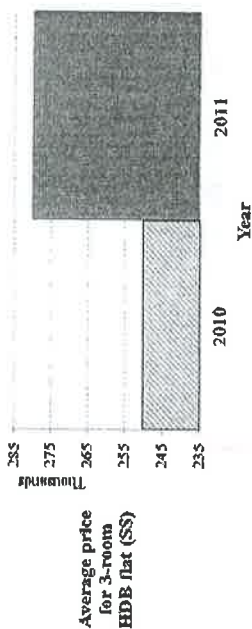
- 14 (a) A factory produces cartons of washing powder. Checks on cartons selected randomly show that one carton is less than 1 kg and the rest of the cartons are more than or equal to 1.5 kg. State, with a reason, whether the mean or the median gives a better representation of the weights of the cartons selected. [1]

Answer

Median gives a better representation because there is a carton that weighs less than 1 kg which is significantly below the mean. 1.5 inclusion will deflate the mean.

- (b) The graph shows the average price for a 3-room HDB flat in 2010 and 2011.

HUGE INCREASE IN PRICE FOR 3-ROOM HDB FLAT IN 2011



One aspect of the graph is misleading. State the aspect and explain how it may lead to a misinterpretation of the graph. [2]

Answer

The vertical axis doesn't start at zero. This may lead to misinterpretation that the price has increased by many times in 2011 as compared to 2010.

- 17 In a scientific experiment, the following six masses, in grams, were recorded.

$$9.6 \times 10^{-5} \quad 1.01 \times 10^{-4} \quad 9.3 \times 10^{-5} \quad 1.04 \times 10^{-4} \quad 1.03 \times 10^{-4} \quad 9.6 \times 10^{-5}$$

Find, giving your answer in standard form,

- (a) the mode,

Answer grams [1]

$$9.6 \times 10^{-5}$$

- (b) the range,

$$\begin{aligned} \text{range} &= 1.04 \times 10^{-4} - 9.3 \times 10^{-5} \\ &= 1.1 \times 10^{-5} \end{aligned}$$

- (c) the upper quartile.

Answer grams [2]

$$9.3 \times 10^{-5}, 9.6 \times 10^{-5}, 9.6 \times 10^{-5}, 1.01 \times 10^{-4}, 1.03 \times 10^{-4}, 1.04 \times 10^{-4}$$

median

1.1×10^{-5}

- 16 Factorise completely

(a) $1 - 4a^2$,

$$(1 + 2a)(1 - 2a)$$

Answer [1]

$$(1 + 2a)(1 - 2a)$$

(b) $4p^2 - 4pq + q^2$,

$$(2p - q)^2$$

Answer [1]

$$(2p - q)^2$$

(c) $2a^2 - 3a - 2ah + 3h$.

$$\begin{aligned} &a(2a - 3) - h(2a - 3) \\ &= (a - h)(2a - 3) \end{aligned}$$

Answer [2]

$$(a - h)(2a - 3)$$

Answer grams [1]

$$1.03 \times 10^{-4}$$

- 19 (a) Find the equation of the line of symmetry of the graph $y = (1-x)(x-2)$.

$$x = \frac{1+1}{2}$$

$$x = \frac{3}{2}$$

$$x = \frac{3}{2}$$

Answer [1]

- (b) (i) Express $x^2 - 2x + 3$ in the form $(x-a)^2 + b$.

$$x^2 - 2x + 3 = \left(x - \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 3$$

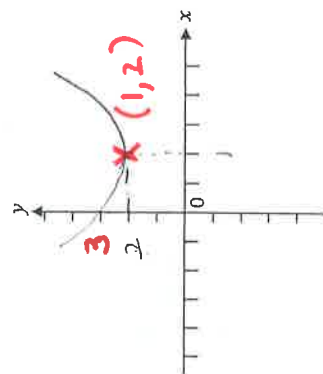
$$= (x-1)^2 + 3 - 1$$

$$= (x-1)^2 + 2$$

Answer $(x-1)^2 + 2$ [1]

- (ii) Sketch the graph of $y = x^2 - 2x + 3$.

Answer



- 18 Spaghetti and lasagne are sold in restaurants X and Y. At each restaurant, the price of each dish is the same, \$5.50 for each plate of spaghetti and \$10.20 for each plate of lasagne. The number of plates of each type of food sold in one day is given in the following table.

	Spaghetti	Lasagne
Restaurant X	215	170
Restaurant Y	155	220

- (a) Evaluate $\begin{pmatrix} 215 & 170 \\ 155 & 220 \end{pmatrix} \begin{pmatrix} 5.5 \\ 10.2 \end{pmatrix}$.

$$\begin{pmatrix} 215 \times 5.5 & 170 \times 10.2 \\ 155 \times 5.5 & 220 \times 10.2 \end{pmatrix}$$

$$= \begin{pmatrix} 2916.5 & 2916.5 \\ 3096.5 & 3096.5 \end{pmatrix}$$

Answer [1]

- (b) Explain what the elements of the matrix in (a) represent.

[1]

Answer

Total amount collected from the sale of spaghetti and lasagne in restaurants X and Y respectively.

- (c) Evaluate $\frac{1}{2} \left[\begin{pmatrix} 215 \\ 170 \end{pmatrix} + \begin{pmatrix} 155 \\ 220 \end{pmatrix} \right]$.

$$\frac{1}{2} \begin{pmatrix} 370 \\ 390 \end{pmatrix}$$

$$= \begin{pmatrix} 185 \\ 195 \end{pmatrix}$$

Answer [1]

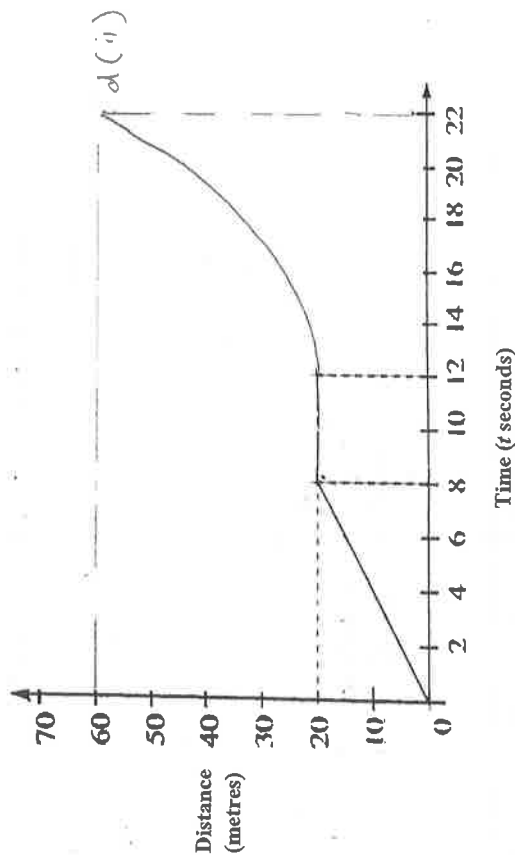
- (d) Explain what the elements of the matrix in (c) represent.

[1]

Answer

Respective average number of plates of spaghetti and lasagne sold by restaurant X and Y

- 23 The diagram shows the distance-time graph of the first 12 seconds of the journey of an object.



- (a) Find the speed during the first 8 seconds.

$$\text{speed} = \frac{20}{8}$$

$$= 2.5$$

Answer 2.5 m/s [1]

- (b) What is the acceleration when $t = 4$?

0

Answer m/s² [1]

- (c) Give a brief description of what happened between $t = 8$ and $t = 12$. [1]

Answer

object is at rest

- 22 One day a farmer collected 300 eggs from his chickens. The table below shows the distribution of the masses of the eggs.

Mass (m grams)	$42 < m \leq 46$	$46 < m \leq 50$	$50 < m \leq 54$	$54 < m \leq 58$
Frequency	50	80	100	70

- (a) Calculate an estimate of the mean mass of an egg.

$$\text{mean} = \frac{50 \times 44 + 80 \times 48 + 100 \times 52 + 70 \times 56}{300}$$

$$= 50 \frac{8}{15}$$

Answer $50 \frac{8}{15}$ grams [2]

- (b) Calculate an estimate of the standard deviation.

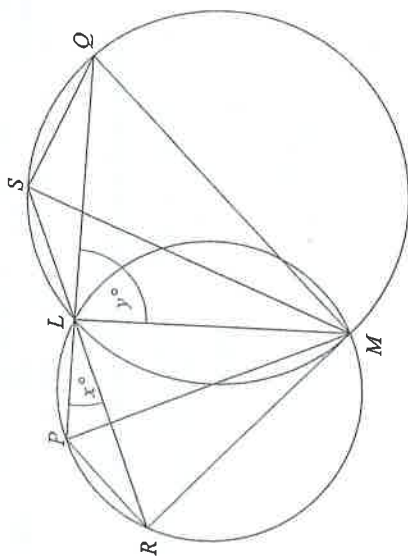
$$SD = \sqrt{\frac{50 \times 44^2 + 80 \times 48^2 + 100 \times 52^2 + 70 \times 56^2}{300} - \left(50 \frac{8}{15}\right)^2}$$

$$= 4.063$$

$$= 4.06$$

Answer 4.06 grams [2]

- 24 Two circles intersect at L and M . R and P are on the circumference of one circle. S and Q are on the circumference of the other circle. PLQ and RLS are straight lines. Angle $PLR = x^\circ$ and angle $MLQ = y^\circ$.



- (a) Stating all reasons, prove that

- (i) angle $SMQ = x^\circ$,

[2]

Answer

$$\angle SLO = x \text{ (vert opp } \angle s)$$

$$\angle SMQ = x \text{ (} \angle s \text{ in the same segment)}$$

- (ii) angle $PRM = y^\circ$.

[1]

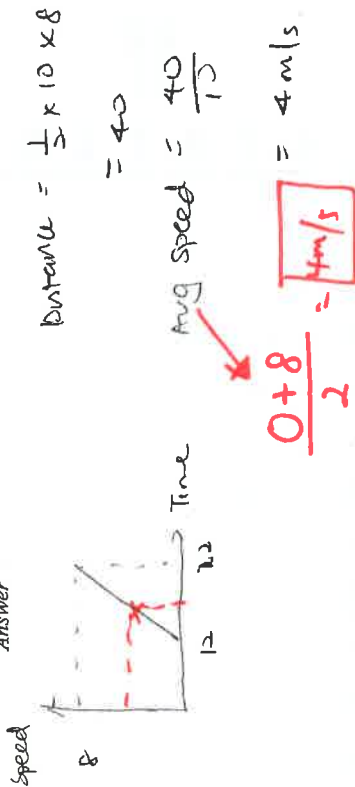
Answer

$$\angle PRM = y \text{ (ext } \angle \text{ of cyclic quad)}$$

- (d) The speed increases uniformly between $t = 12$ and $t = 22$. The speed is 8 m/s when $t = 22$.

- (i) Show that the average speed between $t = 12$ and $t = 22$ is 4 m/s . [2]

Answer



- (ii) On the same distance-time graph diagram on page 18, sketch the distance-time graph for the journey between $t = 12$ and $t = 22$. [1]

- (b) Hence, show that triangles PRM and QSM are similar. [3]

Answer

$\angle PR = \angle PMR = x$ ($\angle$ s in the same segment)

$$\therefore \angle PMR = \angle QMS = x$$

$\angle QSM = \angle MLQ = y$ ($\angle$ s in the same segment)

$$\therefore \angle QSM = \angle PRM = y$$

$\triangle PRM$ and $\triangle QSM$ are similar.
