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Raffles Girls' School

(SECONDARY)

Established in 1879

Name: ______________________________

Class: 4 / ______ Register No: ________

MATHEMATICS 2
YEAR FOUR

End-of-Year Assessment

Monday **4 October 2021** **2 hours 30 minutes**

INSTRUCTIONS TO CANDIDATES

Write your name, class and register number on all the work you hand in.

Write in dark blue or black pen.
You may use a 2B pencil for any diagram or graph.
Do not use paper clips, glue or correction tape.

Answer **all** the questions in the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

Omission of essential working will result in a loss of marks.

You are reminded of the need for clear presentation in your answers.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100** and the weighting is **40%**.

For examiners' use

Question	Marks Obtained
1	/ 5
2	/ 4
3	/ 6
4	/ 10
5	/ 4
6	/ 7
7	/ 7
8	/ 7
9	/ 7
10	/ 8
11	/ 12
12	/ 11
13	/ 12
P, A, U	
Total Marks	100

Parent's / guardian's Name: ______________________________

Signature: ____________________ Date: ________________

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q = 2\sin\frac{1}{2}(P+Q)\cos\frac{1}{2}(P-Q)$$

$$\sin P - \sin Q = 2\cos\frac{1}{2}(P+Q)\sin\frac{1}{2}(P-Q)$$

$$\cos P + \cos Q = 2\cos\frac{1}{2}(P+Q)\cos\frac{1}{2}(P-Q)$$

$$\cos P - \cos Q = -2\sin\frac{1}{2}(P+Q)\sin\frac{1}{2}(P-Q)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

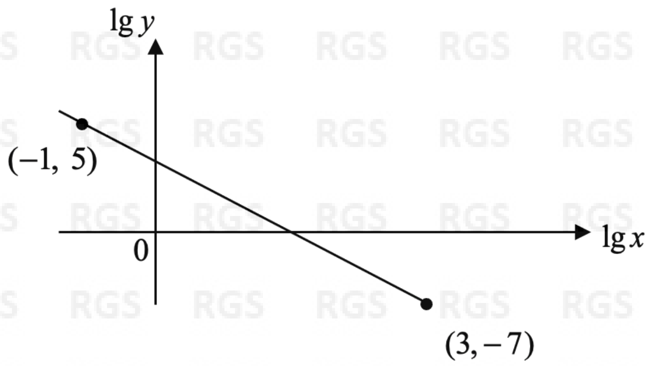
$$\Delta = \frac{1}{2} bc \sin A$$

1 Differentiate each of the following with respect to x, leaving your answers in the simplest form.

(a) $e^{x\sin x}$ [2]

(b) $\dfrac{4x}{\sqrt{2-x^2}}$ [3]

2 The variables x and y are related by an equation of the form $x^a y^b = 100$, where a and b are constants. The diagram shows part of a straight line graph obtained by plotting $\lg y$ against $\lg x$. It is given that the line passes through the points $(-1,\ 5)$ and $(3, -7)$.



Find the value of a and of b. [4]

3 **(i)** By finding the general term in the binomial expansion $\left(\frac{x^3}{2}+\frac{p}{x}\right)^8$, where p is a constant, explain if every term is dependent on x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $\left(\frac{1}{3}x^2+1\right)\left(\frac{x^3}{2}+\frac{p}{x}\right)^8$ is 1701, find the value(s) of the constant p. [3]

4 **(i)** Express $\dfrac{3x^2+4x+5}{(2-x)(x+3)^2}$ in partial fractions. [5]

(ii) Hence find the equation of the curve with gradient function $\dfrac{dy}{dx}=\dfrac{3x^2+4x+5}{(2-x)(x+3)^2}$,

given that the curve passes through the point $(1, -2)$. [5]

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5 Given that $f: x \mapsto \dfrac{3}{1+\sin\left(x-\dfrac{3\pi}{4}\right)}$ for $-\pi \le x \le \pi$, find

(i) the range of f, and [2]

(ii) the value of x for which f is minimum. [2]

6 **(i)** Prove that $\operatorname{cosec} x + \cot x = \cot\frac{x}{2}$. [3]

(ii) Hence solve the equation $\operatorname{cosec} 4\theta + \cot 4\theta = 3\tan 2\theta$ for $0° \le \theta \le 180°$. [4]

7 Given that A and B are acute angles such that $\sin(A-B)=-\dfrac{8}{17}$ and $\sin A\cos B=\dfrac{3}{34}$.

Without finding the angles A and B,

(i) explain why the angle $(A-B)$ lies in the fourth quadrant [1]

(ii) find the exact value of

(a) $\cos(A-B)$. [1]

(b) $\sin(A+B)$. [3]

(c) $\cos 2A-\cos 2B$. [2]

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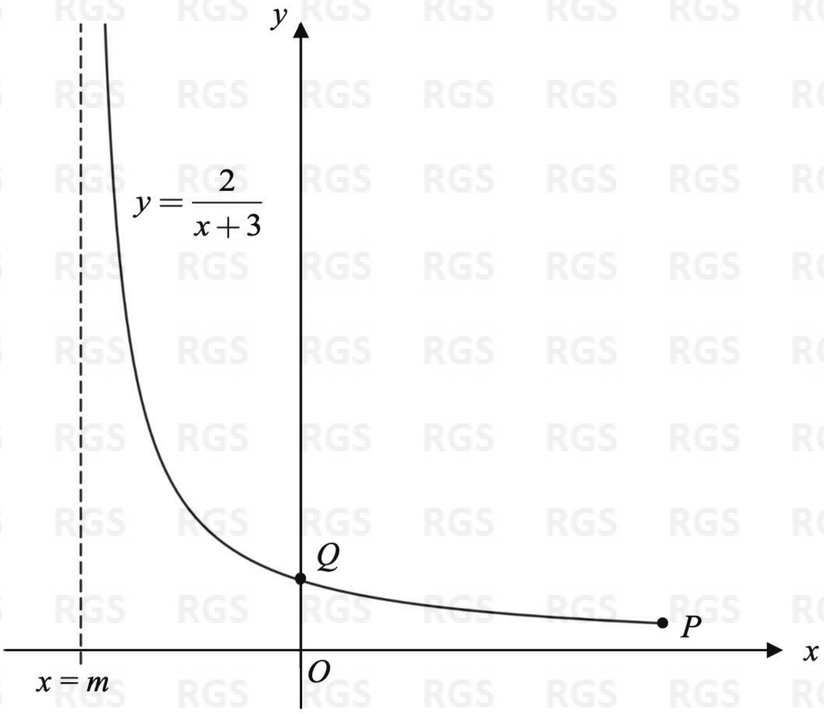
8 The diagram shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2}{x+3}$, for $m < x \leq 5$.

The graph has a vertical asymptote at $x = m$. It also passes through the points P and Q such that Q lies on the y-axis.



(a) State

(i) the value of m. [1]

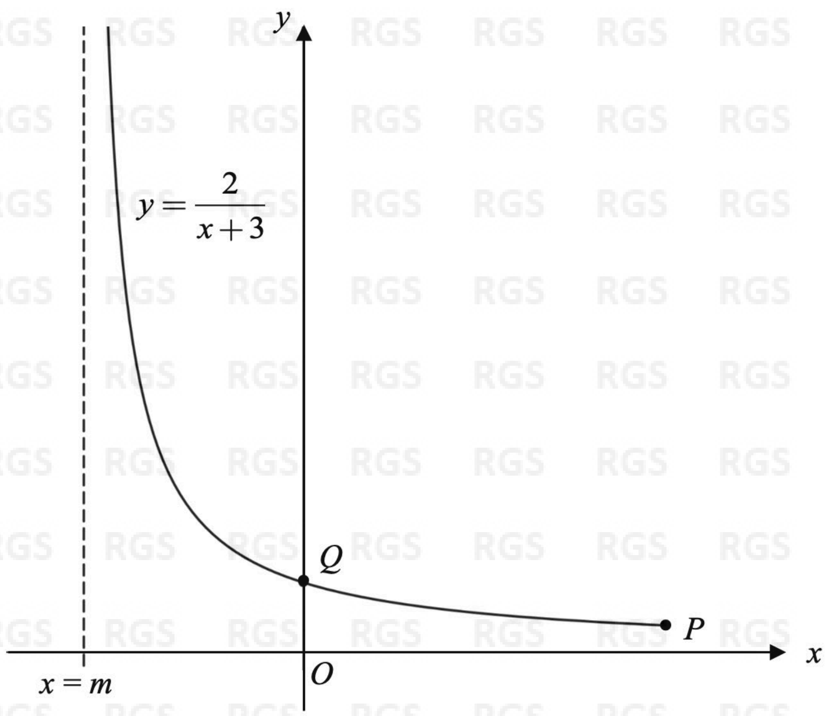
(ii) the coordinates of point Q, [1]

(iii) the range of f. [1]

(b) The equation $(8-x)(x+3)=8$ can be solved by drawing a suitable straight line on the same axes as the graph of $y=\dfrac{2}{x+3}$.

(i) Determine the equation of this line. [2]

(ii) Insert the line on the following diagram for $m < x \leq 5$, indicating clearly its axial intercept(s). [1]



(iii) Hence state the number of real roots of the equation $(8-x)(x+3)=8$, for $m < x \leq 5$. [1]

9 The equation of a line is $y = 2x$ and the equation of a curve is $y = ax^2 + c$, where a and c are constants such that $a \neq 0$.

(i) Given that the curve meets the line, what conditions must apply to the constants a and c? Express these conditions in terms of a and/or c. [3]

(ii) Hence explain why the equation $\frac{1}{p} + 1 = \frac{2}{\sqrt{p}}$ has real roots, for all $p > 0$. [1]

(iii) Solve the equation $\frac{1}{p} + 1 = \frac{2}{\sqrt{p}}$. [3]

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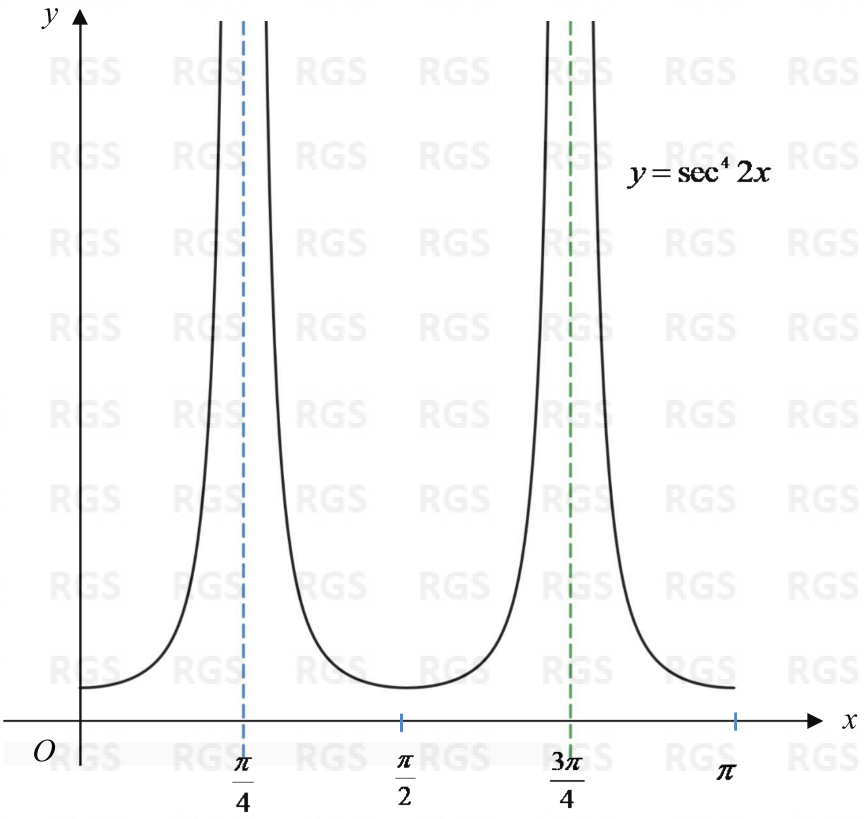
[Turn Over

10 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^3 2x\right) = 6\sec^4 2x - 6\sec^2 2x$. [3]

(ii) Hence evaluate $\int_0^{\frac{\pi}{8}} \sec^4 2x \, \mathrm{d}x$. [4]

(iii) The diagram shows part of the graph of $y = \sec^4 2x$ for $0 \leq x \leq \pi$, where $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ are vertical asymptotes.

Marvin claimed that *'the area of the region bounded by the curve and the x-axis from* $x = 0$ *to* $x = \pi$ *can be represented by the integral* $\int_0^{\pi} \sec^4 2x \, \mathrm{d}x$*'*. Explain, without further calculations, if you agree with Marvin. [1]



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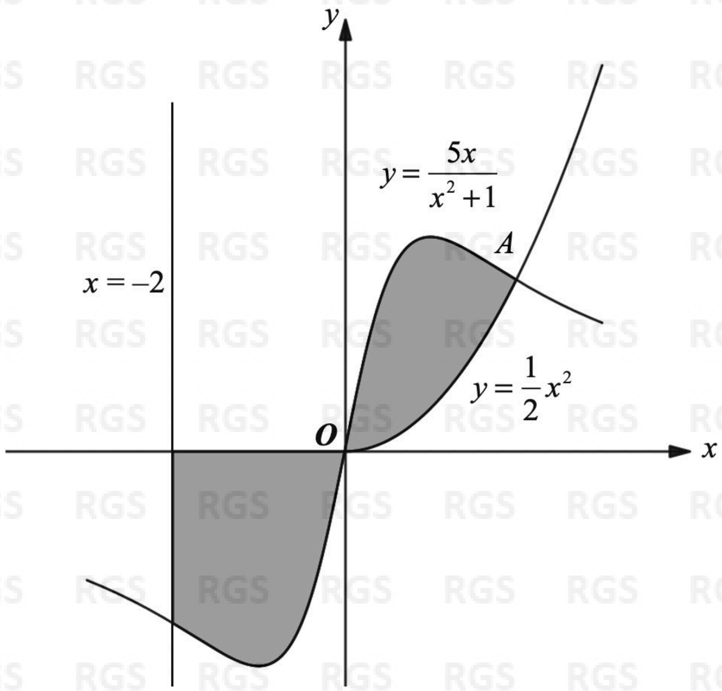
[Turn Over

11 **(a)** Differentiate $\ln\left(x^2+1\right)$ with respect to x. [1]

(b)



The diagram shows parts of the curve $y=\dfrac{5x}{x^2+1}$ intersecting the line $x=-2$, and intersecting another curve $y=\dfrac{1}{2}x^2$ at the origin and point A.

(i) Show that the x-coordinate of A is 2. [5]

(ii) Using your answer in part **(a)**, find the area of the shaded region, correct your answer to 3 significant figures. [6]

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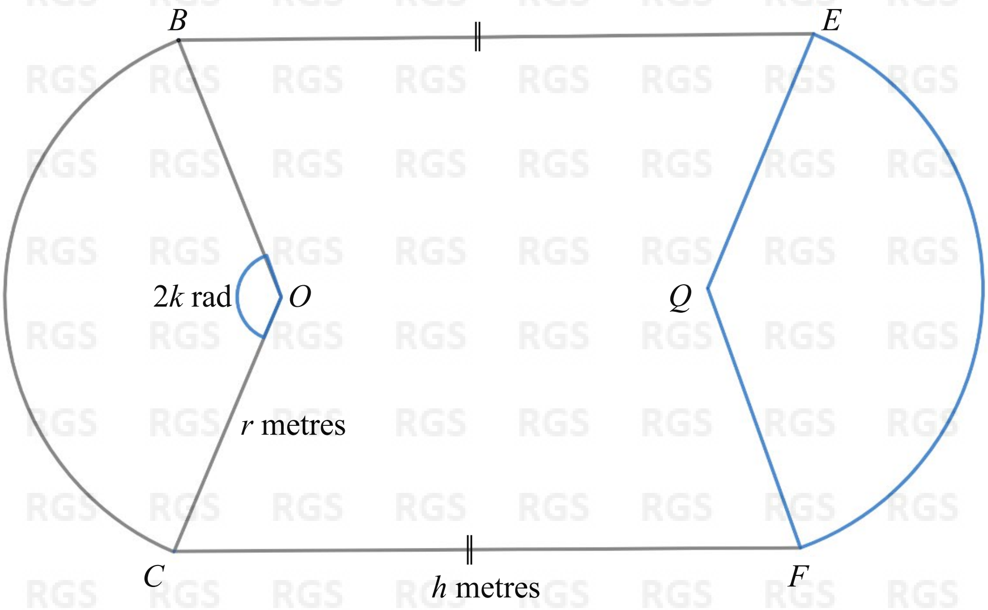
12 Stamford Girls' School is building a new campus, and the following diagram shows the design of the new school field. Sector OBC and sector QEF, with centres at O and Q respectively, are congruent to each other. Given that the sectors are such that $\angle BOC = \angle EQF = 2k$ radians, $OC = r$ m and $BE = CF = h$ m.



(i) Given that the perimeter of the field is 400 metres, express h in terms r. [2]

(ii) Given that the area of the field is A m^2, show that

$$A = \left(2k - \sin 2k - 4k\sin k\right)r^2 + \left(400\sin k\right)r.$$ [4]

(iii) Given that r and h can vary, and that $k = 1$, find the value of r for which A has a stationary value, leaving your answer correct to 3 significant figures. [3]

(iv) Use the second derivative test to determine the nature of the stationary value of A in part **(iii)**. [2]

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13 A curve has the equation $y = \frac{1}{2}\left|x^2 - 5x\right|$ for $0 \le x \le 6$.

(i) Sketch the graph of $y = \frac{1}{2}\left|x^2 - 5x\right|$, clearly indicating the coordinates of turning points, axial intercepts and end points if any. [3]

(ii) The line $y = x$ intersects the curve at two points O (0, 0) and P.

Find the point on the curve which is equidistant from O and P. [7]

(iii) Given that for $\frac{1}{2} < m < k$, the line $y = mx$ will cut the curve at two distinct points for all $0 \le x \le 6$, find largest possible value of k. [2]

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