華僑中學

Hwa Chong Institution
Secondary 3 PAPER H

CANDIDATE NAME					
CLASS				REGISTER NUMBER	

Integrated Mathematics

2 hours 30 minutes

Additional materials:

Writing papers

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work that you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 92.

This document consists of **8** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

2. TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + \mathrm{c}^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

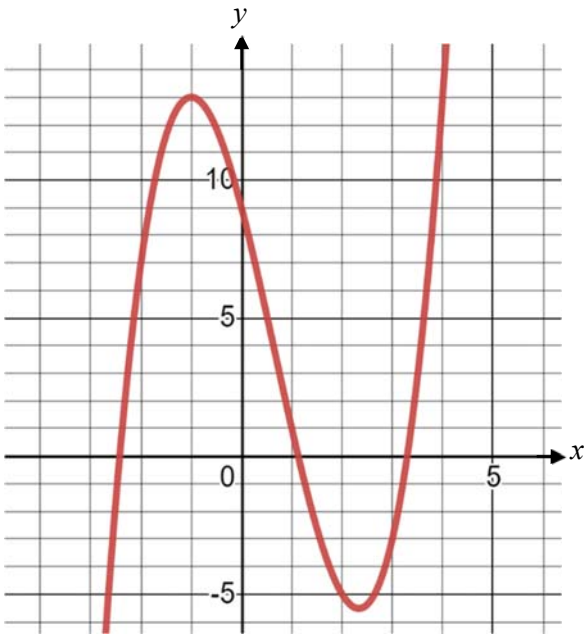
1 **(a)** When $f(x) = x^2 - x - 5$ is divided by $(x - a)$, the remainder is 1. Find the possible values of a. [2]

(b) Simplify $\dfrac{x^3 - 27}{(x-3)(x-4)(x+5)} - \dfrac{2x+29}{x^2+x-20}$. [4]

2 **(a)** The line $x + 3y = 1$ meets the curve $x^2 + 3x - 3y = 4$ at points A and B. Find the distance AB. [5]

(b) Find the range of values of c for which the line $y = 2x + c$ does not intersect the circle $x^2 + y^2 = 9$. [4]

3 The graph of $y = x^3 - 2x^2 - 7x + 9$ is shown below.



(i) Write down the equation of the straight line to be drawn on the same axes in order to solve $x^3 - 2x^2 - 5x + 6 = 0$ graphically. [1]

(ii) Using the graph given or otherwise, find the range of values of x such that $x^3 - 2x^2 - 5x + 6 > 0$. [2]

Please Turn Over

4 A plate of hor fun provides 91 g of carbohydrates, 27 g of fats and 35 g of proteins. A cup of bubble tea provides 75 g of carbohydrates, 4 g of fats and 4 g of proteins. Each gram of carbohydrate, fat and protein provides 4 calories, 9 calories and 4 calories of energy respectively.

(i) Write down a 2×3 matrix to represent the nutrients for a plate of hor fun and a cup of bubble tea. [1]

(ii) Write down a 3×1 matrix to represent the calories that nutrients provide. [1]

(iii) Using matrix operations, calculate the total energy provided by a plate of hor fun and a cup of bubble tea. [3]

5 The population growth of a bacterial sample is represented by $\mathrm{f}(x)=2^x$, with an initial population of one cell and a doubling time of 1 hour, where x is the number of hours after the start of the experiment.

(i) Explain why f^{-1} exists. [1]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [1]

(iii) Comment on the graphs of both $\mathrm{f}(x)$ and $\mathrm{f}^{-1}(x)$ if they were drawn on the same axes with the same scale used. [1]

6 **(a)** Solve the following simultaneous equations.

$$2x^{-\frac{1}{3}}+y^3=28$$
$$6x^{-\frac{1}{3}}-2y^3=-51$$ [4]

(b) Solve the equation $2x=\sqrt{2x+5}+1$. [3]

(c) By considering the valid range of x, without solving the equation, explain why $\sqrt{3x+4}+2\sqrt{x-3}=2$ has no real solution. [2]

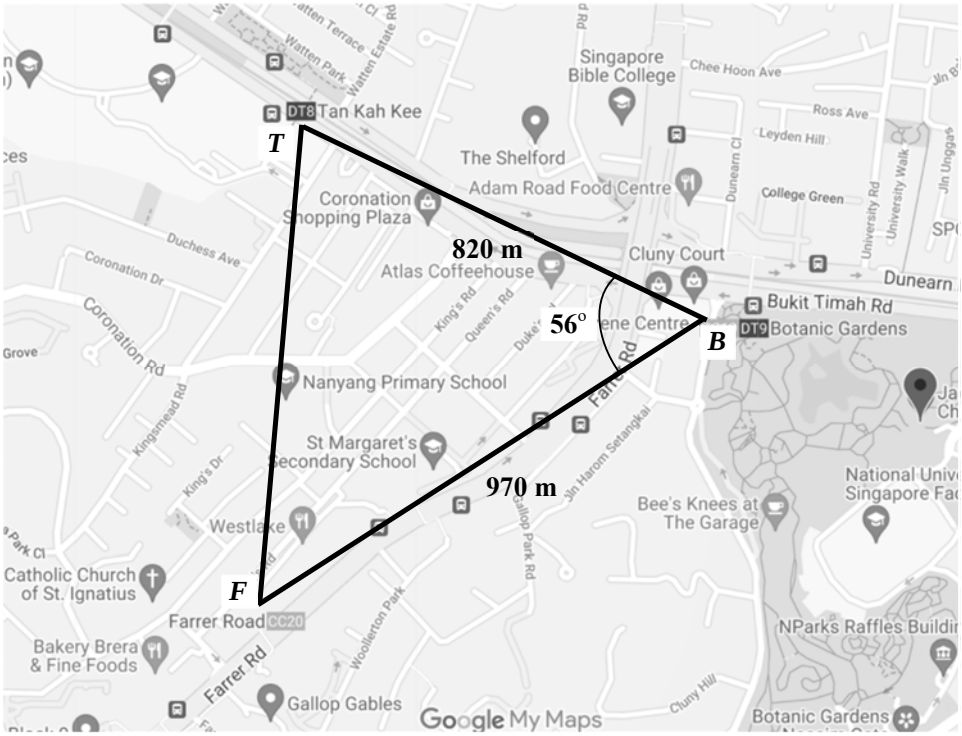
7 The function f is defined as $\mathrm{f}: x\mapsto 2x^2-4x-7,\ -1\le x\le 4$.

(i) Express $\mathrm{f}(x)$ in the form of $a(x+b)^2+c$, where a, b and c are constants. [2]

(ii) Sketch the graph of $y=\left|\mathrm{f}(x)\right|$, indicating clearly the y-intercept, the turning point and the end points. [4]

(iii) Solve $\left|\mathrm{f}(x)\right|=7$ algebraically. [4]

8 The triangle TBF lying on a horizontal plane are marked on the map below. It connects the three MRT stations: Tan Kah Kee Station (T), Botanic Gardens Station (B) and Farrer Road Station (F) respectively. It is given that $TB = 820$ m, $BF = 970$ m, and $\angle TBF = 56°$,



(i) Calculate the distance TF. [2]

A drone D is hovering vertically above B such that the angle of elevation of D from F, $\angle DFB$, is $18°$.

(ii) Calculate the height BD. [2]

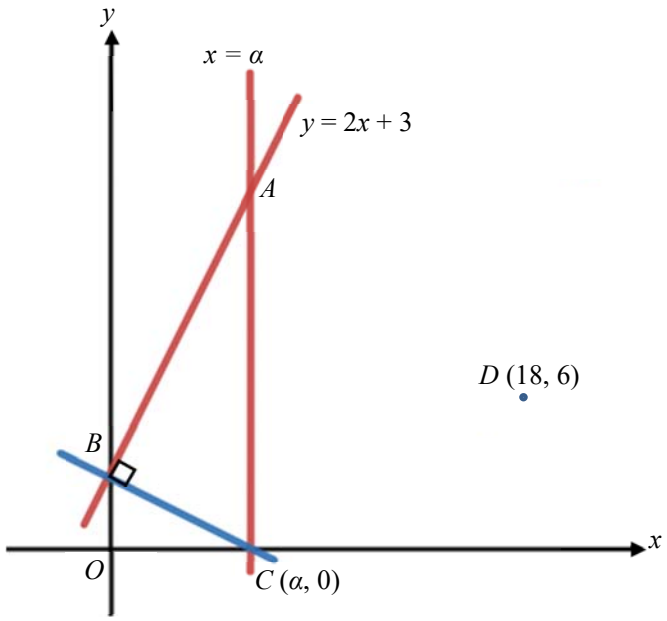
(iii) Without any calculations, state which angle is larger, $\angle DFB$ or $\angle DTB$. [1]

9 **(a)** The graph of $y = \frac{1}{x}$ has undergone two successive transformations. The new graph of $y = \frac{3}{x} - 1$ is obtained. Describe the transformations in order. [2]

(b) The graph of a function is stretched along y-axis by a factor of 2. The new graph of $y = 4\left(3^{2x}\right) - 6$ is obtained. Find the equation of the original function. [1]

Please Turn Over

10 The figure shows a straight line $y = 2x + 3$ intersecting the vertical line $x = \alpha$ at A and the y-axis at B. Point C has the coordinates $(\alpha, 0)$ such that AB is perpendicular to BC.



(i) Show that $\alpha = 6$. [3]

(ii) If the coordinates of D are $(18, 6)$, find the area of quadrilaterial $ABCD$. [3]

(iii) Show that the equation of the perpendicular bisector of CD is $y = -2x + 27$. [3]

(iv) Does point A lie on the perpendicular bisector of CD? Justify your answer. [2]

11 **(a)** Find the value of $\log_5 75 + 2\log_5 1 - \log_5 3$. [2]

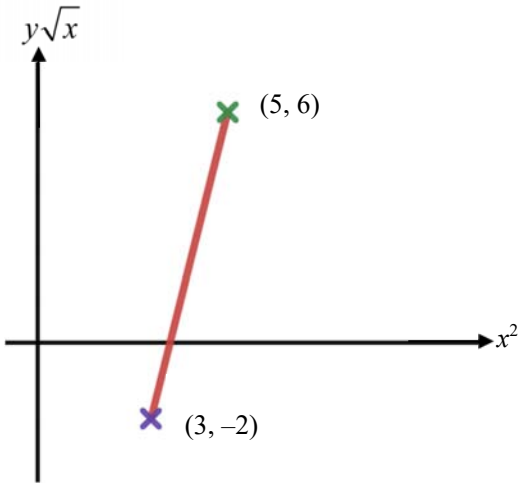
(b) Given that $u = \log_2 x$, express $\log_x\left(2x^2\right)$ in term of u. [3]

(c) Prove that $\log_{a^n} b^n = \log_a b$, where $a, b > 0$. **Hence**, find the value of

$$\left(\log_3 2\right)\left(\log_2 3 + \log_{2^2} 3^2 + \log_{2^3} 3^3 + ... + \log_{2^{2021}} 3^{2021}\right).$$ [5]

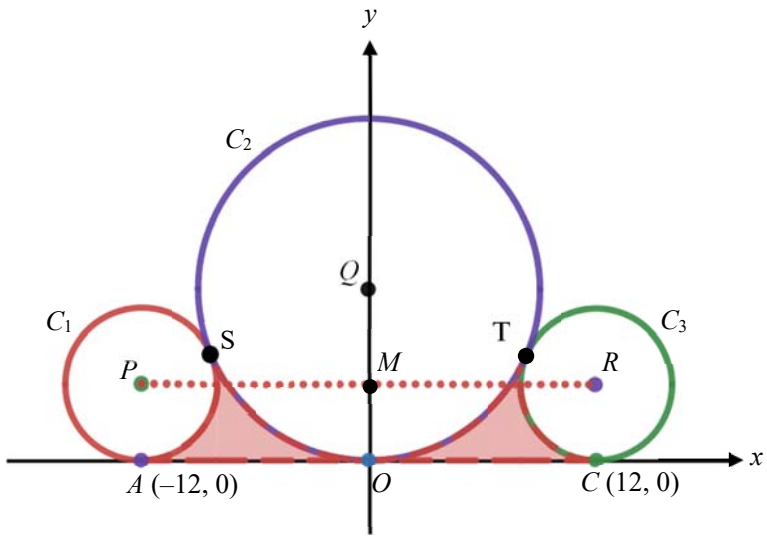
12 **(a)** Express the equation $y^2 3^{x-1} = 81$ where $y > 0$ in the form $\ln y = m(x-1) + c$, where m and c are constants. [3]

(b) The variables x and y are related by the equation $y = ax^{\frac{3}{2}} + bx^{-\frac{1}{2}}$ where a and b are constants. The diagram shows the straight line graph obtained by plotting $y\sqrt{x}$ against x^2. The line passes through the points (3, –2) and (5, 6). Find the values of a and b. [4]



Please Turn Over

13 The diagram below shows three circles, C_1, C_2 and C_3, centred at P, Q and R respectively. The three circles touch the x-axis at A, O and C respectively. Circle C_2 of radius r units touches two smaller circles, C_1 and C_3, of equal radii 4 units externally at S and T. It is also given that $AO = OC = 12$ units. M is the point of intersection of QO and PR.



(i) Find the equation of circle C_1. [2]

(ii) Write down, in term of r, the length of

(a) QR, [1]

(b) QM. [1]

(iii) Hence, show that $r = 9$. [2]

(iv) Show that $\angle QPM$, when rounded to 3 significant figures, is 0.395 radians. [1]

(v) Hence, determine the area of the two shaded regions bounded by the three circles and the x-axis. [4]

End of Paper