Name:	School:	Target Grade:

PARADIGM

MOCK EOY PAPER 2025
SECONDARY 3 AMATH

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a nice comfortable spot without distraction.

2. Be fully focused for the whole duration of the test.

3. Speed is KING. Finish the paper as soon as possible then return-back to Check Your Answers.

4. As you are checking your answers, always find ways to VALIDATE your answer.

5. Avoid looking through line by line as usually you will not be able to see your Blind Spot.

6. If there is no alternative method, cover your answer and REDO the question.

7. Give non-exact answers to 3 significant figures, or 1 decimal place for angles in degree, or 2 decimal place for $$$, unless a different level of accuracy is specified in the question.

Wish you guys all the best in this test.

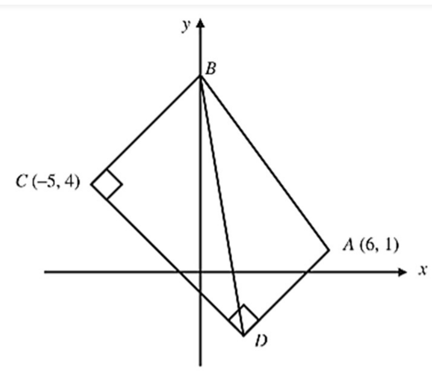
You can do it.

I believe in you.

Team Paradigm

PARADIGM [Turn Over]

1	**[Simultaneous Equation]** Find the coordinates of the points of intersection of the curve $x^2 - xy + y^2 = 16$ and the line $2x - 3y = 4$.	[5]
2	**[Complete The Square]** A dolphin jumps out of the water during a dolphin show. The height, $y\ m$, of the dolphin is given by $y = -4.5t^2 + 9t$, where t is the time in seconds after the dolphin jumps out of the water. (i) Express $y = -4.5t^2 + 9t$ in the form $y = a(t-h)^2 + k$. (ii) How long did it take the dolphin to reach the maximum height and what is the maximum height? (iii) Find the range of values of t where the height of the dolphin is at least 3.375 m.	[2] [2] [2] [4]
3	**[Nature of Roots]** (a) Find the largest integer k for which the line $y = 2x + k$ intersects the curve $x^2 + 2y^2 = 8$ at two distinct points. (b) Show and explain why there are no values of m for which the curve $y = (m-5)x^2 + 2mx + (m+3)$ is always positive.	[5] [4]
4	**[Surds]** A cylinder has volume $\left(\sqrt{45}\ +\ \sqrt{3}\right)$ cm3. If the base area of the cylinder is $\left(\sqrt{45}\ +\ \sqrt{3}\right)$ cm3, find the height of the cylinder. Give your answer in the form $a + b\sqrt{15}$, where a, b and are rational numbers.	[3]
5	**[Exponential]** Solve the equation $2e^{2x} = 7 + 15e^{2x}$ giving your answer correct to 1 significant figure.	[4]
6	**[Exponential]** Explain why$7^{n+1} - 4(7^n) - \frac{1}{7}(7^n)$ is an even number for all positive values of n.	[3]
7	**[Exponential Word Problem]** The mass, m grams, of a radioactive substance remaining, *t* days after being measured is given by $m = 10e^{-0.01t} + 0.2$. (a) Find the initial mass. (b) Sketch the graph $m = 10e^{-0.01t} + 0.2\ for\ t \geq 0$. (c) Find the least number of days it takes before the amount of substance is reduced to 5% of its initial mass. (d) Explain why the mass of the radioactive substance can never be less than 0.2 g.	[1] [2] [3] [2]

8	**[Logarithm]** Solve the following equations. (i) $\log_3(3x+2) - 2\log_3 x = 2$, (ii) $5\log_5 y - 2\log_y 5 = 9$.	 [6] [5]
9	**[Polynomial]** The function $f = x^3 - a^2x + b^2$, where a and b are positive constants, leaves a remainder of 1 when divided by $x + 3$. The function $g(x) = 3ax^3 - 2x^2 + bx - 6$ is exactly divisible by $x - 1$. (a) Find the value of *a* and of *b*.	[5]
10	**[Partial Fraction]** Express $\frac{7x^2-23x+6}{(x-2)(x^2-4)}$ in partial fractions.	[5]
11	**[Binomial Theorem]** (a) Write down the first three terms in the expansion, in ascending powers of *x*, of $(2+ax)^8$, where *a* is a constant. Give the terms in their simplest form. (b) In the expansion of $(1+2x-4x^2)(2+ax)^8$ the coefficient of x^2 is 22 times the coefficient of *x*. Given that *a* is positive, calculate the value of *a*. (c) Explain why there is no even power of *x* in the expansion of $\left(\frac{x^3}{2}+\frac{2}{3x}\right)^{15}$.	
12	**[Coordinate Geometry]** $ABCD$ is a trapezium such that the coordinates of A and C are $(6, 1)$ and $(-5, 4)$ respectively. Angle ADC and angle BCD are right angles and the gradient of AB is $-\frac{4}{3}$.  (i) Show that the coordinates of B are $(0, 9)$. (ii) Show that the coordinates of D are $(2, -3)$. (iii) Find the area of triangle ABD . (iv) Find angle ABC .	[2] [4] [2] [3]

13 **[Circles]**

A circle, centre C, has a chord AB where A is the point $(7, 27)$ and B is the point $(11, 15)$. The normal at A passes through the point D $(3, 3)$.

(a) Showing all your working, find the equation of the circle. [7]

(b) Show that D lies on the circle. [2]

14 **[Linear Law]**

t	1.0	15	2.0	2.5	3.0	3.5
s	4.20	4.65	5.09	5.50	5.89	6.25

An experiment is conducted to find the distance, s metres, travelled by an object over time, t seconds. The table below shows the corresponding values of t and s. It is known that s and t are related by the equation $s = a\sqrt{t} + \frac{b}{\sqrt{t}}$, where a and b are constants.

(a) On the grid below, plot $s\sqrt{t}$ against t and draw a straight line. [2]

(b) Use your graph to estimate a and b. [4]

(c) By drawing a suitable straight line on your graph, find the value of t that satisfies the equation $2s\sqrt{t} + 2t - 10 = at + b$. [3]

15 **[Trigonometry Simplifying & Quadrant]**

(a) Without using a calculator, find the exact value of $\cot\left(\frac{2\pi}{3}\right)$. [1]

(b) Given that $\sin A = -\frac{5}{13}$ and $\tan A > 0$, find without using a calculator, the numerical value of

(i) $\sec A$, [1]

(ii) $\tan(-A)$. [1]

16 **[Trigonometry Graph]**

The function f is defined by $f(x) = -3\cos(0.5x)$.

(i) Write down the period and amplitude of $f(x)$. [2]

(ii) Sketch the graph of $f(x)$ for $0° \leq x \leq 360°$. [2]

17 **[Trigonometry Solving]**

Solve the equation $2\tan^2 y + 5\sec y - 1 = 0$ for $0 \leq y \leq 2\pi$. [3]

Answer Key

1	$x^2 - xy + y^2 = 16$ ----- (1) $2x - 3y = 4$ $x = \frac{4+3y}{2}$ --------- (2) Substitute (2) into (1) $\left(\frac{4+3y}{2}\right)^2 - y\left(\frac{4+3y}{2}\right) + y^2 = 16$ $7y^2 + 16 - 48 = 0$ $(7y-12)(y+4) = 0$ $y = 1\frac{5}{7}$ or -4 $x = 4\frac{4}{7}$ or -4 The coordinates of the points of intersections are $\left(4\frac{4}{7}, 1\frac{5}{7}\right)$ and $(-4, -4)$. Ans: $\left(4\frac{4}{7}, 1\frac{5}{7}\right)$ and $(-4, -4)$	
2	(i) $y = -4.5t^2 + 9t$ $= -4.5(t^2 + 2t)$ $= -4.5(t^2 + 2t + 1 - 1)$ $= -4.5(t-1)^2 + 4.5$ (ii) Time taken to reach maximum height $= 1$ seconds Maximum height $= 4.5$ m (iii) $-4.5t^2 + 9t \geq 3.375$ $4.5t^2 - 9t + 3.375 \leq 0$ $4t^2 - 8t + 3 \leq 0$ $(2t-3)(2t-1) \leq 0$ $\frac{1}{2} \leq t \leq 1\frac{1}{2}$ Ans: (i)$-4.5(t-1)^2 + 4.5$ (ii) 4.5 m (iii) $\frac{1}{2} \leq t \leq 1\frac{1}{2}$	
3	(a) $x^2 + 2(2x+k)^2 = 8$ $9x^2 + 8kx + 2k^2 - 8 = 0$ Line and Curve intersect at 2 distinct points. $(8k)^2 - 4(9)(2k^2 - 8) > 0$ $64k^2 - 72k^2 + 288 > 0$ $8k^2 - 288 < 0$ $k^2 - 36 < 0$ $(k-6)(k+6) < 0$ $-6 < k < 6$ Largest integer k is 5. (b) Since $y > 0$, $(2m)^2 - 4(m-5)(m+3) < 0$ $4m^2 - 4m^2 + 8m + 60 < 0$ $8m + 60 < 0$ $m < -\frac{15}{2}$ But $y > 0$ means the x^2 coefficient must be positive, that is $m - 5 > 0 \Longrightarrow m > 5$. Since m must be $m < -\frac{15}{2}$ and $m > 5$ for $y > 0$, $\therefore$ there are no values of m for which $y > 0$ (is always positive)	

<table>
<tr><td>4</td><td>
$\left(\sqrt{45}-\sqrt{3}\right)\times h=\sqrt{45}+\sqrt{3}$
$\left(3\sqrt{5}-\sqrt{3}\right)\times h=3\sqrt{5}+\sqrt{3}$
$h=\frac{3\sqrt{5}+\sqrt{3}}{3\sqrt{5}-\sqrt{3}}$
$=\frac{3\sqrt{5}+\sqrt{3}}{3\sqrt{5}-\sqrt{3}}\times\frac{3\sqrt{5}+\sqrt{3}}{3\sqrt{5}+\sqrt{3}}$
$=\frac{45+6\sqrt{15}+3}{45-3}$
$=\frac{48+6\sqrt{15}}{42}$
$=\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)$ cm

Ans: $\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)cm$
</td><td></td></tr>
<tr><td>5</td><td>
$2e^{2x}=7+15e^{-2x}$
Let $y=e^{2x}$
$2y=7+\frac{15}{y}$
$2y^2-7y-15=0$
$(2y+3)(y-5)=0$
$y=5$ or $y=-\frac{3}{2}$
$e^{2x}=5$ or $e^{2x}=-\frac{3}{2}$ (rej)
$2x=\ln 5$
$x=\frac{\ln 5}{2}\approx 0.8$

Ans: 0.8
</td><td></td></tr>
<tr><td>6</td><td>
$7^{n+1}-4(7^n)-\frac{1}{7}(7^n)$
$7(7^n)-4(7^n)-\frac{1}{7}(7^n)$
$7^n\left(7-4-\frac{1}{7}\right)$
$\frac{20}{7}(7^n)$
$20(7^{n-1})$
Since 20 is divisible by 2, $7^{n+1}-4(7^n)-7^{n-1}$ is an even number for all positive values of n.

Ans: (a)1.58 $(3sf)$ (b)$x=27$ (c) $20(7^{n-1})$
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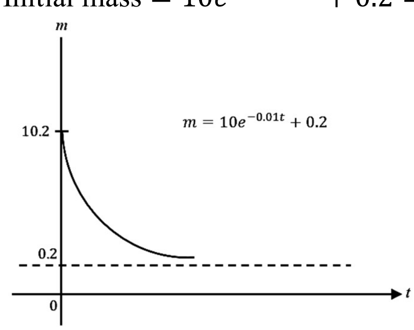
<tr><td>7</td><td>
(a) Initial mass $=10e^{-0.01(0)}+0.2=10.2$ g


(b)
Ans: (a) 10.2g (b)Shown (c) 348 (d) Therefore, the mass of the radioactive substance can never be less than 0.2g.
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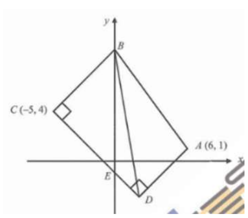
8	(i) $\log_3(3x+2) - 2\log_3 x = 2$ $\log_3(3x+2) - \log_3 x^2 = 2$ $\log_3 \frac{3x+2}{x^2} = 2$ $\frac{3x+2}{x^2} = 9$ $9x^2 - 3x - 2 = 0$ $(3x-2)(3x+1) = 0$ $x = \frac{2}{3} \quad x = -\frac{1}{3}$ (ii) $5\log_5 y - 2\ \log_y 5 = 9$ $5\log_5 y - 2\left(\frac{\log_5 5}{\log_5 y}\right) = 9$ $5(\log_5 y)^2 - 2 = 9\log_5 y$ $5u^2 - 9u - 2 = 0$ $(5u+1)(u-2) = 0$ $u = -\frac{1}{5} \quad u = 2$ $\log_5 y = -\frac{1}{5} \quad \log_5 y = 2$ $y = 5^{-\frac{1}{5}} \quad y = 5^2$ $= 0.725$ (3sf) $= 25$ Ansr: (i) $x = \frac{2}{3}$ (ii) $0.725(3s.f.)$	
9	$f(-3) = -27 + 3a^2 + b^2 = 1$ $3a^2 + b^2 = 28$ $g(1) = 3a - 2 + b - 6 = 0$ $b = 8 - 3a$ $3a^2 + (8-3a)^2 = 28$ $(a-3)(a-1) = 0$ $a = 3$ or $a = 1$ $a = 3, b = -1$ $a = 1, b = 5$ Ans: $a = 1, b = 5$	
10	$\frac{7x^2-23x+6}{(x-2)(x^2-4)} = \frac{7x^2-23x+6}{(x-2)^2(x+2)}$ Let $\frac{7x^2-23x+}{(x-2)^2(x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{c}{x+2}$ $7x^2 - 23x + 6 = A(x-2) + B(x+2) + C(x-2)^2$ Let $x = 2, -12 = 4B$ $B = -3$ Let $x = -2, \quad 80 = 16C$ $C = 5$ Let $x = 0, \quad 6 = -4A - 6 + 20$ $A = 2$ $\frac{7x^2-23x+6}{(x-2)^2(x+2)} = \frac{2}{x-2} - \frac{3}{(x-2)^2} + \frac{5}{x+2}$ Ans: $\frac{2}{x-2} - \frac{3}{(x-2)^2} + \frac{5}{x+2}$	

11	(a) $256 + 102ax + 1792a^2x^2$ (b) Coefficient of $x^2 = 2048a - 1024 + 1792a^2$ Coefficient of $x = 1024a + 512$ $2048a - 1024 + 1792a^2 = 22(1024a + 512)$ $7a^2 - 80a - 48 = 0$ $a = -\frac{4}{7}$ (rejected) or $a = 12$ (c) $(r+1)$ term: $\binom{15}{r}\left(\frac{x^3}{2}\right)^{15-r}\left(\frac{2}{3x}\right)^r$ $\binom{15}{r}\left(\frac{1}{2}\right)^{15-r}\left(\frac{2}{3}\right)^r x^{45-4r}$ 45 is odd and $4r$ is even, odd minus even is odd. So, there is no even power of x. Ans: (a) $256 + 1024ax + 1792a^2x^2$ (b) $a = 12$ (c) 45 is odd and 4r is even, odd minus even is odd. So, there is no even power of x.	
12	(i) $\frac{y-1}{0-6} = -\frac{4}{3}$ $y = 9$ $b(0,9)$ (ii) $m_{CB} = \frac{9-4}{0-(-5)} = 1$ $m_{CD} = -1$ Equation of CD: $y - 4 = -1(x+5)$ $y = -x - 1$ --- (1) $m_{AD} = 1$ Equation of AD: $y - 1 = 1(x - 6)$ $y = x - 5$ ---- (2) (1) = (2) $-x - 1 = x - 5$ $x = 2$ Sub $x = 2$ into (1), $y = -3$ $D(2, -3)$ (iii) area of $\Delta ABC = \frac{1}{2}\begin{vmatrix}0 & 2 & 6 & 0\\ 9 & -3 & 1 & 9\end{vmatrix}$ $\frac{1}{2}[(2+54) - (18-18)]$ 28 units^2 (iv)  $\angle EBA = 90° - \tan^{-1}\left(\frac{4}{3}\right)$ $= 36.870°$ $\angle CBE = 90° - \tan^{-1}(1)$ $= 45°$ $\angle ABC = 45° + 36.870°$ $= 81.9°$ Ans: (iii) $28\ units^2$ (iv) $81.9°(1dp)$	

13

(a) $m_{AD} = \frac{27-3}{7-(-3)} = \frac{12}{5}$

Equation of AD, $y = 3 = \frac{12}{5}(x+3)$

$y = 3 = \frac{12}{5}x + \frac{51}{5}$

$m_{AB} = \frac{27-15}{7-(-11)} = \frac{2}{3}$

Gradient of $\perp$ bisector of $AB = -\frac{3}{2}$

Midpoint of $AB = (-2, 21)$

Equation of perpendicular bisector of AB,

$y - 21 = -\frac{3}{2}(x+2)$

$y = -\frac{3}{2}x + 18$

At C, $\frac{12}{5}x + \frac{51}{5} = -\frac{3}{2}x + 18$

$\frac{12}{5}x + \frac{3}{2}x = 18 - \frac{51}{5}$

$\frac{39}{10}x = \frac{39}{5}$

$x = 2$

$y = 15$

Radius $= BC = 13$ units

Equation of circle, $(x-2)^2 + (y-15)^2 = 169$

(b) $CD = \sqrt{(2+3)^2 + (15-3)^2}$

$= 13$ units = radius of circle

Hence, D lies on the circle

Ans: (a) $(x -)^2 + (y-15)^2 = 169$ (b) Shown

14

(a)

t	1.0	1.5	2.0	2.5	3.0	3.5
$s\sqrt{t}$	4.20	5.70	7.20	8.70	10.20	11.69

(b) $s = a\sqrt{t} + \frac{b}{\sqrt{t}}$

$s\sqrt{t} = at + b$

$a =$ Gradient $= \frac{11.69-1.20}{3.50-0} = 2.997 \approx 3.00$

$b = s\sqrt{t}$ – intercept $= 1.20$

(c) $2s\sqrt{t} + 2t - 10 = at + b$

$2s\sqrt{t} + 2t - 10 = s\sqrt{t}$

$s\sqrt{t} = -2t + 10$

Draw $s\sqrt{t} = -2t + 10$

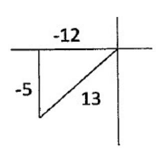
$t = 1.75$

Ans: (a) Shown (b) $b = s\sqrt{t} - intercept = 1.20$ (c) $t = 1.75$

15

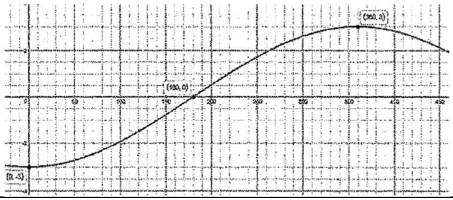
(a) $\cot\left(\frac{2\pi}{3}\right) = \frac{1}{\tan\left(\frac{2\pi}{3}\right)}$

$= -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$



(b)(i) $\sec A = \frac{1}{\cos A} = \frac{1}{-\frac{12}{13}} = -\frac{13}{12}$ or $-1\frac{1}{12}$

$\tan(-A) = -\tan A = -\frac{5}{12}$

16	(i) Period $= \frac{2\pi}{0.5} = 4\pi$ or $720°$ Amplitude $= 3$ (ii) 	
17	$2\ \tan^2 y + 5\sec y - 1 = 0$ $2(\sec^2 y - 1) + 5\sec y - 1 = 0$ $2\sec^2 y + 5\sec y - 3 = 0$ $(\sec y + 3)(2\sec y - 1) = 0$ $\sec y = -3$ or $\frac{1}{2}$ $\cos y = -\frac{1}{3}$ or 2 (rejected) Basic angle $= \cos^{-1}\frac{1}{3} = 1.23096$ $y = \pi - 1.23096, \pi + 1.23096$ $y = 1.91, 4.37$	