華僑中學

Hwa Chong Institution
Secondary 3 Class Test 2024

CANDIDATE NAME

CLASS

REGISTER NUMBER

Additional Mathematics **45 minutes**

Additional materials:

Writing papers

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work that you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 30.

This document consists of **3** printed pages.

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1. (i) Variables x and y are connected such that when $\lg y$ is plotted against x, a straight line is obtained. The straight line passes through the points (2 , 2) and (5 , 11). Express y in terms of x. [2]

(ii) Find the value of y when $x = 1$. [2]

2. The variables x and y are related by the equation $y = ax^3 + bx^2$, where a and b are constants. When the graph of $\frac{y}{x^2}$ against x is drawn, a straight line is obtained. The straight line passes through the points (3, 0) and (6,2).

(i) Find the value of a and of b. [4]

(ii) Find the value of y when $x = 6$. [2]

3. In a chemical reaction, a solid substance slowly dissolves in a liquid. At the start of the reaction, there are M grams of the substance. After t minutes, there are x grams of the substance still to be dissolved. It is known that x and t are related by the equation $x = 8e^{-\frac{t}{5}}$.

left.

(i) Find the value of M. [1]

(ii) Estimate the time taken for half of the substance to be dissolved. [2]

4. (a) Given that $\left(5^{1-x}\right)\left(3^{x+3}\right) = 20$, find the value of $\left(\frac{3}{5}\right)^x$. [2]

(b) Hence, solve the equation $\left(5^{1-x}\right)\left(3^{x+3}\right) = 20$. [1]

5. Solve the equation $2\log_3 x = 1 + 5\log_x 9$. [4]

6. Using long division, find the remainder when $-x^4 - x^3 + x^2 + 2x + 8$ is divided by $x^2 - 3$. [2]

7. It is given that $f(x) = ax^3 - (a+3b)x^2 + 2bx + c$ is divisible by $x^2 - 2x$.

(i) Show that $c = 0$ and $a = 2b$. [2]

When $f(x)$ is divided by $x - 1$, the remainder is 8 **more than** when it is divided by $x + 1$.

(ii) Find the values of a and b. Hence, or otherwise, factorise $f(x)$ completely. [4]

(iii) Sketch the graph of $y = f(x)$, indicate clearly the x and y intercepts. [2]

End of Paper

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MARKS 19

DATE 31/7/24

Q1(i) Let $Y = \lg y$, $X = x$

$Y = mX + c$

Gradient $= \frac{2-11}{2-5} = 3$

$Y = 3X + c$

When $X = 2$, $Y = 2$,

$2 = 3(2) + c$

$c = -4$

$\lg y = 3x - 4$

$y = 10^{3x-4}$ ✓ 2

Q1(ii) When $x = 1$,

$y = 10^{3(1)-4}$

$= \frac{1}{10}$ ✓ 2

Q2(i) $y = ax^3 + bx^2$

$y = x^2(ax + b)$

$\frac{y}{x^2} = ax + b$

Let $Y = \frac{y}{x^2}$, $X = x$

$Y = mX + c$

Gradient $= \frac{0-2}{3-6} = \frac{2}{3}$

$Y = \frac{2}{3}X + c$

When $X = 3$, $Y = 0$,

$0 = \frac{2}{3}(3) + c$

$c = -2$

$\frac{y}{x^2} = \frac{2}{3}x - 2$

$\therefore a = \frac{2}{3}, b = -2$ ✓ 4

Q2(ii) When $x = 6$,

$\frac{y}{6^2} = \frac{2}{3}(6) - 2$

$\frac{y}{36} = 2$

$y = 72$ ✓ 2

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Q3(i) When $t=0$,

$M = 8e^{-\frac{0}{5}}$

$= 8$

Q3(ii) $\frac{1}{2} \times 8 = 4$

$4 = 8e^{-\frac{t}{5}}$

$\ln 4 = \ln 8 - \frac{t}{5}$

$\ln 4 - \ln 8 = -\frac{t}{5}$

$\ln \frac{1}{2} = -\frac{t}{5}$

$-t = 5 \ln \frac{1}{2}$

$t = -5 \ln \frac{1}{2}$

≈ 3.465

$= 3.47$ mins (to 3s.f.) ~~= 4 mins (to nearest minute needed)~~

A

Q4(a) $(5^{1-x})(3^{x+3}) = 20$

$\lg 5^{1-x} + \lg 3^{x+3} = \lg 20$

$(1-x)\lg 5 + (x+3)\lg 3 = \lg 20$

$(-x^2 - 2x + 3)\lg 15 = \lg 20$

$(1-x)(x+3)$

$= x + 3 - x^2 - 3x$

$= -x^2 - 2x + 3$

Q4(a) $(5^{1-x})(3^{x+3}) = 20$

$\left(\frac{5}{5^x}\right)(27)(3^x) = 20$

$\frac{5}{5^x} \times 3^x = \frac{20}{27}$

$5\left(\frac{3^x}{5^x}\right) = \frac{20}{27}$

$\frac{3^x}{5^x} = \frac{4}{27}$

Q4(b) $\lg\left(\frac{3}{5}\right)^x = \lg\frac{4}{27}$

$x = \frac{\lg\frac{4}{27}}{\lg\frac{3}{5}}$

$x = 3.74$ (to 3s.f.)

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NAME Cai Yulong SUBJECT Amath CT MARKS

INDEX NO. 3 CLASS 3i4 DATE 31/7/24

Q5) $2\log_3 x = 1 + 5\log_x 9$

$2\log_3 x = 1 + 10\log_x 3$

$2\log_3 x = 1 + \frac{10}{\log_3 x}$

Let $\log_3 x$ be u.

$2u = 1 + \frac{10}{u}$

$2u^2 = u + 10$

$2u^2 - u - 10 = 0$

$(u+2)(2u-5) = 0$

x	2u	-5
u	$2u^2$	-5u
2	4u	-10

$u = -2$ or $u = 2.5$

$\log_3 x = -2$ $\log_3 x = 2.5$

$x = 3^{-2}$ $x = 3^{2.5}$

$= \frac{1}{9}$ $= 15.6$ (to 3 s.f.)

4

Q6)

$$\begin{array}{r} -x^2 - x - 2 \\ x^2 - 3 \overline{\big) -x^4 - x^3 + x^2 + 2x + 8} \\ -x^4 \quad\quad + 3x^2 \quad\quad\quad \\ \hline -x^3 - 2x^2 + 2x + 8 \\ -x^3 \quad\quad + 3x \quad\quad \\ \hline -2x^2 - x + 8 \\ -2x^2 \quad\quad + 6 \\ \hline -x + 2 \end{array}$$

Remainder $= -x + 2$

2

$x^2-2x = 0(x-$

Q7(i) $f(x) = ax^3 - (a+3b)x^2 + 2bx + c$

$x^2 - 2x = 0$

$x^2 = 2x$

$x = \sqrt{2x}$

$f(\sqrt{2x})$:

$a(\sqrt{2x})^3 - (a+3b)(\sqrt{2x})^2 + 2b(\sqrt{2x}) + c = 0$

$a(\sqrt{2x})^3 - 2xa - 6xb + 2b(\sqrt{2x}) + c = 0$

$a[(\sqrt{2x})^3 - 2x] - b[2(\sqrt{2x}) - 6x] + c = 0$

$a[2\sqrt{2x} - 2x] - b[2\sqrt{2x} - 6x] + c = 0$

(ii) $f(x) = 2bx^3 - 5bx^2 + 2bx$

$= (x-1)(x+1)Q(x) + 2x + y$

$f(-1)$:

$y - z$

$f(1)$:

$z + y = y - z + 8$

$2z = 8$

$z = 4$

$f(x) = (x-1)(x+1)Q(x) + 4x + y$

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NAME ______ SUBJECT ______ MARKS ______

INDEX NO. ______ CLASS ______ DATE ______

Q7(i) Since $f(x)$ is divisible by x^2-2x, it is divisible by x and $x-2$ hence $f(0)=0$ and $f(2)=0$.

$f(0)=c$ hence $c=0$

$f(2) = 8a-4(a+3b)+4b$

$= 4a-8b$

Hence $4a-8b=0 \longrightarrow a=2b$ (shown)

Q7(ii) $f(1)=f(-1)+8$

Hence $-b=(-2a-5b)+8 \longrightarrow 2b=4-a$

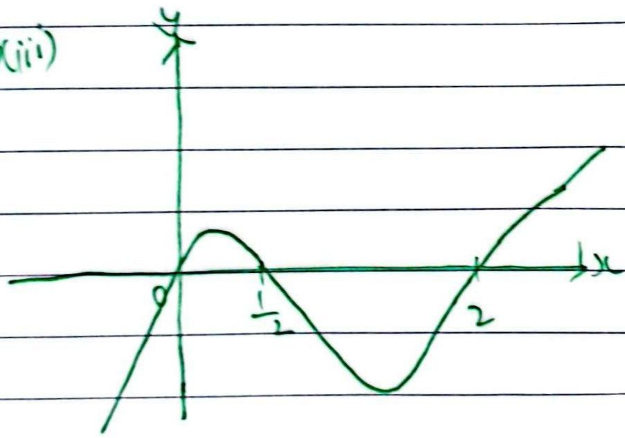
Since $a=2b$, we have $b=1$ and so $a=2$.

Hence, $f(x)=2x^3-5x^2+2x$

$= x(2x^2-5x+2)$

$= x(2x-1)(x-2)$

Q7(iii)



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