Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9758/01

Paper 1

16 September 2025

3 hours

Additional Materials : Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

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1 It is given that $2xy\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - y^2$ where $x > 0$, $y < 0$. Using the substitution $u = \frac{y}{x}$, show that the differential equation can be transformed to $\frac{2u}{1-3u^2}\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$. Hence find the general solution of y in terms of x. [6]

2 A series is given by $\sum_{r=1}^{n} 2(4-3x)^r$ where x is constant.

(a) Explain why this is a geometric series. Determine the range of values of x for the sum to infinity of this series to exist. [3]

(b) Using $x = 1$, and given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find $\sum_{r=0}^{n-1}\left[2(4-3x)^r + (r+1)(2r+5)\right]$, leaving your answer in the form of $n(an^2 + bn + c)$, where a, b and c are constants to be determined. [4]

3 **(a)** Find the first three non-zero terms in the Maclaurin series for $\mathrm{e}^x \sin(x + \pi)$. [3]

(b) It is given that the three terms found in part **(a)** are equal to the first three terms in the series expansion of $ax(1+bx)^c$ for small x, where a, b and c are constants. Find the exact values of a, b and c. Use these values to find the coefficient of x^4 in the expansion of $ax(1+bx)^c$, giving your answer as a simplified rational number. [5]

4 A sequence of real numbers $x_1, x_2, x_3, \ldots$ satisfies the recurrence relation

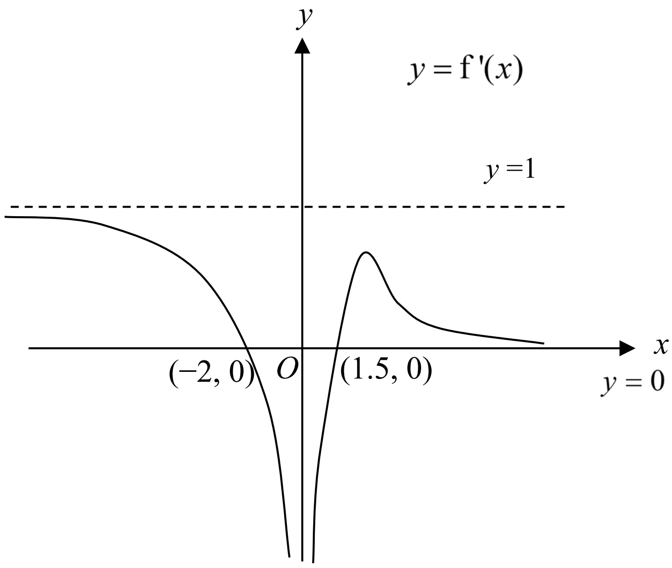
$$x_{n+1} = \frac{5x_n + 2}{2x_n + 3} \quad \text{for all } n \geq 1.$$

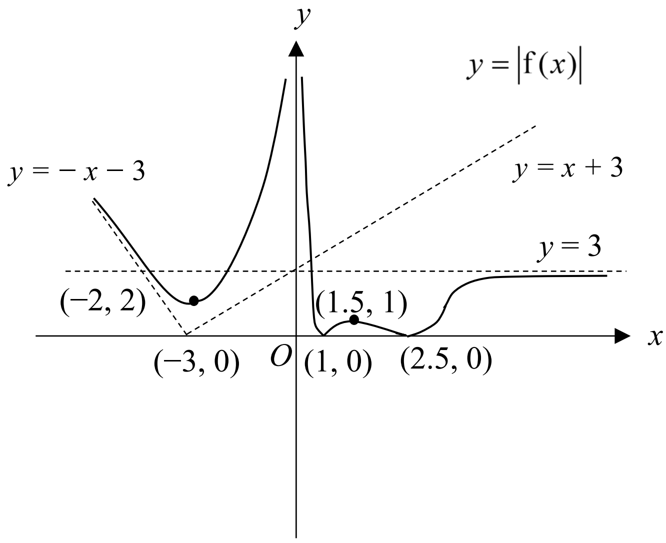
(a) Given that the sequence converges to l, find the possible exact values of l. [3]

(b) Describe how the sequence behaves when $x_1 = 3$. [1]

(c) Given that $x_5 = \frac{3503}{2158}$, find the value of x_1. [3]

5 The graphs of $y = \mathrm{f}'(x)$ and $y = |\mathrm{f}(x)|$ are shown below.





(a) State the nature of all turning point(s) of the graph of $y = \mathrm{f}(x)$. [2]

(b) State the range of values of x where f is decreasing. [2]

(c) Sketch the graph of $y = \mathrm{f}(x)$ indicating clearly the equations of the asymptotes, coordinates of the turning point(s) and the intersections with the axes. [3]

(d) On the copy of the graph of $y = |\mathrm{f}(x)|$ in the Printed Answer Booklet, sketch and label a line $y = kx + 3k$, where k is a constant. Hence state the range of values of k for which there is no real solution to the equation $|\mathrm{f}(x)| = kx + 3k$. [2]

6 A curve C is defined parametrically by

$$x = a\tan t, \qquad y = a\sec^2 t \sin t, \quad -\frac{\pi}{4} < t < \frac{\pi}{4},$$

where a is a positive constant.

(a) Show that $\dfrac{\tan t}{\sqrt{1+\tan^2 t}} = \sin t$. [1]

(b) By using part **(a)** or otherwise, find the cartesian equation of C in the form $y = \mathrm{f}(x)$, simplifying your answer. [2]

(c) Show that $\mathrm{f}(-x) = -\mathrm{f}(x)$. Hence sketch C. [2]

(d) Find the exact area of the region bounded by C, the x-axis and the lines $x = -A$ and $x = A$, where $0 < A < a$, leaving your answer in terms of A and a. [3]

7 The function f is defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^{x} + \frac{1}{2x+2} \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

A function g, defined for $x \in \mathbb{R}, x \geq 1,$ is such that $y \to \infty$ as x increases. It is also given that $\mathrm{g}(1) = -0.5.$

(a) Explain why the composite function fg exists and find the corresponding range of fg. [2]

(b) Given that $\mathrm{fg}(x) = \dfrac{x}{\sqrt{\mathrm{e}}} + \dfrac{1}{2\ln x + 1}$, find an expression for $\mathrm{g}(x)$ in terms of x. [2]

(c) The domain of f is now further restricted to $x > k$. State the least value of integer k for which the function f^{-1} exists. [1]

For the rest of the question, use the value of k found in part **(c)**.

(d) Without finding f^{-1},

(i) sketch, on the same diagram, the graphs of f, f^{-1} and ff^{-1} showing clearly the relationships between the graphs, [2]

(ii) find the gradient of the tangent to the graph of $y = \mathrm{f}^{-1}(x)$ at $x = \mathrm{e} + \dfrac{1}{4}$. [3]

8 A complex number z varies with t such that

$$z = 2\cos t + \mathrm{i}(3\sin t), \text{ where } 0 \leq t < 2\pi.$$

(a) By taking $x = \mathrm{Re}(z)$ and $y = \mathrm{Im}(z)$, sketch on an Argand diagram the curve that shows the positions of the points representing the complex number z. Find the cartesian equation of this curve. [3]

Two complex numbers z_1 and z_2 for two distinct values of t are such that $|z_1| = |z_2|$ and $0 < \arg(z_1) < \dfrac{\pi}{2}$.

(b) By referring to the Argand diagram in part **(a)**, find the possible values of $\arg(z_1 + z_2)$. [2]

It is given further that z_1 and z_2 are roots to the quadratic equation $z^2 + \alpha z + \beta = 0$.

(c) Explain whether it is necessary for α to be real. [2]

(d) Given that α is not real and $|z_1| = |z_2| = \dfrac{\sqrt{26}}{2}$, find the values for α and β. [4]

9 The point A has coordinates $(1, -2, 4)$ and the plane π_1 has equation $2x - y + 2z = 5$.

(a) Find the exact shortest distance between A and π_1. [2]

The plane π_2 has equation $x+3y-az=3$, where a is a constant.

(b) Find the vector equation of line l, which is the line of intersection of π_1 and π_2, in terms of a. [4]

The plane π_3 has equation $bx-2y+4z=3$, where b is a constant.

(c) Show that $(a-6)(b-4)=0$ if l is parallel to π_3. [2]

(d) State the conditions that a and b must follow for the three planes to form a triangular prism, where all the planes are non-parallel and they do not have any point in common. Justify your answer. [4]

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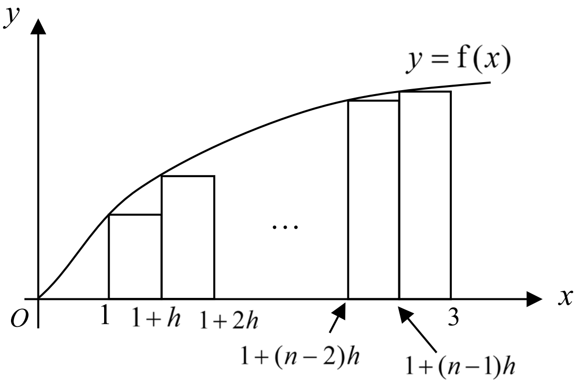


The diagram above shows a sketch of the graph of $y = \mathrm{f}(x)$ for $x \geq 0$. There are n rectangles each of width h drawn under the curve for $1 \leq x \leq 3$. Each rectangle when rotated through 2π radians about the x-axis, will result in a cylindrical disc. The total volume of the n cylindrical discs V_1, can be used to estimate the volume V, which is the actual volume generated when the region bounded by the curve, the lines $x = 1$ and $x = 3$, and the x-axis is rotated through 2π radians about the x-axis.

(a) Show that V_1, the total volume of the n cylindrical discs, is given by $V_1 = \pi h \sum_{r=0}^{n-1} \left[\mathrm{f}(1+rh)\right]^2$.
State the value of h in terms of n. [3]

(b) From the above diagram, it can be observed that $V_1 < V$. Write down V_2, a similar expression as V_1 where $V_2 > V$. [1]

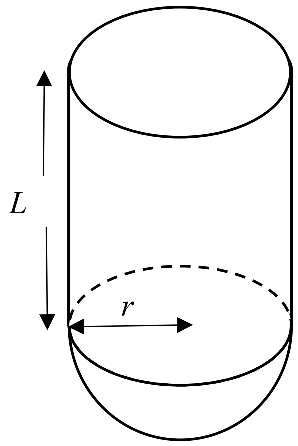
It is now given that $\mathrm{f}(x) = \dfrac{x^2}{\sqrt{1+\mathrm{e}^x}}$ for $x \geq 0$.

(c) Find the value of $\lim_{n \to \infty} V_1$. [2]

(d) Find the area of the region bounded by $y = \mathrm{f}(x)$ and another curve with equation $(x-1)^2 + (y-3)^2 = 9$, for $y \leq 3$. [4]

11 [The surface area and volume of a sphere are given by $4\pi r^2$ and $\frac{4}{3}\pi r^3$ respectively.]

An aquarist who has a fixed budget of \$$k$ wants to make a goldfish sanctuary. His design of the tank consists of a circular cover, a cylinder of length L cm and a hemispherical base of radius r cm neatly fitted together. The costs of the material per cm^2 used for the cover, the cylindrical surface and hemispherical base are \$20, \$10 and \$30 respectively. Assume that the tank is made of material with negligible thickness.

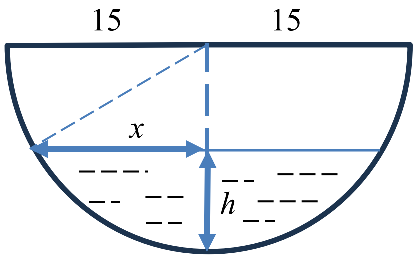


(a) Show that the volume of the tank is V cm^3, where $V = \frac{k}{20}r - \frac{10}{3}\pi r^3$. [3]

(b) As r varies, find the cost of the material used to make the hemispherical base in terms of k when V is at its maximum value. You need to show that V is a maximum. [3]

The aquarist has fixed the radius of the cylinder to be 15 cm. Initially, there is some water in the hemispherical part of the tank. However, due to a defect in the tank, water is leaking at a constant rate of 20 cm^3 per minute.

The depth of the water at the hemispherical bottom is denoted by h cm, and the radius of the water surface is x cm. The volume of water in the hemispherical bottom is given by W cm^3, where $W = \frac{\pi h\left(3x^2 + h^2\right)}{6}$.



(c) By formulating a relationship between x and h, find, when $h = 3$, the rate of change of

(i) the depth of water, [3]

(ii) the radius of the water surface. [3]

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