Bedok View Secondary School
Department of Mathematics — Mark Scheme

Year	2025	**Level & Stream**	4E
Type of Assessment	Preliminary Exam	**Subject & Paper**	Additional Math P2

Qns	**Working**

1a

$$\mathrm{N}(x) = x^2 \ln\left(\frac{2}{x}\right)$$

let $u = x^2$ and $v = \ln\left(\frac{2}{x}\right)$

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 2x \qquad v = \ln 2 - \ln x$$

$$\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{x}$$

$$\mathrm{N}'(x) = x^2.\left(-\frac{1}{x}\right) + \ln\left(\frac{2}{x}\right).2x$$

$$= -x + 2x\ln\left(\frac{2}{x}\right)$$

b

stationary point $\mathrm{N}'(x) = 0$

$$-x + 2x\ln\left(\frac{2}{x}\right) = 0$$

$$x\left(-1 + 2\ln\left(\frac{2}{x}\right)\right) = 0$$

$$x = 0 \text{ (rej)} \quad \text{or} \quad -1 + 2\ln\left(\frac{2}{x}\right) = 0$$

$$2\ln\left(\frac{2}{x}\right) = 1$$

$$\ln\left(\frac{2}{x}\right) = \frac{1}{2}$$

$$e^{0.5} = \frac{2}{x}$$

$$x = 2 \div e^{0.5}$$

$$x = 1.21306$$

x	1.1	1.21306	1.3
Sign of N’(x)	+ve	0	−ve
Sketch of tangent			

$\therefore$ max absorption rate at x = 1.21 (3 s.f.)

Qns	Working
2a	$T = 22 + Ae^{kt}$ when $t = 0,\ T = 90$ $90 = 22 + Ae^{k(0)}$ $90 = 22 + A$ $A = 68$
b	when $t = 3.5,\ T = 79$ $79 = 22 + 68e^{k(3.5)}$ $57 = 68e^{k(3.5)}$ $\frac{57}{68} = e^{k(3.5)}$ $\ln\left(\frac{57}{68}\right) = k(3.5)$ $k = \ln\left(\frac{57}{68}\right) \div 3.5$ $k = -0.050416$ $k = -0.0504$ (3 s.f.)
c	$T = 22 + 68e^{-0.050416t}$ when $t = 10$, $T = 22 + 68e^{-0.050416(10)}$ $= 63.0728$ Since 63.0728 °C is between 55 °C and 70 °C, the coffee is suitable to be served.
d	After a long time, T approaches 22 °C.

Qns	Working
3a	$\cos\theta = \frac{ED}{14}$ $ED = 14\cos\theta$ Let G be the point on FD such that $BG \perp FD$ $\sin\theta = \frac{EG}{3}$ $EG = 3\sin\theta$ $FD = 14\cos\theta + 3\sin\theta + 6$ (shown)
b	$R = \sqrt{14^2 + 3^2}$ $= \sqrt{205}$ $\tan\alpha = \frac{3}{14}$ $\alpha = 12.0947$ $FD = \sqrt{205}\cos(\theta - 12.1°) + 6$ (3 s.f.)
c	$\sqrt{205}\cos(\theta - 12.0947°) + 6 = 12$ $\cos(\theta - 12.0947°) = \frac{6}{\sqrt{205}}$ $\theta - 12.0947° = 65.2248°$ $\theta = 77.3195°$ $\theta = 77.3°$ (1 d.p.)
d	Maximum $\sqrt{205}\cos(\theta - 12.0947°) + 6$ $= \sqrt{205} + 6$ $= 20.317$ $\therefore$ it is possible for FD to be 15 cm Or $\sqrt{205}\cos(\theta - 12.0947°) + 6 = 15$ $\cos(\theta - 12.0947°) = \frac{9}{\sqrt{205}}$ $\theta - 12.0947° = 51.054°$ $\theta = 63.1°$ (1 d.p.) FD is 15 cm when $\theta = 63.1°$

Qns	Working
4a	

Qns	Working
b	$N = a10^{kt}$ $\lg N = \lg a10^{kt}$ $\lg N = \lg a + \lg 10^{kt}$ $\lg N = \lg a + kt$ $\lg N = kt + \lg a$ $k =$ gradient $= 0.232$ (3 s.f.) ± 0.1 $\lg a =$ vertical axis $-$ intercept $\lg a = 2.5 \pm 0.05$ $a = 10^{2.5}$ $=316$ (3 s.f.)
5a	$-2(x+1)(x-2)(x-1.5)$ $= (x^2 - x - 2)(-2x + 3)$ $= -2x^3 + 3x^2 + 2x^2 - 3x + 4x - 6$ $= -2x^3 + 5x^2 + x - 6$
b	$$\begin{array}{r} x-3 \\ x^2+0x-1 \overline{\smash{)}\, x^3-3x^2-2x+5} \\ -\left(x^3+0x^2-x\right) \\ \hline -3x^2-x+5 \\ -\left(-3x^2+0x+3\right) \\ \hline -x+2 \end{array}$$ $\therefore$ Quotient $= x - 3$

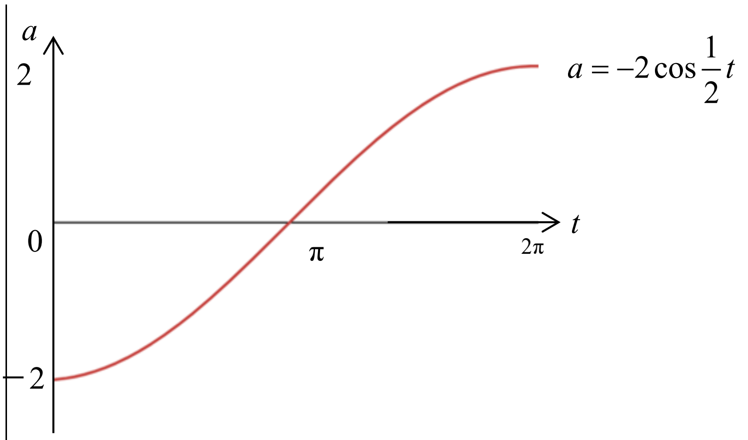
Qns	Working

c	$hx^3-7x^2+kx-2=\left(3x^2-5x-2\right)\times\text{Quotient}$
	$hx^3-7x^2+kx-2=\left(3x+1\right)\left(x-2\right)\times\text{Quotient}$
	sub $x=-\frac{1}{3}$, $h\left(-\frac{1}{3}\right)^3-7\left(-\frac{1}{3}\right)^2+k\left(-\frac{1}{3}\right)-2=0$
	$-\frac{1}{27}h-\frac{1}{3}k=\frac{25}{9}$----------(1)
	sub $x=2$, $h\left(2\right)^3-7\left(2\right)^2+k\left(2\right)-2=0$
	$8h+2k=30$----------(2)
	From (2), $4h+k=15$
	$k=15-4h$----------(3)
	sub (3) into (1), $-\frac{1}{27}h-\frac{1}{3}\left(15-4h\right)=\frac{25}{9}$
	$-\frac{1}{27}h-5+\frac{4}{3}h=\frac{25}{9}$
	$\frac{35}{27}h=\frac{70}{9}$
	$h=6$
	sub $h=6$ into (3), $k=15-4(6)$
	$k=-9$

Qns	Working
6a	$AB = AE$ (tangent from external point) $AC = AD$ (tangent from external point) $\frac{AB}{AC} = \frac{AE}{AD}$ (ratio of corresponding sides are equal) $\angle BAE = \angle CAD$ (common angle) ΔABE is similar to ΔACD (ratio of 2 pairs of corresponding sides and a pair of included $\angle$s are equal)
b	$\angle ABE = \angle ACD$ (ΔABE is similar to ΔACD) $\angle ACD = \angle ADC$ (base $\angle$ of isosceles Δ) $\angle EBC = 180° - \angle ABE$ (adjacent $\angle$s on a straight line) Since $\angle ADC + \angle EBC = 180°$, by the converse of angles in opposite segments, a circle can be drawn to pass through $BEDC$.
c	Let $\angle CBF = x$, $\angle BEF = x$ (alternate segment thm) $BE // CD$ since ΔABE is similar to ΔACD $\angle ECD = x$ (alternate angles, $BE // CD$) $\angle EDF = x$ (alternate segment thm) $\therefore \angle CBF = \angle EDF$ (proven)

Qns	Working
7a	General term of $\left(1-\frac{x}{5}\right)^8$ $=\binom{8}{r}(1)^{8-r}\left(-\frac{x}{5}\right)^r$ $=\binom{8}{r}\left(-\frac{x}{5}\right)^r$ $(7+2x)\left(1-\frac{x}{5}\right)^8$ $=(7+2x)\left(...+\binom{8}{3}\left(-\frac{x}{5}\right)^3+\binom{8}{2}\left(-\frac{x}{5}\right)^2+...\right)$ x^3 term $=7\binom{8}{3}\left(-\frac{x}{5}\right)^3+2x\binom{8}{2}\left(-\frac{x}{5}\right)^2$ $=7(56)\left(-\frac{x^3}{125}\right)+2x(28)\left(\frac{x^2}{25}\right)$ $=-\frac{392}{125}x^3+\frac{56}{25}x^3$ Coefficient of $x^3=-\frac{112}{125}$
b	$\binom{n}{2}3^{n-2}\left(\frac{1}{2}x\right)^2=\binom{n}{2}3^{n-2}\frac{1}{4}x^2$ $\binom{n}{3}3^{n-3}\left(\frac{1}{2}x\right)^3=\binom{n}{3}3^{n-3}\frac{1}{8}x^3$ $\binom{n}{2}3^{n-2}\frac{1}{4}=3\times\binom{n}{3}3^{n-3}\frac{1}{8}$ $\binom{n}{2}3^{n-2}.2=\binom{n}{3}3^{n-2}$ $\binom{n}{2}.2=\binom{n}{3}$ $\frac{n(n-1)}{2}.2=\frac{n(n-1)(n-2)}{2\times 3}$ $6=n-2$ $n=8$

Qns	Working
8a	$\cos 2A = \frac{1}{3}$ $1 - 2\sin^2 A = \frac{1}{3}$ $\sin^2 A = \frac{1}{3}$ $\sin A = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$ (rej) $\text{cosec } A = \frac{1}{\sin A}$ $= 1 \div \frac{1}{\sqrt{3}}$ $= \sqrt{3}$
b	$\sin 105°$ $= \sin(45° + 60°)$ $= \sin 45° \cos 60° + \cos 45° \sin 60°$ $= \frac{1}{\sqrt{2}} . \frac{1}{2} + \frac{1}{\sqrt{2}} . \frac{\sqrt{3}}{2}$ $= \frac{1+\sqrt{3}}{\sqrt{2}.2}$ $\text{c osec } A - \sqrt{2} \sin 105°$ $= \sqrt{3} - \sqrt{2} . \frac{1+\sqrt{3}}{\sqrt{2}.2}$ $= \sqrt{3} - \frac{1+\sqrt{3}}{2}$ $= \frac{\sqrt{3}-1}{2}$

Qns	Working
9a	
b	$v=\int -2\cos\frac{1}{2}t\ \mathrm{d}t$ $=\dfrac{-2\sin\frac{1}{2}t}{\frac{1}{2}}+c$ $=-4\sin\frac{1}{2}t+c$ Given $t=0$ and $v=3$, find c $3=-4\sin\frac{1}{2}(0)+c$ $c=3$ $v=-4\sin\frac{1}{2}t+3$ find t when $v=0$, $-4\sin\frac{1}{2}t+3=0$ $\sin\frac{1}{2}t=\frac{3}{4}$ basic $\angle$ of $\frac{1}{2}t=0.84806$ $t=1.6961$ $t=1.70$ (3 s.f.)

Qns	Working
c	basic $\angle$ of $\frac{1}{2}t = 0.84806$ since $\sin\frac{1}{2}t$ is +ve, $\frac{1}{2}t$ is also in 2nd quad. $\frac{1}{2}t = \pi - 0.84806$ $\frac{1}{2}t = 2.2935$ $t = 4.5870$ Displacement $=\int\left(-4\sin\frac{1}{2}t+3\right)\mathrm{d}t$ $=-4.\dfrac{-\cos\frac{1}{2}t}{\frac{1}{2}}+3t+C_1$ $=8\cos\frac{1}{2}t+3t+C_1$ Distance travelled in first 5 min $=\left\|\left[8\cos\frac{1}{2}t+3t\right]_0^{1.6961}\right\|+\left\|\left[8\cos\frac{1}{2}t+3t\right]_{1.6961}^{4.5870}\right\|+\left\|\left[8\cos\frac{1}{2}t+3t\right]_{4.5870}^{5}\right\|$ $=\|10.379-8\|+\|8.4696-10.379\|+\|8.5908-8.4696\|$ $=2.3798+1.9100+0.1212$ $=4.41$ m

Qns	Working
10a	$y = 2x - 1 - - - - - (1)$ $y = -x^{\frac{3}{2}} - - - - - - - - (2)$ fr (2), $y^2 = x^3 - - - (3)$ sub (1) into (3), $(2x-1)^2 = x^3$ $4x^2 - 4x + 1 = x^3$ $0 = x^3 - 4x^2 + 4x - 1$ $(x-1)$ is a factor since $A(1,1)$ is intersection $0 = (x-1)(Ax^2 + Bx + C)$ $x^3 - 4x^2 + 4x - 1 = (x-1)(Ax^2 + Bx + C)$ compare x^3 : $1 = A$ compare constant: $-1 = -C$ $1 = C$ sub $x = 2$, $-1 = 2^2 + 2B + 1$ $-3 = B$ $(x-1)(x^2 - 3x + 1) = 0$ $x - 1 = 0$ or $x^2 - 3x + 1 = 0$ $x = 1$ $\quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$ $= 2.6180$ or 0.381966 sub $x = 0.381966$, $y = 2x - 1$ $y = 2(0.381966) - 1$ $= -0.236068$ $\therefore C = (0.38197, -0.23607)$ (shown)

Qns	Working
b	sub $y = 0$ to find $x-$coordinate of B $y = 2x - 1$ $0 = 2x - 1$ $x = 0.5$ Let A be the shaded area above the x-axis Let B be the shaded area below the x-axis $A = \left\| \int_0^1 x^{\frac{3}{2}} \mathrm{d}x \right\| - \frac{1}{2}(0.5)(1)$ $= \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^1 - 0.25$ $= \frac{2}{5} - 0.25$ $= 0.15$ units^2 $B = \int_0^{0.38197} -x^{\frac{3}{2}} \mathrm{d}x + \frac{1}{2}(0.5 - 0.38197)(0.23607)$ $= \left\| \left[-\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^{0.38197} \right\| + 0.013931$ $= 0.03606089 + 0.0139391$ $= 0.049999$ units^2 Total shaded area $= 0.15 + 0.049999$ $= 0.200$ units^2 (3 s.f.)