RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Additional Practice Questions for Chapter S2B: Binomial Distribution (Solutions)

1 Published articles in medical journals indicate that, on average, 35 out of 100 patients having a lumbar puncture will suffer SSH ('Severe Spinal Headache'). Twelve patients are given a lumbar puncture.

(i) Using a binomial model, find the expected number of patients who will suffer SSH, and find also the standard deviation. [3]

(ii) Find the probability that four or more of the twelve patients will suffer SSH. [2]

Solution:

(i) Let X be the number of patients, out of 12, who will suffer SSH.

Assumptions: (1) Whether a patient suffers SSH is independent of another.

(2) The probability of a patient suffering SSH remains constant at 0.35.

Then $X \sim \mathrm{B}(12, 0.35)$

Expected number of patients who will suffer SSH = 12(0.35) = 4.2

Standard deviation = $\sqrt{12(0.35)(1-0.35)} = 1.65$ (3.s.f)

(ii) $\mathrm{P}(X \geq 4) = 1 - \mathrm{P}(X \leq 3)$ =0.653 (3.s.f)

2 The random variable $X \sim \mathrm{B}\,(16, p)$, where $p < 0.5$. If the variance of X is 3.36, find the value of p. Find also the probability that X is less than its mean. [4]

Solution:

Variance = $(16)(p)(1-p) = 3.36$, which gives $p = 0.3$ or 0.7 (rejected since $p < 0.5$)

Hence, $p = 0.3$

Mean = 16(0.3) = 4.8

$\mathrm{P}(X < 4.8) = \mathrm{P}(X \leq 4) = 0.450$ (3 s.f.)

3 TPJC Prelim 2007/02/Q5

A factory produces chocolate which are packed into boxes of 20 and delivered to shops for sale. A chocolate will not meet the minimum criteria for packing for sale if it weighs less than 20 grams. On average, 2% of the chocolate produced did not meet the minimum criteria.

(i) Find the probability that a randomly chosen box contain at least 1 chocolate that does not meet the minimum criteria. [2]

(ii) Find the probability that out of 4 randomly chosen boxes of chocolate, there are exactly 2 boxes with at least 1 chocolate that does not meet the minimum criteria. [2]

Solution:

(i) Let X be the number of chocolates, out of 20, not meeting the minimum criteria.

Assumptions: (1) Whether a chocolate does not meet the minimum criteria is independent of another.

(2) The probability of a chocolate not meeting the minimum criteria remains constant at 0.02.

Then $X \sim \mathrm{B}(20, 0.02)$

$\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) \approx 0.33239 = 0.332$ (3 s.f.)

(ii) Let Y be the number of boxes of chocolates, out of 4, with at least 1 chocolate that doesn't meet the minimum criteria. Then $Y \sim \mathrm{B}(4, 0.33239)$

$\mathrm{P}(Y = 2) = 0.295$ (3 s.f.)

4 CJC Prelim 2006/02/Q25

In a multi-national company with a large population, 13.5% of the staff owns a vehicle.

(i) Find the probability that in a random sample of 30 staff, exactly 5 of them will own a vehicle. [2]

(ii) The probability that there is at least one staff who owns a vehicle in a random sample of size n is greater than 0.95. Find the least value of n. [3]

Solution:

(i) Let X be the number of staff, out of 30, who owns a vehicle.

Assumptions: (1) Whether a staff owns a vehicle is independent of another.

(2) The probability of a staff owning a vehicle remains constant at 0.135.

Then $X \sim \mathrm{B}(30, 0.135)$

$\mathrm{P}(X = 5) = 0.170$ (3 s.f.)

(ii) Let Y be the number of staff, out of n, who owns a vehicle.

$Y \sim \mathrm{B}(n, 0.135)$

Then we have

$\mathrm{P}(Y \geq 1) > 0.95$

$\Rightarrow 1 - \mathrm{P}(Y = 0) > 0.95$

$\Rightarrow \mathrm{P}(Y = 0) < 0.05$

$\Rightarrow (1 - 0.135)^n < 0.05$

$\Rightarrow n > \dfrac{\ln 0.05}{\ln 0.865} = 20.7$

$\Rightarrow n \geq 21$

or using a GC,

When $n = 20$, $\mathrm{P}(Y=0) \approx 0.054995 > 0.05$

When $n = 21$, $\mathrm{P}(Y=0) \approx 0.047571 < 0.05$

Least value of n is 21.

5 **RVHS Prelim 2014/02/Q9 (part)**

Market research showed that 3 out of 10 households in a housing estate subscribe to fibre broadband internet services.

(a) 20 households from a particular block of flats in the estate were surveyed.

(i) Show that the probability of more than 3 and less than 9 of the households surveyed subscribe to fibre broadband internet services is 0.780, correct to 3 decimal places. [2]

(ii) Find the least value of k such that the probability that at most k of the households surveyed subscribe to fibre broadband internet services is at least 0.75. [2]

(b) There are a total of 50 blocks of flats in the estate. 20 households from each block of flats were surveyed.

Find the probability that there are exactly 39 blocks of flats with more than 3 and less than 9 households surveyed that subscribe to fibre broadband services. [2]

Solution:

(a)(i)

Let X be the number of households that subscribe to broadband internet services out of 20 households.

Assumptions: (1) Whether a household subscribe to broadband internet services is independent of another.

(2) The probability of a household subscribing to broadband internet services remain constant at 0.3.

$X \sim \mathrm{B}(20, 0.3)$

$\mathrm{P}(3 < X < 9)$

$= \mathrm{P}(4 \le X \le 8)$

$= \mathrm{P}(X \le 8) - \mathrm{P}(X \le 3)$

$= 0.77958 \text{ (5 d.p.)}$

$= 0.780 \text{ (3 d.p.)} \text{ (Shown)}$

(ii) $\mathrm{P}(X \le k) \ge 0.75$

Using the GC to set up a table of values,

$\mathrm{P}(X \le 6) \approx 0.608 < 0.75$

$\mathrm{P}(X \le 7) \approx 0.772 > 0.75$

The least value of k is 7.

(b)

Let Y be the number of blocks, with more than 3 and less than 9 households subscribing to broadband internet services, out of 50 blocks each with 20 households surveyed.

$Y \sim \mathrm{B}(50, 0.780)$

$\mathrm{P}(Y = 39) = 0.135 \text{ (3 s.f.)}$

6 RI CT2/H1/2018/Q5

It is known that 36% of the customers of a certain supermarket will bring their own environmental friendly bags. On a certain day, there are 3 cashiers and each cashier has 5 customers in queue.

(i) Find the probability that among all the customers in queue, at least 4 of them brought their own environmental friendly bags. [2]

(ii) If exactly 4 customers in queue brought their own environmental friendly bags, find the probability that each cashier will have at least 1 customer bringing his or her own environmental friendly bag. [4]

Solution

(i)	Let X denote the number of customers (out of 15) who brought their own environmental friendly bags. $X \sim \text{B}(15, 0.36)$ $\text{P}(X \geq 4) = 1 - \text{P}(X \leq 3) = 0.84694 = 0.847$ (3 sf)
(ii)	Let Y denote the number of customers (out of 5) who brought their own environmental friendly bags. $Y \sim \text{B}(5, 0.36)$ P(Each cashier will have at least 1 customer bringing his or her own environmental friendly bag given that $X = 4$) $= \dfrac{3 \times \text{P}(Y_1 = 1) \times \text{P}(Y_2 = 1) \times \text{P}(Y_3 = 2)}{\text{P}(X = 4)}$ $= \dfrac{3 \times (0.30199)^2 \times 0.33974}{0.16917} = 0.54945 = 0.549$ (3 sf)

7 MJC Prelim 2010/02/Q8 (modified)

A factory produces watches. The probability that a watch is defective is 0.02. The watches are packed into boxes which contains 55 watches each before being shipped.

(i) State, in context, two assumptions for the number of defective watches in a box to be well modelled by a binomial distribution. [2]

(ii) Show that the probability for a box to contain at least 1 defective watch is 0.671. [2]

(iii) A customer ordered 40 boxes of watches. He will be compensated if more than 19 boxes contain defective watches. Find the probability that the customer will be compensated. [3]

(iv) Find the least number of watches to be added to a box of 55 watches such that the probability of finding at least 53 non-defective watches is more than 0.99. [3]

Solution:

(i) Whether a watch is defective is independent of another watch being defective.
The probability of a watch being defective is constant at 0.02.

(ii) Let X be the number of defective watches in a box of 55.

$X \sim \mathrm{B}(55, 0.02)$

$\mathrm{P}\left(X \geq 1\right) = 1 - \mathrm{P}\left(X = 0\right) \approx 1 - 0.32918 = 0.67082 (5\text{ s.f.}) = 0.671 (3\text{ s.f.}) \left(\text{Shown}\right)$

(iii) Let W be the number of boxes out of 40, which contain defective watches.

$W \sim \mathrm{B}(40, 0.671)$

$\mathrm{P}\left(W > 19\right) = 1 - \mathrm{P}\left(W \leq 19\right) = 0.992 \text{ (3s.f.)}$

(iv) Let Y be the number of non-defective watches in a box of $55+n$ watches.

$Y \sim \mathrm{B}(55 + n, 0.98)$

$\mathrm{P}(Y \geq 53) > 0.99$

$\Rightarrow 1 - \mathrm{P}(Y \leq 52) > 0.99$

$\Rightarrow \mathrm{P}(Y \leq 52) < 0.01$

From GC,

For $n = 1$, $\mathrm{P}(Y \leq 52) = 0.0258 > 0.01$

For $n = 2$, $\mathrm{P}(Y \leq 52) = 0.00566 < 0.01$

Therefore, the least number of watches to be added is 2.

8 DHS Prelim 9758/2018/02/Q6

A test consists of 15 multiple choice questions, where each question has n possible options, of which only one is correct. A student took the test by randomly choosing an answer to each question. It is known that the probability of answering exactly 3 questions correctly is the same as the probability of answering exactly 4 questions correctly.

(i) By forming an equation in terms of n, find the value of n. [3]

Each correct answer is awarded 3 marks and each incorrect answer carries a penalty of 1 mark. The score is the total marks awarded based on the number of correct and incorrect answers.

(ii) Find the expected score, s, obtained by the student. [3]

(iii) Find the probability that the score obtained by the student is within 4 marks of s. [2]

Solution

(i)

Let X be the random variable denoting the number of questions answered correctly out of 15.

$$X \sim \mathrm{B}\left(15, \frac{1}{n}\right)$$

$$\mathrm{P}(X=3) = \mathrm{P}(X=4)$$

$$\binom{15}{3}\left(\frac{1}{n}\right)^3\left(\frac{n-1}{n}\right)^{12} = \binom{15}{4}\left(\frac{1}{n}\right)^4\left(\frac{n-1}{n}\right)^{11}$$

$$\left(\frac{n-1}{n}\right) = 3\left(\frac{1}{n}\right)$$

$$n = 4$$

<u>Using your GC to check the answer</u>

From GC, $n = 4$

X	Y1	Y2
1	0	0
2	.01389	.04166
3	.12988	.19482
4	.2252	.2252
5	.25014	.1876
6	.23626	.14175
7	.20862	.10431

(ii)

$$X \sim \mathrm{B}\left(15, \frac{1}{4}\right)$$

<u>Method 1</u>

Let T be the score for 15 questions.

Then $T = 3X - (15 - X) = 4X - 15$

$$s = \mathrm{E}(T) = 4\mathrm{E}(X) - 15$$
$$= 4\left[15\left(\frac{1}{4}\right)\right] - 15 = 0$$

	Method 2 Let A be the score for 1 question. $\mathrm{E}(A)=3\left(\frac{1}{4}\right)-1\left(\frac{3}{4}\right)=0$ $s=\mathrm{E}(T)=\mathrm{E}(A_1+A_2+\ldots+A_{15})=15\mathrm{E}(A)=0$ **Method 3** Let Y be the random variable denoting the number of questions answered incorrectly out of 15. $Y\sim\mathrm{B}\left(15,\frac{3}{4}\right)$. $s=3\mathrm{E}(X)-\mathrm{E}(Y)$ $=3\left[15\left(\frac{1}{4}\right)\right]-\left[15\left(\frac{3}{4}\right)\right]=0$
(iii)	$\mathrm{P}(-4<T<4)=\mathrm{P}(-4<4X-15<4)$ $=\mathrm{P}(2.75<X<4.75)$ $=\mathrm{P}(3\le X\le 4)$ $=\mathrm{P}(X\le 4)-\mathrm{P}(X\le 2)$ or $2\mathrm{P}(X=4)$ or $2\mathrm{P}(X=3)$ $=0.450$

9 **9758/2017/02/Q5**

A bag contains 6 red counters and 3 yellow counters. In a game, Lee removes counters at random from the bag, one at a time, until he has taken out 2 red counters. The total number of counters Lee removes from the bag is denoted by T.

(i) Find $\mathrm{P}(T = t)$ for all possible values of t. [3]

(ii) Find $\mathrm{E}(T)$ and $\mathrm{Var}(T)$. [2]

Lee plays this game 15 times.

(iii) Find the probability that Lee has to take at least 4 counters out of the bag in at least 5 of his games. [2]

(i) Let T be the total number of counters Lee removes from the bag.

$T = 2,3,4,5$

$$P(T=2)=\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)=\frac{5}{12}$$

$$P(T=3)=2\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\left(\frac{5}{7}\right)=\frac{5}{14}$$

$$P(T=4)=3\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right)\left(\frac{5}{6}\right)=\frac{5}{28}$$

$$P(T=5)=4\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)\left(\frac{5}{5}\right)=\frac{1}{21}$$

(ii) $\mathrm{E}(T)=2\left(\frac{5}{12}\right)+3\left(\frac{5}{14}\right)+4\left(\frac{5}{28}\right)+5\left(\frac{1}{21}\right)=\frac{20}{7}$

$$\mathrm{Var}(T)=\mathrm{E}(T^2)-\mathrm{E}(T)^2=2^2\left(\frac{5}{12}\right)+3^2\left(\frac{5}{14}\right)+4^2\left(\frac{5}{28}\right)+5^2\left(\frac{1}{21}\right)-\left(\frac{20}{7}\right)^2=\frac{75}{98}$$

(iii) P(taking at least 4 counters) = $\mathrm{P}(T = 4) + \mathrm{P}(T = 5) = \frac{19}{84}$

Let X be the no of games out of 15 games where he needs to take at least 4 counters.

$X \sim \mathrm{B}(15, \frac{19}{84})$

$P(X \geq 5) = 1 - P(X \leq 4) = 0.238$

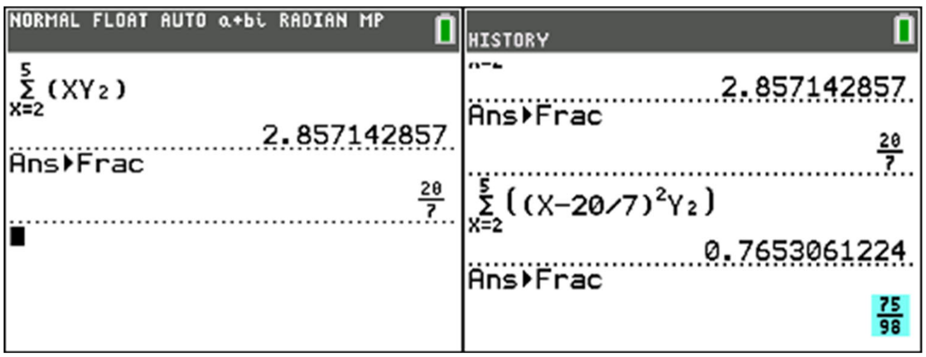
Alternative solution to (i)

$$P(T=t)=P(1\ R\text{ and }t-2\ Y)P\left(R\text{ on }t^{\text{th}}\text{ draw}|1\ R\text{ and }t-2\ Y\right)$$

$$=\frac{\binom{6}{1}\binom{3}{t-2}}{\binom{9}{t-1}}\left[\frac{\binom{5}{1}}{\binom{10-t}{1}}\right]$$

$$=\frac{30}{10-t}\frac{\binom{3}{t-2}}{\binom{9}{t-1}}$$

Enter the expression for P(T=t) in graphing window as Y_2: we can calculate E(T) and Var(T).



10 N1982/01/Q13

The random variable X is the number of successes in n independent trials of an experiment in which the probability of a success in any one trial is p. Show that

$$\frac{\mathrm{P}(X=k+1)}{\mathrm{P}(X=k)}=\frac{(n-k)p}{(k+1)(1-p)}, \quad k=0,1,2,...,(n-1)$$

Find the most probable number of successes when $n=10$ and $p=\frac{1}{4}$.

X~B(n, p)

$$\frac{\mathrm{P}(X=k+1)}{\mathrm{P}(X=k)}=\frac{{}^{n}C_{k+1}p^{k+1}(1-p)^{n-(k+1)}}{{}^{n}C_{k}p^{k}(1-p)^{n-k}}$$

$$=\frac{\frac{n!}{(n-k-1)!(k+1)!}p}{\frac{n!}{(n-k)!k!}(1-p)}$$

$$=\frac{(n-k)p}{(k+1)(1-p)}, \quad k=0,1,2,...,(n-1)$$

When n = 10, $p=\frac{1}{4}$,

$$\frac{\mathrm{P}(X=k+1)}{\mathrm{P}(X=k)}=\frac{(10-k)\frac{1}{4}}{(k+1)\left(\frac{3}{4}\right)}=\frac{10-k}{3k+3}$$

We want to know the values of k such that $\mathrm{P}(X=k+1)\geq \mathrm{P}(X=k)$.

$$\text{Consider } \frac{10-k}{3k+3}\geq 1 \quad \Leftrightarrow 10-k\geq 3k+3$$

$$\Leftrightarrow k\leq \frac{7}{4}$$

Largest k is 1, where $\mathrm{P}(X=2)>\mathrm{P}(X=1)$.

This implies that P(X=0) < P(X=1) < P(X=2)

We also have $\mathrm{P}(X=k+1)\leq \mathrm{P}(X=k)$ whenever $k\geq \frac{7}{4}$

Smallest k is 2, where $\mathrm{P}(X=2)>\mathrm{P}(X=3)$

Which implies P(X=2)> P(X=3) > …> P(X=10).

Hence the most probable number of successes (mode) is 2.

11 NYJC Prelim/9758/2018/02/Q6

A game is played using a fair six-sided die, a pawn and a simple board as shown below.

S	1	2	3	4	5	**E**

Initially, the pawn is placed on square **S**. The game is played by throwing the die and moving the pawn in the following manner:

S 1 2 3 4 5 **E** 5 4 3 2 1 2 3 4 5 **E**………

Thus, for example, if the first and second throw of the die gives a "5" and "4" respectively, the final position of the pawn will be on square "3".

The game will stop when the pawn stops at square **E**.

Let X be the random variable denoting the number of throws of the die required to move the pawn such that it stops at square **E**.

(i) Show $\mathrm{P}(X = 2) = \frac{5}{36}$. [1]

(ii) Find the probability that more than two throws of the die are needed for the pawn to stop at square **E** given that the first throw of the die gives an even number. [3]

It is now given that for each game, a player has a maximum of 3 throws of the die and a special prize is given to any player who uses not more than two throws for the pawn to stop at square **E**.

(iii) Find the probability of a player winning a special prize in at least three but not more than eight games out of ten games. [3]

(i) Find the least number of games needed so that the probability of winning at least a special prize is at least 0.998. [3]

(i)	Let Di be the random variable denoting the score of the ith throw of the die for i = 1, 2 $\mathrm{P}(X=2)$ $= \mathrm{P}(D_1=1, D_2=5)+\mathrm{P}(D_1=2, D_2=4)+\mathrm{P}(D_1=3, D_2=3)$ $+\mathrm{P}(D_1=4, D_2=2)+\mathrm{P}(D_1=5, D_2=1)$ $= \frac{1}{6}\left(\frac{1}{6}\right)5 = \frac{5}{36}$
(ii)	$\mathrm{P}(X>2 \mid D_1 \text{ is even})$

	$= \dfrac{\mathrm{P}(X > 2 \text{ and } D_1 \text{ is even})}{\mathrm{P}(D_1 \text{ is even})}$ $= \dfrac{\mathrm{P}(D_1 = 2, D_2 \neq 4) + \mathrm{P}(D_1 = 4, D_2 \neq 2)}{\mathrm{P}(D_1 \text{ is even})}$ $= \dfrac{\frac{1}{6}\left(\frac{5}{6}\right) + \frac{1}{6}\left(\frac{5}{6}\right)}{\frac{3}{6}} = \dfrac{5}{9}$
(iii)	Let W be the random variable denoting number of games, out of 10, that a special prize is won. P(winning a special prize) $= \mathrm{P}(X = 1) + \mathrm{P}(X = 2) = \dfrac{1}{6} + \dfrac{5}{36} = \dfrac{11}{36}$ $W \sim \mathrm{B}\left(10, \dfrac{11}{36}\right)$ Probability required $= \mathrm{P}(3 \leq W \leq 8) = \mathrm{P}(W \leq 8) - \mathrm{P}(W \leq 2)$ $= 0.63173 = 0.632$
(iv)	Let n be the number of games needed. Let Y be the random variable denoting number of games, out of n, that a special prize is won. $Y \sim \mathrm{B}\left(n, \dfrac{11}{36}\right)$ $\mathrm{P}(Y \geq 1) \geq 0.998 \Rightarrow 1 - \mathrm{P}(Y = 0) \geq 0.998$ Using GC, <table><tr><td>n</td><td>$1 - \mathrm{P}(Y = 0)$</td></tr><tr><td>17</td><td>0.99797 < 0.998</td></tr><tr><td>18</td><td>0.99859 > 0.998</td></tr></table>Least n = 18

12 9758/2020/2/Q9 part

A factory produces ballpoint pens. On average 6% of the pens are faulty. The pens are packed in boxes of 100 for sale to retail outlets. It should be assumed that the number of faulty pens in a box of 100 pens follows a binomial distribution.

For quality control purposes a random sample of 10 pens from each box is tested. If 2 or fewer faulty pens are found in the sample of 10, the box is accepted for sale. Otherwise the box is rejected.

(i) Explain what is meant by a random sample in this context. [1]

(ii) Find the probability that a randomly chosen box of 100 pens is accepted for sale. [1]

(iii) One morning 75 boxes are tested in this way. Find the probability that more than 5% of these boxes are rejected. [4]

An alternative testing procedure is trialled in which a random sample of 5 pens is initially taken from a box and tested.

- If there are no faulty pens in this sample of 5, the box is accepted.
- If there are 3 or more faulty pens in this sample of 5, the box is rejected.
- If there are 1 or 2 faulty pens in this sample, a second random sample of 5 pens is taken from the box. When the second sample has been tested, the box is accepted if the total number of faulty pens found in the combined sample of 10 is 2 or fewer and rejected otherwise.

(iv) Find the probability that a randomly chosen box of 100 pens is accepted for sale when the alternative testing procedure is used. [5]

(v) Explain why the factory manager might prefer to use the alternative testing procedure. [1]

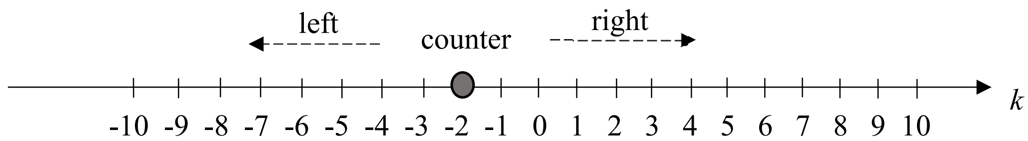
(i)	All pens have equal chance of being choosen and the selection of one pen does not affect the selection of another.
(ii)	X = no. of faulty pens out of 10 $X \sim \mathrm{B}(10, 0.06)$ $\mathrm{P}(X \leq 2) = 0.98116 \approx 0.981$
(iii)	Y = no. of faulty boxes out of 75 $Y \sim \mathrm{B}(75, 1-0.98116) = \mathrm{B}(75, 0.018838)$ $\mathrm{P}(Y > 5\% \text{ of } 75)$ $= \mathrm{P}(Y > 3.75)$ $= 1 - \mathrm{P}(Y \leq 3)$ $= 0.053454$ ≈ 0.0535
(iv)	T = no. of faulty pens out of 5

	$T \sim \mathrm{B}(5, 0.06)$ $\mathrm{P}(\text{a box is accepted})$ $= \mathrm{P}(T_1 = 0) + \mathrm{P}(T_1 = 1)\mathrm{P}(T_2 \le 1) + \mathrm{P}(T_1 = 2)\mathrm{P}(T_2 = 0)$ $= \mathrm{P}(T = 0) + \mathrm{P}(T = 1)\mathrm{P}(T \le 1) + \mathrm{P}(T = 2)\mathrm{P}(T = 0)$ $= 0.983$
(v)	Probability in (iv) is similar or slightly higher than (ii) with less testing.

13 DHS Prelim 9758/2021/02/Q8

In a computer game, a counter moves along a straight line and is originally placed at $k = 0$. At each stage, it takes one step to the right with probability p or one step to the left with probability q, where $q = 1 - p$. Each step is of length 1 unit and the step taken at each stage is independent of one another.

For illustration purpose, the counter is seen to be at $k = -2$ in the diagram below.



The counter takes 10 consecutive steps.

(i) Explain why the counter cannot possibly end at $k = 7$. [1]

(ii) Show that the probability that the counter ends at $k = 6$ is $45p^8q^2$. [2]

(iii) Given that the most probable end-point of the counter is $k = 6$, find exactly the range of values of p in the form $p_1 < p < p_2$ where p_1 and p_2 are constants to be determined. [4]

(iv) Write down the most probable end-point(s) of the counter if $p = 1 - p_1$. [1]

	Let R and L be the number of right and left steps taken respectively.
(i)	Note that since counter starts at 0, $R-L$ gives the number of the ending position. For the counter to end at $k=7$, $R - L = 7 \Rightarrow R = L + 7$ Also, since game is played for 10 stages, $R + L = 10$. Hence, $(L+7)+L = 10 \Rightarrow L = \frac{3}{2}$ (Or show that $R = \frac{17}{2}$) But L must be integer, hence it's not possible for counter to end at $k = 7$. **Alternative** Case 1: R is odd, L is odd (since must add up 10) $\Rightarrow$ $R-L$ is even. Case 2: R is even, L is even $\Rightarrow$ $R-L$ is even Hence, one can never end at an odd numbered position with 10 steps starting at 0.
(ii)	For the counter to end at $k=6$, $R - L = 6$ and $R + L = 10$ Hence $R=8$ and $L = 2$. Any combination of 8 right steps and 2 left steps occurs with probability p^8q^2. Number of such combinations is $\binom{10}{8} = 45$. Hence the probability that the counter ends at $k = 6$ is $45p^8q^2$.
(iii)	$R \sim \mathrm{B}(10, p)$

	For the most probable end-point to be $k=6$, the mode of R is 8. As R is binomial, it suffices to ensure the two inequalities below are satisfied: $\binom{10}{7}p^7q^3 < \binom{10}{8}p^8q^2 \quad \ldots(1)$ $\binom{10}{9}p^9q^1 < \binom{10}{8}p^8q^2 \quad \ldots(2)$ From (1), $8(1-p)<3p \Rightarrow p>\frac{8}{11}$ From (2) $2p<9(1-p) \Rightarrow p<\frac{9}{11}$ Hence we have $\frac{8}{11}<p<\frac{9}{11}$ i.e. $p_1=\frac{8}{11}$, $p_2=\frac{9}{11}$.
(iv)	Note that when $p=p_1$, the modes of R are 7 and 8 i.e. most probable end-points for the counter are $k=4$ and $k=6$. When $p=1-p_1$ (complement probability), most probable end-points for the counter will be $k=-4$ and $k=-6$. (by symmetry since interchange right with left) **Alternative** $p=1-\frac{8}{11}=\frac{3}{11}$ $R \sim \mathrm{B}\left(10, \frac{3}{11}\right)$ Modes of $R=2$ and 3 The two most probable end-points for the counter will be $k=-4$ and $k=-6$.