Practice Paper C Sec 3 Math Marking Scheme

Q1(a) $(x^{\frac{1}{3}}-y^{\frac{1}{3}})(x^{\frac{1}{3}}+y^{\frac{1}{3}})\ \times$

$(x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}})(x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}})$ **M1**

$=(x-y)(x+y)$ **A1**

$=x^2-y^2$

Q1(b) $\dfrac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}+\dfrac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}$

$=\dfrac{(x+a)-2\sqrt{x+a}\sqrt{x-a}+(x-a)+(x+a)+2\sqrt{x+a}\sqrt{x-a}+(x-a)}{(x+a)-(x-a)}$ **M1**

$=\dfrac{2x+2a+2x-2a}{2a}$ **M1**

$=\dfrac{2x}{a}$ **A1**

Award M1 for rationalising *y*

Q1(c) $\log_5 x+1=\dfrac{6}{\log_5 x}$ **M1**

$(\log_5 x)^2+\log_5 x-6=0$ **M1**

$(\log_5 x+3)(\log_5 x-2)=0$

$\log_5 x=-3$ or $\log_5 x=2$

$x=\dfrac{1}{125}$ or $x=25$ **A1**

Q2 Given that $\tan A=\dfrac{3}{4}\ \Rightarrow\ A$ is in the 1st & 3rd Q

$\sin B=-\dfrac{4}{5}\ \Rightarrow\ B$ is in the 3rd & 4th Q **M1**

When A is in the 1st Q, $\sin A=\dfrac{3}{5}$,

When A is in the 3rd Q, $\sin A=-\dfrac{3}{5}$. **M1**

When B is in the 3rd Q, $\tan B=\dfrac{4}{3}$,

When B is in the 4th Q, $\tan B=-\dfrac{4}{3}$. **M1**

When A and B are in different quadrants :

Case (i) A is in the 1st Q and B is in the 3rd Q,

$2\sin A-\tan B=2(\dfrac{3}{5})-\dfrac{4}{3}=\dfrac{18-20}{15}=-\dfrac{2}{15}$ **A1**

Case (ii) A is in the 1st Q and B is in the 4th Q,

$2\sin A-\tan B=2(\dfrac{3}{5})-(-\dfrac{4}{3})=\dfrac{18+20}{15}=2\dfrac{8}{15}$ **A1**

Case (iii) A is in the 3rd Q and B is in the 4th Q,

$2\sin A-\tan B=2(-\dfrac{3}{5})-(-\dfrac{4}{3})=\dfrac{-18+20}{15}=\dfrac{2}{15}$ **A1**

-1 for additional case

Q3(i) $\mathbf{A}^{-1}=\begin{bmatrix}1&-1.5\\-1&2\end{bmatrix}$ and $\mathbf{B}^{-1}=\begin{bmatrix}1&-4\\-1&5\end{bmatrix}$ **B2**

(ii) $\mathbf{M}=\begin{bmatrix}1&-1.5\\-1&2\end{bmatrix}\begin{bmatrix}4&0\\6&2\end{bmatrix}\begin{bmatrix}1&-4\\-1&5\end{bmatrix}$ **M1**FT

$=\begin{bmatrix}-5&-3\\8&4\end{bmatrix}\begin{bmatrix}1&-4\\-1&5\end{bmatrix}$ **M1** FT

$=\begin{bmatrix}-2&5\\4&-12\end{bmatrix}$ **A1**

Q4(a) $x-h=hx^2+7x\quad\Rightarrow\quad hx^2+6x+h=0$

As the line meets the curve i.e. $b^2-4ac\geq 0$ **M1**

$\Rightarrow\quad 6^2-4h^2\geq 0$

$h^2\leq 9$ **M1**

$-3\leq h\leq 3, h\neq 0$ **A1**

No M1 for $b^2-4ac>0$

No M1 for $b^2-4ac=0$

(b) As y is always positive and $a>0$

$\Rightarrow\ b^2-4ac<0$

where $a=1,\ b=k-4$ and $c=1$

$(k-4)^2-4<0$ **M1**

$k^2-8k+12<0$

$(k-2)(k-6)<0$ **M1**

$\Rightarrow\ 2<k<6$ **A1**

No marks for this question if student considers $b^2-4ac>0$

Q5 As $x^2+x-6=(x-2)(x+3)$ **M1**

Let $x=2$ and subs into the eqn. **M1**

$\Rightarrow\quad a-b=13\quad ----\ (1)$

Let $x=-3$ and subs into the eqn.

$\Rightarrow\quad 4a+b=67\quad ----\ (2)$ **M1**

Solve the above two equations, $a=16$ and $b=3$ **A1**

Method marks are awarded for substituting $x=2$ and $x=-3$

Q6(a)

$\dfrac{(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)}{(a+b)(b+c)(a+c)}$ **M1**

$=\dfrac{a^2-b^2+b^2-c^2+c^2-a^2}{(a+b)(b+c)(a+c)}$ **M1**

$=\dfrac{0}{(a+b)(b+c)(a+c)}=0$ **A1**

Students should simplify to obtain 0 as the answer

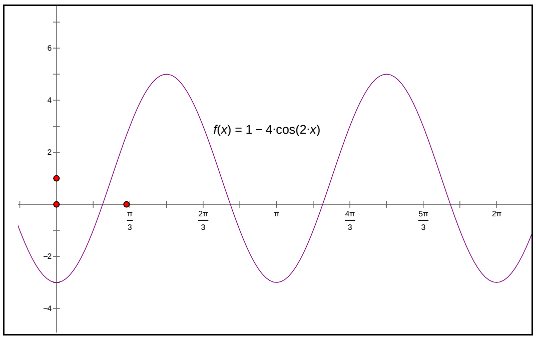
(b) $\dfrac{2(x^2-3x+2)}{(x-1)(x-5)} \times \dfrac{(x-5)(x+4)}{3(x^2+2x-8)}$

$= \dfrac{2(x-1)(x-2)}{(x-1)(x-5)} \times \dfrac{(x-5)(x+4)}{3(x-2)(x+4)}$ **M1 M1**

$= \dfrac{2}{3}$ **A1**

If constants "2" and "3" are missing from factorisation, award 2 method marks

Q7



[G1] for correct period
[G1] for correct amplitude
[G1] for smoothness and correct shape of the graph

Q8(a)

$\sin 3x \;=\; \sin(90° - 2x)$ **M1**

$\Rightarrow\; 3x = 90° - 2x,\; 180° - (90° - 2x),$ **M1**
$360° + (90° - 2x),\; 720° + (90° - 2x)$

$\Rightarrow\; x = 18°,\; 90°,\; 90°,\; 162°$

Hence $x = 18°,\; 90°,\; 162°$ **A1**

Award 2 method marks if students obtained $x = 18°$

(b) Let
$\tan(2x - 40°) = 2.718 \;\Rightarrow\; x = 69.800°$ (B.A.)
M1

Hence $2x - 40° = 180° - 69.80°,$
$360° - 69.80°$ **M1**

$\Rightarrow\; x \simeq 75.1°,\; 165.1°$ **A1**

Deduct A1 for additional x values

Q9(a)

LHS$=\sin^2 x(\sin^2 x + \cos^2 x) + \cos^2 x$ **M1**

$\equiv \sin^2 x + \cos^2 x \equiv 1 = \text{RHS}$ **M1**

Alternatively
LHS$=(\sin^2 x)^2 + (1-\cos^2 x)\cos^2 x + \cos^2 x$

$\equiv (1-\cos^2 x)^2 + (1-\cos^2 x)\cos^2 x + \cos^2 x$
M1

$\equiv 1 - 2\cos^2 x + \cos^4 x + \cos^2 x$ **M1**
$- \cos^4 x + \cos^2 x$

Alternatively

LHS$=\dfrac{2 - \dfrac{1}{\sin^2 x}}{\dfrac{1}{\sin^2 x} + \dfrac{2\cos x}{\sin x}} \equiv \dfrac{\dfrac{2\sin^2 x - 1}{\sin^2 x}}{\dfrac{1 + 2\sin x\cos x}{\sin^2 x}}$ **M2**

$\equiv \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x + 2\sin x\cos x + \cos^2 x}$

$\equiv \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{(\sin x + \cos x)^2} \equiv \dfrac{\sin x - \cos x}{\sin x + \cos x}$

$= \text{RHS}$

M2

Q10(a)

$z \;\alpha\; \dfrac{x^2}{\sqrt{y}} \quad\Rightarrow\quad z = \dfrac{kx^2}{\sqrt{y}}$ **M1**

When $x = 2$ and $y = 4$ then $z = 6$,

$6 = \dfrac{k2^2}{\sqrt{4}} = 2k$

$\Rightarrow\; k = 3$ **A1**

Hence $z = \dfrac{3x^2}{\sqrt{y}}$. Therefore $25 = \dfrac{3x^2}{\sqrt{9}}$ **M1**^{FT}

$\Rightarrow\; x = 5$ or $x = -5$ (rej) **A1**

(b) Let $x_1 = 2x,\; y_1 = \dfrac{y}{4}$

then $z_1 = \dfrac{k(2x)^2}{\sqrt{\dfrac{y}{4}}} = \dfrac{8kx^2}{\sqrt{y}} = 8z$ **M1**^{FT}

Acceptable answers:
Hence z is increased to 800%
Hence z is increased by 700%
% change is 700% **A1**

Q11

Given $\left|x^2 - 3\right| \;=\; 2x$

$\Rightarrow\; x^2 - 3 = 2x$ or $x^2 - 3 = -2x$ M1

Case 1 : $x^2 - 3 = 2x$
or $x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$

$\equiv 1 = \text{RHS}$

(b) $\text{LHS} = \dfrac{2-(1+\cot^2 x)}{1+\cot^2 x + 2\cot x}$ **M1**

$\equiv \dfrac{1-\cot^2 x}{(1+\cot x)^2}$ **M1**

$\equiv \dfrac{1-\cot x}{1+\cot x}$ **M1**

$\equiv \dfrac{1-\dfrac{\cos x}{\sin x}}{1+\dfrac{\cos x}{\sin x}} \equiv \dfrac{\sin x - \cos x}{\sin x + \cos x} = \text{RHS}$

M1

$x = 3$ or -1 M1

Case 2 : $x^2 - 3 = -2x$

or $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or 1 M1

But $\left| x^2 - 3 \right| = 2x \geq 0$

Hence after verifying, the answer is:

$x = 3$ or $x = 1$ A]

Alternative Solution

$(x^2-3)^2 = (2x)^2$ M1

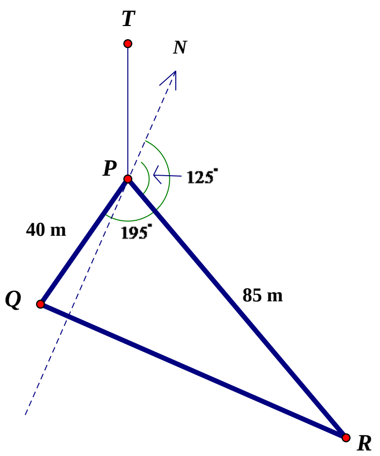
$x^4 - 10x^2 + 9 = 0$ M1

$(x^2-9)(x^2-1) = 0$ M1

$x = \pm 3, \pm 1$

$\therefore x = 3, 1$ A1

Q12



(a) In ΔPQR, $\angle QPR = 195^\circ - 125^\circ = 70^\circ$

$QR = \sqrt{40^2 + 85^2 - 2(40)(85)\cos 70^\circ}$ **M1**

$= \sqrt{1600 + 7225 - 2325.736}$

$= \sqrt{6499.24} = 80.61 \simeq 80.6$ m **A1**

(b) Area of the triangular field

$\Delta PQR = \dfrac{1}{2} \times 40 \times 85 \times \sin 70^\circ$ **M1**

$= 1597.47 \simeq 1600 \text{ m}^2$ **A1**

(c) In ΔPQR, $\dfrac{40}{\sin \angle PRQ} = \dfrac{80.61}{\sin 70^\circ}$ **M1^FT^**

$\Rightarrow \angle PRQ = 27.79^\circ$

Hence the shortest distance from P to QR is

$d = 85 \sin 27.79^\circ = 39.629 \simeq 39.6$ m **A1**

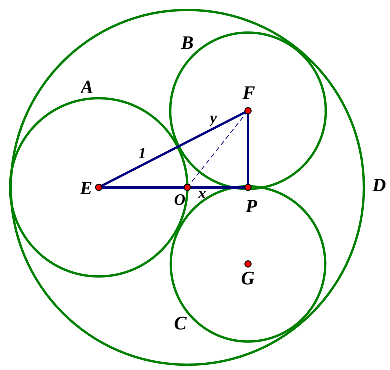
(iii) The coordinates of the centre of the circle after the reflection on the line $y = x$ are (-1, 5)

Hence the equation of the new circle formed is

$(x+1)^2 + (y-5)^2 = 5^2$

$\Rightarrow x^2 + y^2 + 2x - 10y + 1 = 0$ **B1**

Q14



Let P be the contact point of circle B and circle C.

Let $OP = x$ and join OF, $OF = 2 - y$. **M1**

In right-angled triangle ΔEFP ,

$(1+y)^2 = y^2 + (1+x)^2$ **M1**

$\Rightarrow y = x + \dfrac{x^2}{2}$ ---- (1)

In right-angled triangle ΔFOP ,

$(2-y)^2 = x^2 + y^2$ **M1**

(d) In ΔPTR, $\tan 20° = \dfrac{h}{85}$

where h is the height of the building **M1**

Hence $h = 85\tan 20° = 30.93 \simeq 30.9$ m

A1

(e) The biggest angle of elevation α is

$\alpha = \tan^{-1}(\dfrac{30.93}{39.62}) = 37.97° \simeq 38.0°$ **M1 FT,A1**

Q13

(i) Centre is (5, -1) and the radius is

$r = \sqrt{5^2 + (-1)^2 - 1} = 5$ units **B1,B1**

(ii) The gradient of the straight line $x - y + 2 = 0$ is m = 1 so the gradient of the diameter through A is -1. **M1**

The equation of the diameter passes through A is

$y - 5 = -1(x+1) \quad \Rightarrow \quad x + y = 4$ **A1**

$\Rightarrow y = 1 - \dfrac{x^2}{4}$ ---- (2)

Solving eqns (1) and (2)

$x + \dfrac{x^2}{2} = 1 - \dfrac{x^2}{4}$

$\Rightarrow 3x^2 + 4x - 4 = 0$ **M1**

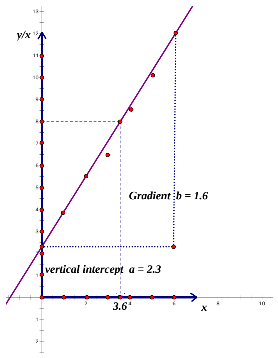
$(3x-2)(x+2) = 0$

i.e. $x = \dfrac{2}{3}$ or $x = -2$ (rej) **A1**

hence $y = 1 - \dfrac{(\frac{2}{3})^2}{4} = 1 - \dfrac{1}{9} = \dfrac{8}{9}$ (the radius of circle B). **A1**

Q15

x	1	2	3	4	5
y	3.8	11.2	20.5	33.2	50.6
$\dfrac{y}{x}$	3.8	5.6	***6.8***	8.3	***10.1***



(a) **(i)** Plot $\dfrac{y}{x}$ against x. From the diagram,

M1 for plotting
M1 for best fit line

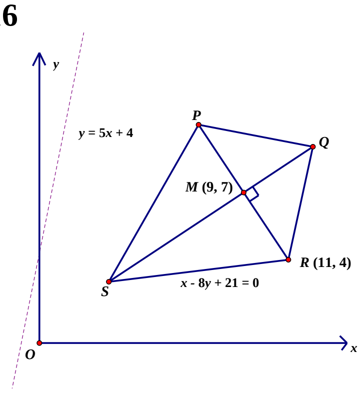
The gradient b = 1.6 (± 0.2) and **A1**

the vertical intercept is a = 2.3. (± 0.1) **A1**

(ii) When $y = 8x \Rightarrow \dfrac{y}{x} = 8$.

From the diagram, the value of x is 3.6 (± 0.1) **A1**

Q16



(i) let $P = (s, t)$, since M is the mid-point PR

$(\dfrac{s+11}{2}, \dfrac{t+4}{2}) = (9,7) \quad \Rightarrow \quad s = 7$ and $t = 10$

$P(7,10)$

M1, A1

(ii) Gradient of the given line is 5 and RQ//to the given line

So the equation of QR is $y - 4 = 5(x - 11)$ or $y = 5x - 51$ **M1**

Gradient of $RM = \dfrac{7-4}{9-11} = -\dfrac{3}{2}$ **A1**

Gradient of $QS = \dfrac{2}{3}$.

So the equation of QS is

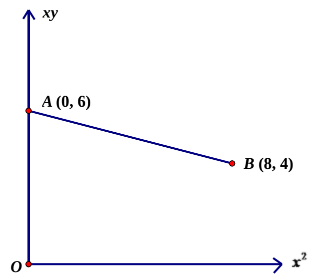
$y - 7 = \dfrac{2}{3}(x-9)$ or $2x - 3y + 3 = 0$

M1, A1

(iii) Coordinates of Q : Solve Equation QR : $y = 5x - 51$ and Equation QS : $2x - 3y + 3 = 0$

M1

(b)



xy – intercept is 6

gradient $\dfrac{6-4}{0-8}=-\dfrac{1}{4}$ **M1**

equation of straight line is of the form "$y = mx + c$"

i.e. $xy = mx^2 + c$ **M1**

$$xy=-\frac{1}{4}x^2+6$$

Hence $y=-\dfrac{1}{4}x+\dfrac{6}{x}$ or $-\dfrac{x^2+24}{4x}$ **A1**

$\Rightarrow \quad x=12$ and $y=9$ $\therefore$ $Q(12,9)$ **A1**

Coordinates of S : Solve Equation RS :
$x-8y+21=0$ and

Equation QS :
$2x-3y+3=0$

or $y=\dfrac{2}{3}x+1$

$\Rightarrow \quad x=3$ and $y=3$ $\therefore$ $S(3,3)$ **A1**

(iv) Area of $PQRS$ $=\dfrac{1}{2}\times\begin{vmatrix}3 & 11 & 12 & 7 & 3\\ 3 & 4 & 9 & 10 & 3\end{vmatrix}$ **M1**

$=\dfrac{1}{2}\times(12+99+120+21-33-48-63-30)$

= 39 square unit **A1**

End-Of-Paper