2023 Additional Mathematics Preliminary Examination Paper 2 Solutions

1

$y = 2x + 5 \quad \ldots (1)$

$y^2 = px \quad \ldots (2)$

Sub (1) into (2):

$(2x + 5)^2 = px$

$4x^2 + 20x + 25 = px$

$4x^2 + (20 - p)x + 25 = 0$

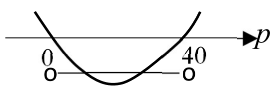
Since the line and curve do not meet, D < 0

$\therefore (20 - p)^2 - 4\times4\times25 < 0$

$400 - 40p + p^2 - 400 < 0$

$p^2 - 40p < 0$

$p\,(p - 40) < 0$



$0 < p < 40$

$\therefore$ greatest integer value of $p = \underline{39}$ [4]

2 $y = -3x^2 + 4.5x + 10$

(a) When $x = 0$, $y = 10$.

$\therefore$ Gabriel is 10 m above the water when he first leave the diving board. [1]

(b) $y = -3x^2 + 4.5x + 10$

$= -3\left(x^2 - 1.5x\right) + 10$

$= -3\left[\left(x - \frac{1.5}{2}\right)^2 - \left(\frac{1.5}{2}\right)^2\right] + 10$

$= -3\left[(x - 0.75)^2 - 0.562\,5\right] + 10$

$= -3(x - 0.75)^2 + 1.687\,5 + 10$

$\therefore y = -3(x - 0.75)^2 + 11.687\,5$ [2]

(c) Since $-3(x - 0.75)^2 \le 0$,

$y \le 11.687\,5$

Thus, Gabriel will not reach 12 m when executing the dive. [2]

3 $y = \dfrac{3x}{\sqrt{5 - 4x}}$

(a) $\dfrac{dy}{dx} = \dfrac{3\sqrt{5-4x} - 3x \times \frac{1}{2}(5-4x)^{-\frac{1}{2}}(-4)}{5 - 4x}$

$= \dfrac{3\sqrt{5-4x} + \frac{6x}{\sqrt{5-4x}}}{5 - 4x}$

$= \dfrac{3(5-4x) + 6x}{(5-4x)\sqrt{5-4x}}$

$= \dfrac{15 - 6x}{\sqrt{(5-4x)^3}}$ [3]

(b) When $x = 1$, $y = 3$ and $\dfrac{dy}{dx} = 9$.

$\therefore$ gradient of the normal $= -\dfrac{1}{9}$

$\therefore$ Equation of the normal is

$y - 3 = -\dfrac{1}{9}(x - 1)$

$y = -\dfrac{1}{9}x + \dfrac{28}{9}$ [2]

4 $\dfrac{d^2y}{dx^2} = \sin\left(6x - \dfrac{\pi}{2}\right)$

$\dfrac{dy}{dx} = \int \sin\left(6x - \dfrac{\pi}{2}\right) dx$

$= -\dfrac{1}{6}\cos\left(6x - \dfrac{\pi}{2}\right) + c$

When $x = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = \dfrac{1}{2}$,

$\therefore \dfrac{1}{2} = -\dfrac{1}{6}\cos\pi + c$

$\therefore c = \dfrac{1}{3}$

$\dfrac{dy}{dx} = -\dfrac{1}{6}\cos\left(6x - \dfrac{\pi}{2}\right) + \dfrac{1}{3}$

$y = \int -\dfrac{1}{6}\cos\left(6x - \dfrac{\pi}{2}\right) + \dfrac{1}{3}\, dx$

$y = -\dfrac{1}{36}\sin\left(6x - \dfrac{\pi}{2}\right) + \dfrac{1}{3}x + c_1$

When $x = \dfrac{\pi}{4}$, $y = \dfrac{13\pi}{12}$,

$\dfrac{13\pi}{12} = -\dfrac{1}{36}\sin\pi + \dfrac{\pi}{12} + c_1$

$\therefore c_1 = \pi$

$\therefore$ Equation of the curve is

$y = -\dfrac{1}{36}\sin\left(6x - \dfrac{\pi}{2}\right) + \dfrac{1}{3}x + \pi$ [6]

5 $y = k\sin(5\pi t) + 100$

(a) Since max blood pressure is 120 mmHg,

$k + 100 = 120$

$\Rightarrow \underline{k = 20}$

and min blood pressure = –20 + 100

= 80 mmHg [2]

(b) Period of $y = \dfrac{2\pi}{5\pi} = \underline{0.4 \text{ min}}$

In $0 \le t \le 0.6$, there are 1.5 cycle. [2]

(c) $20\sin(5\pi t)+100=100$

$\sin(5\pi t)=0$

$5\pi t=0,\ \pi,\ 2\pi$

$t=0,\ 0.2,\ 0.4$

Thus for y to be at least 100 mmHg,

$0\le t\le 0.2$

$\therefore$ Duration = 0.2 min [2]

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6 (a) LHS $=\dfrac{3\cos x+\cos 2x-1}{\cos^2 x+2\cos x}$

$=\dfrac{3\cos x+(2\cos^2 x-1)-1}{\cos x(\cos x+2)}$

$=\dfrac{2\cos^2 x+3\cos x-2}{\cos x(\cos x+2)}$

$=\dfrac{(2\cos x-1)(\cos x+2)}{\cos x(\cos x+2)}$

$=\dfrac{2\cos x-1}{\cos x}$

$=\dfrac{2\cos x}{\cos x}-\dfrac{1}{\cos x}$

$=2-\sec x$ [3]

(b) $\dfrac{3\cos x+\cos 2x-1}{\cos^2 x+2\cos x}=\tan^2 x+\sec x$

$2-\sec x=\tan^2 x+\sec x$

$2-\sec x=(\sec^2 x-1)+\sec x$

$\sec^2 x+2\sec x-3=0$

$(\sec x+3)(\sec x-1)=0$

$\sec x=-3$ or $\sec x=1$

$\cos x=-\frac{1}{3}$ or $\cos x=1$

basic angle = 70.528° or basic angle = 0°

For $-180°\le x\le 180°$,

$x=\pm(180°-70.528°)$ or $x=0°$

$\therefore\ x=-109.5°,\ 0°$ or $109.5°$ [3]

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7 (a) $3^{n+2}-3^n=\dfrac{5^{n+1}}{25^n}$

$3^n\times 3^2-3^n=\dfrac{5^n\times 5}{25^n}$

$3^n\left(3^2-1\right)=\left(\dfrac{5}{25}\right)^n\times 5$

$3^n\times 8=\dfrac{5}{5^n}$

$3^n\times 5^n=\dfrac{5}{8}$

$15^n=\dfrac{5}{8}$ [3]

(b) $\log_2(3x-2)=\log_4\left(x^2+1\right)+\dfrac{2}{\log_{\sqrt{3}}3}$

$\log_2(3x-2)=\dfrac{\log_2\left(x^2+1\right)}{\log_2 4}+\dfrac{2}{\log_{\sqrt{3}}\left(\sqrt{3}\right)^2}$

$\log_2(3x-2)=\dfrac{\log_2(x^2+1)}{2}+\dfrac{2}{2}$

$2\log_2(3x-2)=\log_2(x^2+1)+2$

$\log_2\dfrac{(3x-2)^2}{x^2+1}=2$

$\dfrac{(3x-2)^2}{x^2+1}=2^2$

$9x^2-12x+4=4(x^2+1)$

$5x^2-12x=0$

$x(5x-12)=0$

$x=0$ or $x=\dfrac{12}{5}$

But $x>\dfrac{2}{3}$, $\therefore\ x=\dfrac{12}{5}$ [4]

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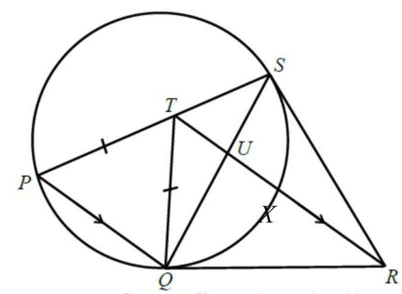


(a) $\angle TUQ=\angle SUR$ (vertically opposite $\angle$s)

$\angle RSU=\angle SPQ$ (alternate segment thm)

$=\angle TQP$ (base $\angle$s of isosceles Δ)

$=\angle QTU$ (alternate angles)

$\therefore \Delta TQU$ and ΔSRU are similar Δs (AA) [3]

(b) Since T is the midpoint of PS and $PQ//TU$, then by the converse of Midpoint Theorem, U is the midpoint of QS and $PQ=2TU$ and U is the midpoint of TX $\Rightarrow TX=2TU=PQ$

i.e. $TX \parallel PQ$ and $TX=PQ$

$\therefore PQXT$ is a parallelogram. [2]

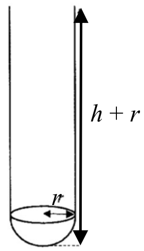
(c) From (a), $\angle RSU=\angle QTU$, this indicates the property of angles in the same segment. Thus, there is a circle passing through the points Q, R, S and T. [1]

9 **(a)** Surface area of test tube:

$2\pi rh + 2\pi r^2 = 12\pi$

$rh + r^2 = 6$

$h = \dfrac{6 - r^2}{r}$ [2]



(b) Volume of test tube,

$V = \frac{2}{3}\pi r^3 + \pi r^2 h$

$V = \frac{2}{3}\pi r^3 + \pi r^2\left(\dfrac{6-r^2}{r}\right)$

$V = \frac{2}{3}\pi r^3 + \pi r(6 - r^2)$

$V = 6\pi r - \frac{\pi}{3} r^3$ [2]

(c) $\dfrac{dV}{dr} = 6\pi - \pi r^2 = \pi(6 - r^2)$

For stationary value of V, $\dfrac{dV}{dr} = 0$.

$\therefore\ \pi(6 - r^2) = 0$

$\pi(\sqrt{6} + r)(\sqrt{6} - r) = 0$

Since $r > 0$, $\therefore\ r = \sqrt{6}$ cm

$\therefore$ Stationary value of V

$= 6\pi\sqrt{6} - \frac{\pi}{3}\sqrt{6^3}$

$= 6\pi\sqrt{6} - \frac{\pi}{3}\times 6\sqrt{6}$

$= 4\pi\sqrt{6}\ \text{cm}^3$

$\dfrac{d^2V}{dr^2} = -2\pi r$

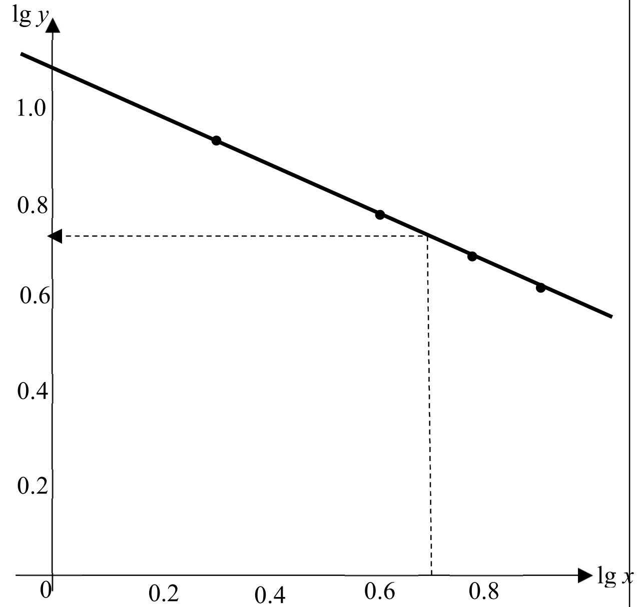
< 0 since $r > 0$

Thus, V is a maximum. [4]

10 **(a)** $x^n y = k \Rightarrow n \lg x + \lg y = \lg k$

$\therefore\ \lg y = -n \lg x + \lg k$

lg x	0.30	0.60	0.78	0.90
lg y	0.93	0.78	0.69	0.63



(b) $-n$ = gradient of line

$= \dfrac{1.08 - 0.62}{0 - 0.9}$

$= -0.511$

$\therefore\ n = 0.51$

$\lg k = \lg y$ – intercept

$= 1.08$

$\therefore\ k = 10^{1.08} = 12.02$ [3]

(c) When $x = 5$, $\lg x = 0.70$

$\lg y = 0.72$

$\therefore\ y = 10^{0.72} = 5.25$ [2]

11 **(a)** Let the centre of the circles be (a, a) since it lies on the line $y = x$.

Distance of centre from $(0, -3) = \sqrt{5}$

$\sqrt{a^2 + (a+3)^2} = \sqrt{5}$

$a^2 + a^2 + 6a + 9 = 5$

$2a^2 + 6a + 4 = 0$

$a^2 + 3a + 2 = 0$

$(a+1)(a+2) = 0$

$a = -1$ or $a = -2$

$\therefore$ The centres of C_1 and C_2 are $(-1, -1)$ and $(-2, -2)$ respectively. [3]

(b) $C_1 : (x+1)^2 + (y+1)^2 = 5$

gradient of line joining centre and (1, 0)

$= \dfrac{0+1}{1+1} = \dfrac{1}{2}$

$\therefore$ gradient of tangent at $(1, 0) = -2$

$\therefore$ equation of tangent at (1, 0) is

$y - 0 = -2(x - 1)$

$y = -2x + 2$ [2]

(c) $C_1 : (x+1)^2 + (y+1)^2 = 5$

$C_2 : (x+2)^2 + (y+2)^2 = 5$

Since the point of intersection lies on the x-axis, $y = 0$.

$\therefore\ (x+1)^2 + 1 = 5 \Rightarrow (x+1)^2 = 4$

$\therefore\ (x+2)^2 + 4 = 5 \Rightarrow (x+2)^2 = 1$

$\therefore\ x + 1 = \pm 2 \Rightarrow x = 1$ or -3

$\therefore\ x + 2 = \pm 1 \Rightarrow x = -1$ or -3

$\therefore$ the point of intersection is $(-3, 0)$. [3]

(d) Greatest distance between P and Q

$= \text{distance between the 2 centres} + 2\sqrt{5}$

$= \sqrt{(-1+2)^2 + (-1+2)^2} + 2\sqrt{5}$

$= \underline{\underline{\sqrt{2} + 2\sqrt{5} \text{ units}}}$. [2]

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12 $v = \dfrac{36}{(t-3)^2} - 9$

(a) $a = \dfrac{dv}{dt} = 36(-2)(t-3)^{-3}$

$= -\dfrac{72}{(t-3)^3}$

When $t = 0$, $a = -\dfrac{72}{(-27)} = \dfrac{8}{3}$

$\therefore$ Initial acceleration = $\underline{\underline{\dfrac{8}{3}}}$ m/s^2 [2]

(b) When the particle is at rest, $v = 0$.

$\therefore \dfrac{36}{(t-3)^2} - 9 = 0$

$36 - 9(t-3)^2 = 0$

$(t-3)^2 = 4$

$t - 3 = \pm 2$

$\Rightarrow t = 5 \text{ or } t = 1$

$\therefore$ The particle is at instantaneous rest when $t = 1$ or 5 s. [2]

(c) $s = \displaystyle\int \frac{36}{(t-3)^2} - 9 \, dt$

$= -36(t-3)^{-1} - 9t + c$

$= -\dfrac{36}{t-3} - 9t + c$

When $t = 0$, $s = 0$.

$\therefore\ 0 = 12 + c \Rightarrow c = -12$

$\therefore\ s = -\dfrac{36}{t-3} - 9t - 12$

$s = 0 \Rightarrow -\dfrac{36}{t-3} - 9t - 12 = 0$

$\Rightarrow (3t+4)(t-3) = -12$

$\Rightarrow 3t^2 - 5t - 12 = -12$

$\Rightarrow 3t^2 - 5t = 0$

$\Rightarrow t(3t-5) = 0$

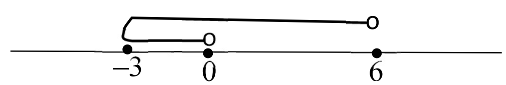
$\therefore\ t = 0 \text{ or } t = \dfrac{5}{3}$

Thus the particle returns to O only once at $t = \dfrac{5}{3}$. [3]

(d) $t = 0$: $s = 0$

$v = 0, t = 1$: $s = -\dfrac{36}{-2} - 9 - 12 = -3$ m

$t = 2$: $s = -\dfrac{36}{-1} - 18 - 12 = 6$ m



Distance travelled in the first 2 seconds

$= 3 \times 2 + 6$

$= \underline{\underline{12 \text{ m}}}$ [3]

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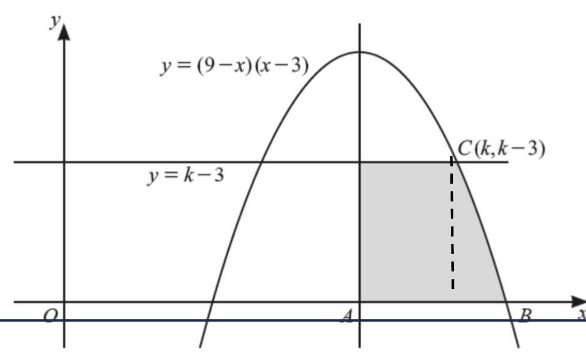

At $C(k, k-3)$,

$k - 3 = (9 - k)(k - 3)$

$(k-3)(1 - 9 + k) = 0$

$(k-3)(k-8) = 0$

Since $k > 0$, $k = 8$.

$A = (6, 0)$

$B = (9, 0)$

$\therefore$ Area of shaded region

= Area of rectangle + Area under curve

$= (8-6)\times(8-3) + \displaystyle\int_8^9 (9-x)(x-3)\, dx$

$= 2 \times 5 + \displaystyle\int_8^9 -x^2 + 12x - 27 \, dx$

$= 10 + \left[-\dfrac{x^3}{3} + 6x^2 - 27x\right]_8^9$

$= 10 + [-243 + 6\times81 - 27\times9]$

$\quad - \left[-\dfrac{512}{3} + 6\times64 - 27\times8\right]$

$= 10 + 0 - \left(-\dfrac{8}{3}\right)$

$= \underline{\underline{\dfrac{38}{3}}}$ units^2 [8]