Name: ________________________________ Class: __________ Date: ___________

Sec 3 Physics

Worksheet 10: Work, energy and power

SECTION A: CONCEPTUAL QUESTIONS

1 How does friction produce internal energy? Explain what is happening at the molecular level to produce this form of energy.

When the surface molecules 'rubbed' against one another, they are displaced from their equilibrium positions and subsequently oscillate strongly. This work done by friction is transferred to a molecular level and is manifested as internal energy (thermal energy) of the molecules.

2 Can the total work done on an object during a displacement be negative? Explain.

If the total work is negative, can its magnitude be greater than the initial kinetic energy of the object? Explain.

Yes, the work done can be negative. This happens when the direction of the applied force is opposite that of the direction of displacement. *e.g. during the slowing down of a car, the displacement is forward while the acceleration (force) is backward*.

It is important to note that work done on a system is the amount of energy transferred to the system by an external force. If the system speeds up as a result, then positive work is done on the system. If the system slows down, then negative work is done on the system.

Care should be taken to distinguish the fact that negative work is different from, say, negative velocity because, unlike velocity, work is a scalar quantity. There are no spatial directions involved.

As for the second question, consider this: when the negative work is equal to the positive initial kinetic energy, the object stops. At this point, the external force would have slowed down the object to a complete halt. Further application of the external force will result in positive work done because the object will now move in the same direction as the force. Therefore, the negative work magnitude can't be greater than the initial kinetic energy.

3 An elevator is hoisted by its cables at a constant speed. Is the *total work* done on the elevator positive, negative or zero? Explain.

Since the elevator is moving at a constant speed, the resultant force on the elevator is zero, and hence the total work done on the elevator is zero.

You can think of it as the positive work done by the tension in the cables is negated by the negative work done by gravity, and the resultant work is zero. (*the tension in the cable is stuffing energy into the elevator while the gravity is taking energy out at the same rate.*)

4 A rope tied to an object is pulled, causing the object to accelerate. According to Newton's Third Law, the object pulls back on the rope with equal and opposite force.

Is the net work done on the object zero?
Is there a change to the kinetic energy of the object?

No, there is non-zero net work done on the object. (*Recall that the third law says that the two forces act on different objects; the rope acts on the object while the object acts on the rope, and so the two forces do not act on the same body, and they cannot cancel each other*.)

There is a change in the kinetic energy as the object is accelerating. This is consistent with the work-energy theorem as the work done by the force is changed into the object's kinetic energy.

5 When you use a jack to lift a car, the force you exert on the jack is much less than the car's weight. Does this mean that less work is done on the car than if the car is lifted directly?

While the applied force is smaller than the weight, the work done (applied energy) cannot be smaller than the gravitational potential energy. This idea is discussed in the section on machines.

In the ideal case, the amount of work required to lift the car is equal to the gain in GPE. In reality, the amount of work will be higher than GPE because part of the work done is used to overcome resistive forces (friction).

6 A compressed spring is clamped in its compressed position and is then dissolved in acid. What becomes of the spring's elastic potential energy?

The elastic potential energy is dissipated in the form of thermal energy in the acid. In other words, dissolving a compressed spring in acid will make the resulting solution slightly hotter than dissolving an identical spring in the relaxed state.

No amount of energy is lost in the process (conservation of energy).

7 You bounce on a trampoline, going a little higher with each bounce. Explain how you increase your total mechanical energy.

If you drop a stone on the trampoline, you will find that the stone's rebound height will decrease after each rebound. This is expected as resistive forces present in the system will remove energy from the system.

A man can increase his height after each rebound because the man is supplying mechanical energy into the system through the work done by his leg muscles. If you look at the big picture, you can say that the man is converting chemical potential energy in his body into mechanical energy of the bouncing system.

Extra: You should note that there is an optimum position for the leg to exert an upward force, and that position is at the lowest point of the trajectory, i.e. just when the man is about to bounce upward. This is not a coincidence; it is a well-studied phenomenon called <u>resonance</u>. We will be discussing more resonance when we learn of waves and oscillations.

8 When you jump from the ground into the air, where does your kinetic energy come from? What force does work on you to lift you into the air?

The energy source is you. The chemical potential energy in your leg muscles is converted into elastic potential energy in a tensed position. When you released that tension, the foot exerts a normal contact force on the ground that is greater than the weight of your body. (During the push-off phase, the total force your feet exert on the ground is your weight + the extra push-off force).

According to Newton's third law, the ground then exerts an equal but opposite force on you. Hence, during that push-off phase, the total force on you is a normal contact force from the ground minus your weight, and the resultant is directed upward. According to the work-energy principle, the work done by the resultant upward force is converted to the kinetic energy of your body.

The upward force is the normal contact force from the ground on you.

9 Hydroelectric energy comes from gravity pulling down water through dams in rivers. Explain how such energy is just a form of solar energy by tracing how the Sun's energy can get the water from the ocean to the reservoir behind the dam.

This question is asking you the basic principle behind the water cycle. First, the thermal energy of the Sun causes water to evaporate and form clouds in the atmosphere. Later, the water is precipitated in the rain on higher grounds, flowing down in rivers, and eventually stored behind the dam.

Let's take a closer look at the energy cycle in the whole process. The energy delivered by the Sun is converted to the <u>*latent heat of vaporisation*</u> and <u>*work done against the atmosphere*</u> during the initial evaporation. The latent heat of vaporisation is the amount of energy required to convert a liquid into a gas. Work done against the atmosphere is the energy needed to expand the volume occupied when the water converts from liquid to gas at atmospheric pressure.

When the water vapour rises to a height to form clouds, the latent heat of vaporisation is released back to the environment as the water condenses in the form of droplets. The work done against the atmosphere is converted to gravitational potential energy because the water has risen to a reasonable height. When the water droplets fall in the rain, some GPE is lost because there is a drop in height but not entirely lost because the water is still on higher ground. Whatever residue GPE left in the process is what the power generators at the hydroelectric power plants are trying to retrieve. (i.e. only a fraction of the original solar energy that the Sun shone on the oceans is left in the water behind a dam.)

I have described an idealised system where effects like air resistance are ignored. In real life, air resistance on rain or wind effects on clouds can affect the system's energy.

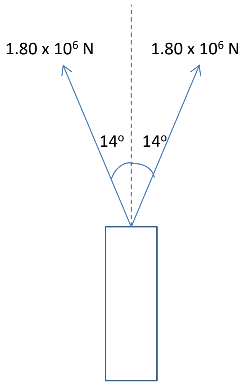
10 Does the kinetic energy of a car change more when the car speeds up from 10 km/h to 15 km/h or from 15 km/h to 20 km/h?

You can work this solution out quite quickly. Just take the difference in KE = $\frac{1}{2}m\left(v_{final}^2 - v_{initial}^2\right)$ for each case. Going from 15 km/h to 20 km/h will take more energy.

In real life, trying to speed up at high speed is also very difficult because more work is to be done against greater air resistance.

SECTION B: STRUCTURED QUESTIONS

1 Two tugboats pull a disabled supertanker. Each tug exerts a constant force of 1.80 x 10^6 N, one 14° west of north and the other 14° east of north, as they pulled the tanker 0.75 km toward the north. What is the total work they do on the supertanker?



$$\text{Total force acting on the tanker} = 2\left(1.80\times10^6 \times \cos14^\circ\right)$$
$$= 3.49\times10^6 \text{ N}$$

$$\text{Total work done} = 3.49\times10^6 \times 750$$
$$= 2.62\times10^9 \text{ J}$$

2 The driver of an 1800 kg car (including passengers) travelling at 23 m s^{-1} slams on the brakes, locking the wheels on dry pavement. The total friction between the rubber wheels and the pavement is 1.2 x 10^4 N.

a Use the work-energy theorem to calculate how far the car will travel before stopping.

work done by force = change in KE (work-energy theorem)

$$F_{friction} \times d = \frac{1}{2}mv^2$$
$$1.2\times10^4 d = \frac{1}{2}(1800)(23)^2$$
$$d = 39.6 \text{ m}$$
$$= 40 \text{ m} \qquad (2 \text{ s.f.})$$

b How far would the car travel if it is moving twice as fast?

As the previous equation has shown, twice the velocity would result in 4 times the kinetic energy. Therefore, the car will travel 4 times the distance before stopping. (160 m)

c What happened to the car's original kinetic energy?

The car's KE is converted to the internal energy of the car and the environment. (Both the car and the environment get hotter.

3 About 50 000 years ago, a meteor crashed into the earth near present-day Flagstaff, Arizona. Recent (2005) measurements estimate that this meteor has a mass of 1.4×10^8 kg and hits the earth at 12 km s^{-1}.

a How much kinetic energy did this meteor deliver to the ground?

$$\text{KE of meteor} = \frac{1}{2}\left(1.4\times10^{8}\right)\left(1.2\times10^{4}\right)^{2}$$
$$= 1.01\times10^{16}\ \text{J}$$
$$= 1.0\times10^{16}\ \text{J} \qquad \text{(2 s.f.)}$$

b How does this compare to the energy produced in one day by a standard coal-fired power plant, which generates about 1 billion joules per second?

Time taken for a coal-fired power plant to produce this energy

$$= \frac{1.0\times10^{16}}{1\times10^{9}}$$
$$= 1\times10^{7}\ \text{s}$$
$$= 100\ \text{days}\ \ \text{(1 sf)}$$

4 A block of ice with a mass of 2.00 kg slides 0.750 m down an inclined plane. The slope is downward at an angle of 36.9º below the horizontal. If the block of ice starts from rest, what is its final speed? Ignore friction for this question.

$$F_{gravity} \times d = \frac{1}{2}mv^2$$

$$mg\sin 36.9^\circ \times 0.750 = \frac{1}{2}mv^2$$

$$v = \sqrt{2(9.81)(\sin 36.9^\circ)(0.750)}$$

$$= 2.97\ \text{ms}^{-1}$$

5 To stretch a certain spring by 2.5 cm from its equilibrium position requires 8.0 J of work.

a What is the force constant of this spring?

$$EPE = \frac{1}{2}kx^2$$

$$8.0 = \frac{1}{2}k(0.025)^2$$

$$k = 2.56 \times 10^4\ \text{Nm}^{-1}$$

$$= 2.6 \times 10^4\ \text{Nm}^{-1} \quad \text{(2 s.f.)}$$

b What is the minimum force required to stretch it by that distance?

$$F = kx$$

$$= 2.6 \times 10^4 \times 0.025$$

$$= 6.4 \times 10^2\ \text{N}$$

6 A spring of force constant 300.0 Nm^{-1} and unstretched length 0.240 m is stretched by two forces, pulling in opposite directions at opposite ends of the spring, increasing to 15.0 N.

a What is the final length of the spring?

The correct answer is

$$F_{el} = kx$$
$$15.0 = 300.0x$$
$$x = 0.0500 \text{ m} \quad \Rightarrow \quad \text{New length} = 0.290 \text{ m}$$

The following answer seems to use the work-energy theorem, but it is not correct. Why?

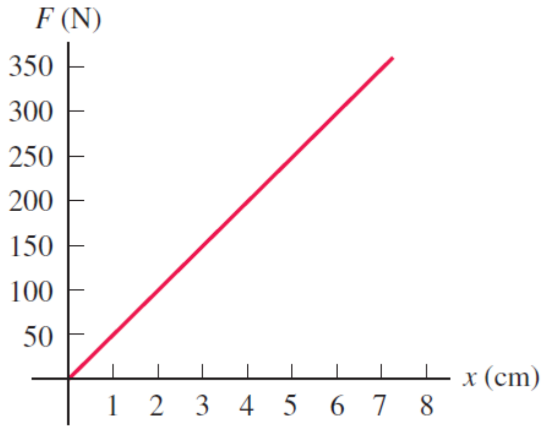
$$F_{el} \times d = \frac{1}{2}kx^2$$
$$15.0 \times d = \frac{1}{2}(300.0)(d)^2$$
$$d = 0.100 \text{ m} \quad \Rightarrow \quad \text{New length} = 0.340 \text{ m} \quad \text{Wrong}$$

Note that the F_{el} applied in the equation is 15.0 N, not 30.0 N and certainly not 0.0 N. Can you figure out why?

b How much work is needed to stretch the spring that distance?

$$\text{Work done to stretch the spring} = \frac{1}{2}kx^2$$
$$= \frac{1}{2}(300.0)(0.0500)^2$$
$$= 0.375 \text{ J}$$

7 The graph in the accompanying figure shows the magnitude of the force exerted by a given spring as a function of the distance *x* the spring is stretched. How much work is needed to stretch this spring:



a a distance of 5.0 cm, starting unstretched, and

$$\begin{aligned}\text{Work done} &= \text{area under F-x graph} \\ &= \frac{1}{2}(250)(0.05) \\ &= 6.25 \text{ J}\end{aligned}$$

b from x = 2.0 cm to x = 7.0 cm?

$$\begin{aligned}\text{Work done} &= \frac{1}{2}(100+350)(0.05) \\ &= 11.25 \text{ J}\end{aligned}$$

8 The *food calorie*, equal to 4186 J, measures how much energy is released when the body metabolises food. A certain brand of the fruit-and-cereal bar contains 140 food calories per bar.

a Suppose a 65-kg hiker eats one of these bars. How high a mountain must he climb to work off the calories, assuming all the food energy goes only into increasing the gravitational potential energy?

$$\begin{aligned} GPE &= \text{energy of bar} \\ (65)(9.81)(h) &= 140(4186) \\ h &= 920 \text{ m} \end{aligned}$$

b If, as typical, only 20% of the food calories go into mechanical energy, what would be the answer to part (a)?

$$\begin{aligned} (65)(9.81)(h_{new}) &= (0.20)140(4186) \\ h_{new} &= 180 \text{ m} \end{aligned}$$

Note: In this and all other problems, we assume that 100% of the food calories that are eaten are absorbed and used by the body. This is not true. A person's "metabolic efficiency" is the percentage of calories eaten are used; the body eliminates the rest. Metabolic efficiency varies from person to person

9 Although the altitude may vary considerably, hailstones sometimes originate around 500 m above the ground.

a Neglecting air drag, how fast will these hailstones be moving when they reach the ground, assuming that they started from rest?

By the conservation of energy,

$$\begin{aligned} \text{Work done by gravity} &= \text{gain in KE} \\ (m)(9.81)(500) &= \frac{1}{2}(m)v^2 \\ v &= 99.0 \text{ m s}^{-1} \end{aligned}$$

b From your experience or intuition, are hailstones falling that fast when they reach the ground? Why not? What has happened to most of its initial potential energy?

In reality, hailstones do not fall at close to 100 ms^{-1}. The air resistance is sufficient to impede the motion such that the hailstone will reach its terminal velocity during the drop. Most of the initial potential energy is used to do work against air resistance.

(*Answers such as original PE is converted to thermal energy are accepted*)

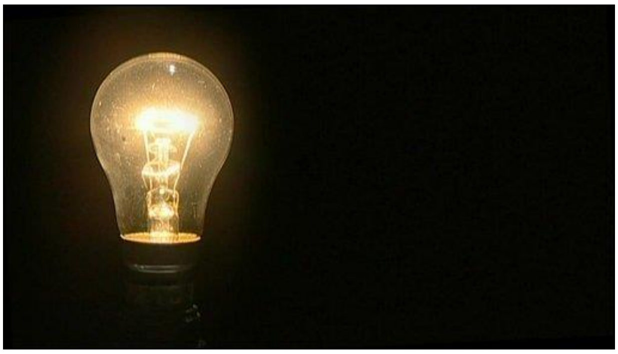
10 a How many joules of energy does a 100-watt lightbulb use every hour?

$$\text{Energy used} = (100)(60)(60)$$
$$= 3.6 \times 10^5 \text{ J}$$
$$= 0.10 \text{ kWh}$$

b Express the answer in part (a) in terms of kilowatt-hour (kWh).

As above.

c The electrical tariff from Singapore Power from 1 Jul to 30 Sep is 23.38 cents per kWh. How much would it cost to operate the lightbulb for a full day?



$$\text{Cost} = (24) \times (0.10 \text{ kWh}) \times (\$0.2338)$$
$$= \$0.56$$

"We will make electricity so cheap that only the rich will burn candles."
Thomas Edison, in a statement to a reporter during the first public demonstration of his incandescent (31 December 1879), as quoted in *Chronology of Americans and the Environment* (2011) by Chris J. Magoc, p. 46.

11 The Sun transfers energy to the earth by radiation at the rate of approximately 1.0 kW per m^2 of the surface.

a If this energy could be collected and converted into electrical energy with 25% efficiency, how large an area would be required to collect energy to be used in Singapore during the daylight hours?

Given that Singapore power consumption in 2008 is 37.11 billion kWh and assuming that power consumption is evenly spread throughout the entire year. You may assume that Singapore enjoys 12 hours of daylight a day.

$$\text{Solar energy collected} = 37.11\times10^{9}\times\left(3.6\times10^{6}\right)$$

$$25\%(\mathit{Intensity})(\mathit{Area})(365\mathit{days})(12h)(60\,\text{min})(60s) = 1.33\times10^{17}\ \text{J}$$

$$(0.25)\left(1.0\times10^{3}\right)(A)(365)(12)(60)(60) = 1.33\times10^{17}$$

$$A = 3.4\times10^{7}\ \text{m}^2$$

$$= 34\ \text{square km}$$

b If the solar collectors are arranged in a square array, what would be its length in km? Does an array of these dimensions seem feasible?

Each length will be $\sqrt{34} = 5.8$ km. Such a dimension is physically feasible.

(*In the above calculations, we have assumed ideal conditions again. The amount of sunlight can be affected by the angle of inclination of the Sun during the different times of the day and the presence of clouds. Nonetheless, it is much preferred environmentally to use solar power than generating electricity using fossil fuels*.)

12 When its 75-kW engine is generating full power, a small single-engine aeroplane with a mass of 700 kg gains altitude at a rate of 2.5 ms^{-1}. What fraction of the engine power is being used to make the aeroplane climb? (The remainder is used to overcome the effects of air resistance and inefficiencies in the propeller and engine.)

$$\begin{aligned}\text{Climbing power} &= Fv\\ &= mgv\\ &= (700)(9.81)(2.5)\\ &= 1.7\times10^{4}\ \text{W}\end{aligned}$$

$$\begin{aligned}\text{Fraction of input power used for climbing} &= \frac{17}{75}\\ &= 23\%\end{aligned}$$

13 At the wind farm site in North Dakota, the average wind speed is 9.3 ms^{-1}, and the average density of air is 1.2 kg m^{-3}.

a Calculate how much kinetic energy the wind contains, per cubic metre, at this location.

$$\begin{aligned}\text{KE of wind per m}^3 &= \frac{1}{2}(1.2)(9.3)^2\\ &= 51.9\ \text{J m}^{-3}\\ &= 52\ \text{J m}^{-3}\end{aligned}$$

b No wind turbine can capture all the energy contained in the wind, the main reason being that capturing all the energy would require stopping the wind completely, meaning that air will stop flowing through the turbine. Suppose a particular turbine has blades with a radius of 41 m and is able to capture 35% of the available wind energy. What would be the power output of this turbine, under average wind conditions?

Consider a cylindrical wind tube with circular cross section of radius 41 m, then the volume of wind flowing through the cross section per second is

$$\pi(41)^2\times9.3 = 4.9\ \times10^{4}\ \text{m}^3\text{s}^{-1}$$

$$\begin{aligned}\text{Power output} &= 0.35\times4.9\times10^{4}\left(\frac{\text{m}^3}{\text{s}}\right)\times52\left(\frac{\text{J}}{\text{m}^3}\right)\\ &= 8.9\times10^{5}\ \text{W}\end{aligned}$$

14 A 75.0 kg mountain climber is holding his 60.0 kg partner over a cliff when he suddenly steps on frictionless ice at the horizontal top of the cliff, as shown in the accompanying figure. The rope has negligible mass and is held horizontally by the climber. There is no appreciable friction at the icy edge of the cliff. Use energy methods to calculate the speed of the climbers after the lower one has descended 1.50 m starting from rest.

Work done by gravity on 60.0 kg climber $=$ gain in KE of the system of two climbers

$$(60.0)(9.81)(1.50) = \frac{1}{2}(60.0 + 75.0)v^2$$

$$v = 3.65 \text{ m s}^{-1}$$

15 The spring of a spring gun has a force constant k = 400 N/m and negligible mass. The spring is compressed 6.00 cm, and a ball with a mass of 0.0300 kg is placed in the horizontal barrel against the compressed spring. The spring is then released, and the ball is propelled out of the barrel of the gun. The barrel is 6.00 cm long, so the ball leaves the barrel at the same point it loses contact with the spring. The gun is held so that the barrel is horizontal.

a Calculate the speed with which the ball leaves the barrel if you ignore friction.

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400}{0.0300}}(0.0600) = 6.93 \text{ m s}^{-1}$$

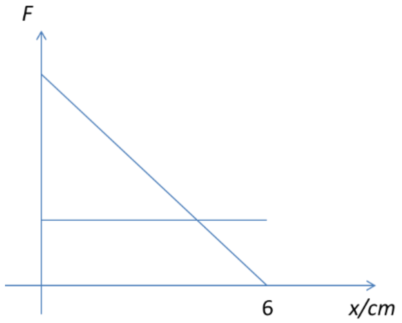
b Calculate the speed of the ball as it leaves the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel.

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + fx$$

$$\frac{1}{2}(400)(0.0600)^2 = \frac{1}{2}(0.0300)v^2 + (6.00)(0.0600)$$

$$v = 4.90 \text{ m s}^{-1}$$

c For the situation in part (b), at what position along the barrel does the ball have the greatest speed, and what is that speed? (In this case, the maximum speed does not occur at the end of the barrel.)



If we consider the work done by the two forces, essentially the areas under the graph, we can see that the maximum work done occurs when the spring force equal the resistive force.

$$kx = 6.00$$

$$x = \frac{6.00}{400} = 0.0150 \text{ m}$$

Remember x is the compressed length. When the compressed length is 0.015 m, the actual position of the ball is at 0.0600 – 0.0150 = 0.0450 m or 4.50 cm.