2025 RI Prelims H3 Physics Solutions

1 **(a)** **(i)**

1 $$F = 1 - \frac{\frac{1}{2}mv_{n+1}^2}{\frac{1}{2}mv_n^2} = 1 - \frac{v_{n+1}^2}{v_n^2}$$

Taking up as positive, sphere starts out with initial velocity v_{n+1} after the nth impact and hits the plate with a final velocity of $-v_{n+1}$ just before the (n + 1)th impact.

2 Using $v = u + at$

$$-v_{n+1} = v_{n+1} + (-g)t$$

$$2v_{n+1} = gt^2 \quad \Rightarrow t = \frac{2v_{n+1}}{g}$$

(ii) Time taken from release to 1st impact:

$$-v_1 = 0 + (-g)t_1 \quad \Rightarrow t_1 = \frac{v_1}{g}$$

From **(a) (ii)** Time taken between 1st and 2nd impact: $t_{1\to2} = \frac{2v_2}{g}$

From **(a) (i)** $F = 1 - \frac{v_{n+1}^2}{v_n^2} \quad \Rightarrow v_{n+1} = \sqrt{(1-F)}v_n$

Hence, time taken between 1st and 2nd impact

$$t_{1\to2} = \frac{2v_2}{g} = \frac{2v_1}{g}\sqrt{(1-F)} = \frac{2v_1}{g}(1-F)^{\frac{1}{2}}$$

Total time

$$\begin{aligned}
T &= t + t_{1\to2} + t_{2\to3} + t_{3\to4} + \ldots \\
&= \frac{v_1}{g} + \frac{2v_2}{g} + \frac{2v_3}{g} + \frac{2v_4}{g} + \ldots \\
&= \frac{v_1}{g} + \frac{2v_1}{g}(1-F)^{\frac{1}{2}} + \frac{2v_1}{g}(1-F)^{\frac{2}{2}} + \frac{2v_1}{g}(1-F)^{\frac{3}{2}} + \frac{2v_1}{g}(1-F)^{\frac{4}{2}} + \ldots \\
&= \frac{v_1}{g} + \frac{2v_1}{g}\left[(1-F)^{\frac{1}{2}} + 2(1-F)^{\frac{2}{2}} + 2(1-F)^{\frac{3}{2}} + 2(1-F)^{\frac{4}{2}} + \ldots\right] \\
&= \frac{v_1}{g} + \frac{2v_1}{g}\left[\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}}\right]
\end{aligned}$$

Since the sphere is dropped from height *h* initially, using $v^2 = u^2 + 2as$

$v_1^2 = 0 + 2gh \quad \Rightarrow v_1 = \sqrt{2gh}$

Sub into total time

$$T = \frac{\sqrt{2gh}}{g} + \frac{2\sqrt{2gh}}{g}\left[\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}}\right]$$

$$= \frac{\sqrt{2gh}}{g}\left(1+2\left[\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}}\right]\right)$$

$$= \sqrt{\frac{2h}{g}}\left(1+2\left[\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}}\right]\right)$$

(iii)

$$T = \sqrt{\frac{2h}{g}}\left(1+2\left[\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}}\right]\right)$$

$$8.2 = \sqrt{\frac{2(1.2)}{9.81}}\left(1+2\left[\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}}\right]\right)$$

$$\frac{(1-F)^{\frac{1}{2}}}{1-(1-F)^{\frac{1}{2}}} = 7.7892$$

$$(1-F)^{\frac{1}{2}} = 7.7892\left[1-(1-F)^{\frac{1}{2}}\right]$$

$$8.7892(1-F)^{\frac{1}{2}} = 7.7892 \quad \Rightarrow (1-F) = \sqrt{\frac{6.7892}{8.7892}} \quad \Rightarrow F = 0.2146 \approx 0.215$$

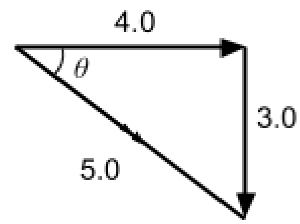
1 (b) (i) $$\vec{v}_{CM} = \frac{m_A\vec{v}_A + m_B\vec{v}_B}{m_A + m_B}$$

The vector sum $m_A\vec{v}_A + m_B\vec{v}_B$ can be found as follows:

$m_A\vec{v}_A = 1.0(4.0) = 4.0$ kg m s^{-1} to the right

$m_B\vec{v}_B = 3.0(1.0) = 3.0$ kg m s^{-1} downwards

$\left|m_A\vec{v}_A + m_B\vec{v}_B\right| = \sqrt{4.0^2 + 3.0^2} = 5.0$ kg m s^{-1}



$$\frac{\left|m_A\vec{v}_A + m_B\vec{v}_B\right|}{m_A + m_B} = \frac{5.0}{1.0+3.0} = \frac{5}{4} \text{ m s}^{-1}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$$

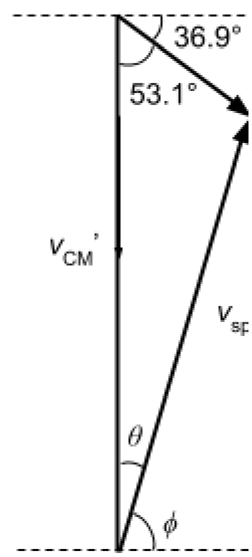
Hence, v_{CM} is $\frac{5}{4}$ m s^{-1} at 36.9° clockwise below the horizontal.

(ii) The combined meteorite of A and B must move off at the velocity v_{CM} because in the absence of external forces, the total momentum of the system $(m_A + m_B)v_{CM}$ is conserved, and hence v_{CM} is conserved before and after the collision.

Therefore, once A and B are combined into a single mass, it must move at the velocity v_{CM}.

(iii) $\vec{v}_{CM}{}' = \vec{v}_{CM} - \vec{v}_{spaceship}$

Using a vector diagram, one can see that if v_{CM}', the velocity of the combined meteorite relative to the spaceship has to be vertically downwards, the velocity of the spaceship has to be angled as shown in the diagram.



Using sine rule,

$$\frac{\sin\theta}{v_{CM}} = \frac{\sin 53.1°}{v_{spaceship}}$$

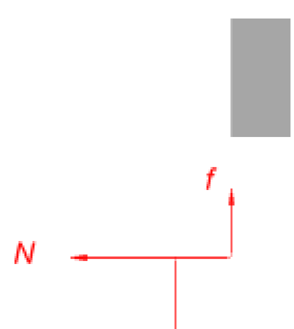
$$\frac{\sin\theta}{5/4} = \frac{\sin 53.1°}{9.0}$$

$$\theta = \sin^{-1}\left(\frac{5}{4}\frac{\sin 53.1°}{9.0}\right)$$

$$= 6.4°$$

$$\phi = 90 - 6.4 = 83.6°$$

2 **(a)**



(b) Resolving horizontally:

$N = F\cos\theta$ ---------- (1)

Resolving vertically:

$f + F\sin\theta - Mg = 0 \Rightarrow f = Mg - F\sin\theta$ ---------- (2)

Since $f \le \mu N$

$$f \le \mu F\cos\theta$$

$$Mg - F\sin\theta \le \mu F\cos\theta$$

$$F\sin\theta + \mu F\cos\theta \ge Mg$$

$$F \ge \frac{Mg}{\sin\theta + \mu\cos\theta}$$

(c) To minimize *F*, the denominator must be maximized.

Let $y = \sin\theta_0 + \mu\cos\theta_0$

$$\frac{dy}{d\theta} = \cos\theta_0 - \mu\sin\theta_0 = 0 \;\Rightarrow \tan\theta_0 = \frac{1}{\mu}$$

Subst $\sin\theta_0 = \dfrac{1}{\sqrt{1+\mu^2}}$ and $\cos\theta_0 = \dfrac{\mu}{\sqrt{1+\mu^2}}$ into **(b)**

$$F_{\min} = \frac{Mg}{\sin\theta_0 + \mu\cos\theta_0}$$

$$= \frac{Mg}{\dfrac{1}{\sqrt{1+\mu^2}} + \mu\left(\dfrac{\mu}{\sqrt{1+\mu^2}}\right)}$$

$$= \frac{Mg}{\dfrac{1+\mu^2}{\sqrt{1+\mu^2}}}$$

$$= \frac{Mg}{\sqrt{1+\mu^2}} \text{ (shown)}$$

3 **(a)** Well-insulated ⇒ adiabatic process

$$pV^{\gamma} = \text{constant} \Rightarrow p_f V_f^{\gamma} = p_i V_i^{\gamma}$$

$$\frac{p_f}{p_i} = \left(\frac{V_f}{V_i}\right)^{-\gamma}$$

$$p_f = p_0 + y\rho g \quad , \quad V_f = (h-y)A$$

$$p_i = p_0 \quad , \quad V_i = hA$$

$$\frac{p_0 + y\rho g}{p_0} = \left(\frac{(h-y)A}{hA}\right)^{-\gamma} = \left(\frac{(h-y)}{h}\right)^{-\gamma}$$

$$1 + \frac{y\rho g}{p_0} = \left(1 - \frac{y}{h}\right)^{-\gamma}$$

$$\left(1 - \frac{y}{h}\right) = \left(1 + \frac{y\rho g}{p_0}\right)^{-1/\gamma}$$

$$\frac{y}{h} = 1 - \left(1 + \frac{y\rho g}{p_0}\right)^{-1/\gamma}$$

$$h = \frac{y}{1 - \left(1 + \frac{y\rho g}{p_0}\right)^{-1/\gamma}} \quad \text{(shown)}$$

(b) Isothermal process ⇒ $pV = \text{constant} \Rightarrow p_f V_f = p_i V_i$

$$\mathrm{d}W = p\,dV = \frac{p_i V_i}{V} dV$$

$$W = p_i V_i \int_{V_i}^{V_f} \frac{dV}{V}$$

$$= p_i V_i \ln \frac{V_f}{V_i}$$

$$= p_0 (hA) \ln \frac{(h-y)A}{hA}$$

$$= p_0 hA \ln\left(1 - \frac{y}{h}\right)$$

4 **(a)**

$$h = R - R\cos\theta$$
$$= R(1-\cos\theta)$$
$$= R(1-[1-2\sin^2\frac{\theta}{2}])$$
$$= R(1-1+2\sin^2\frac{\theta}{2})$$
$$= 2R\left(\frac{\theta}{2}\right)^2 \qquad \text{[since } \theta \text{ is small, } \sin\frac{\theta}{2} \approx \frac{\theta}{2}\text{]}$$
$$= \frac{1}{2}R\theta^2 \quad \text{(shown)}$$

(b)

Ray 2 has a phase shift of π radians at Q.

$$\text{path difference} = 2h + \frac{1}{2}\lambda = \left(m + \frac{1}{2}\right)\lambda$$
$$2h = m\lambda$$

(c) **(i)**

$$r = R\sin\theta$$

Since θ is small, $\sin\theta \approx \theta$

$\therefore r = R\theta$ ----- (1)

From (a), $h = \frac{1}{2}R\theta^2$

$$(R\theta)^2 = 2hR \quad \text{-----(2)}$$

Sub (2) into (1)

$$r^2 = 2hR$$
$$= m\lambda R$$
$$r = \sqrt{m\lambda R} \quad \text{(shown)}$$

(ii)

$$r = \sqrt{m\lambda R}$$
$$= \sqrt{\left(16 \times 650 \times 10^{-9} \times 2.8\right)}$$
$$= 5.396 \times 10^{-3} \text{ m}$$
$$\approx 5.4 \times 10^{-3} \text{ m}$$

(iii) Testing of optical lenses.

If the lens surface is ground to a perfectly symmetrical curvature, a circular diffraction pattern as shown in Fig. 4.2 will be obtained.

Variations from symmetry will result in fringes that are not smooth circular shape. These variations indicate that the lens must be reground and repolished to remove imperfections.

5 **(a)**

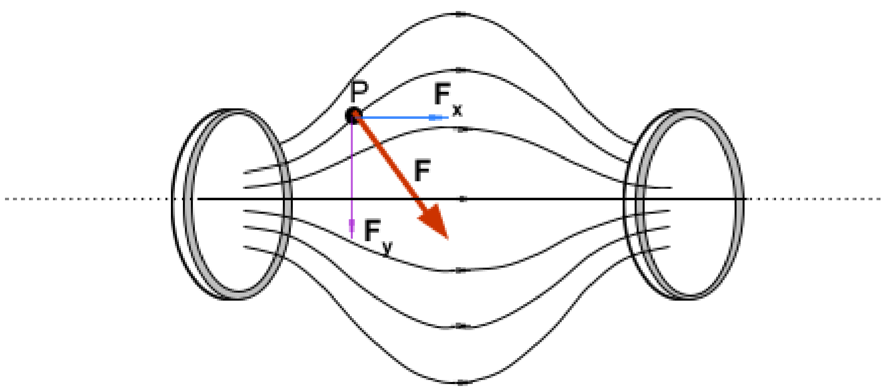
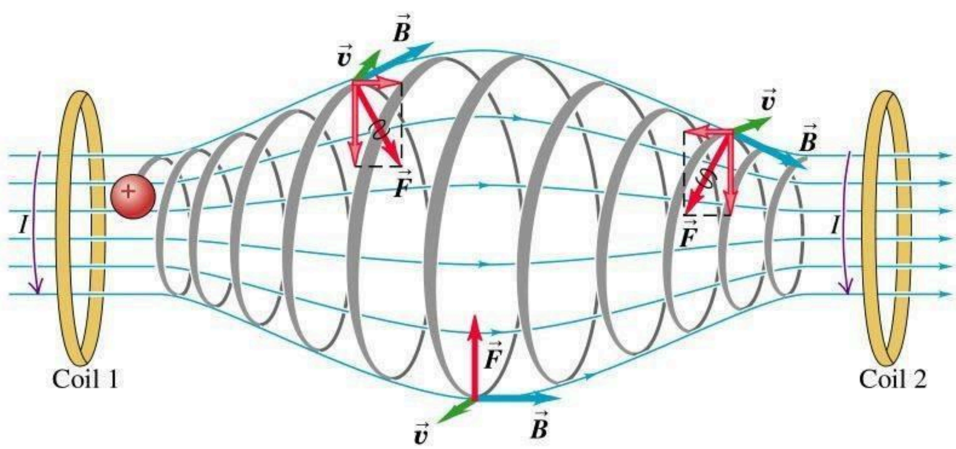


Fig. 5.2

(b) **(i)** The vertical component $\mathbf{F_y}$ (is perpendicular to the velocity of the proton and this provides the centripetal force which) causes it to move in a circular path in the vertical plane.

The horizontal component $\mathbf{F_x}$ causes the proton to accelerate towards the right where the magnetic field is stronger.

(ii)



- <u>helical path</u> around the magnetic field lines, and <u>pitch</u> of helical path is <u>wider in the region of weaker magnetic field</u> than in the region of stronger magnetic field.
- (when the proton crosses over the mid-point of the magnetic field to the stronger field region on the right, the direction of the horizontal component

of the magnetic force is reversed, causing it to) accelerate back towards the left. (This process repeats itself and the proton spirals back and forth between the two coils and is unable to leave the magnetic field).

(iii) As the charged particles will be trapped (or confined) within the magnetic field region between the coils, this set-up can have further applications. [This setup is called a magnetic bottle, and the two coils are magnetic mirrors, reflecting the charged particles back and forth.]

6 **(a)** KE = loss of electric $PE = q\,\Delta V = (1.60\times10^{-19})\,(15\,000) = 2.40\times10^{-15}$

$$v = \sqrt{\frac{2(KE)}{m}}$$

$$= \sqrt{\frac{2(2.40\times10^{-15})}{9.11\times10^{-31}}}$$

$$= 7.258\times10^{7}$$

$$= 7.26\times10^{7}\ \text{m s}^{-1}$$

(b) $\Delta p_y\,\Delta y \gtrsim h$

$m\,\Delta v_y\,\Delta y \gtrsim h$

$\Delta v_y \gtrsim h\,/\,(m\,\Delta y)$

$$= \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(0.500\times10^{-3})}$$

= 1.455
= 1.46 m s^{-1}

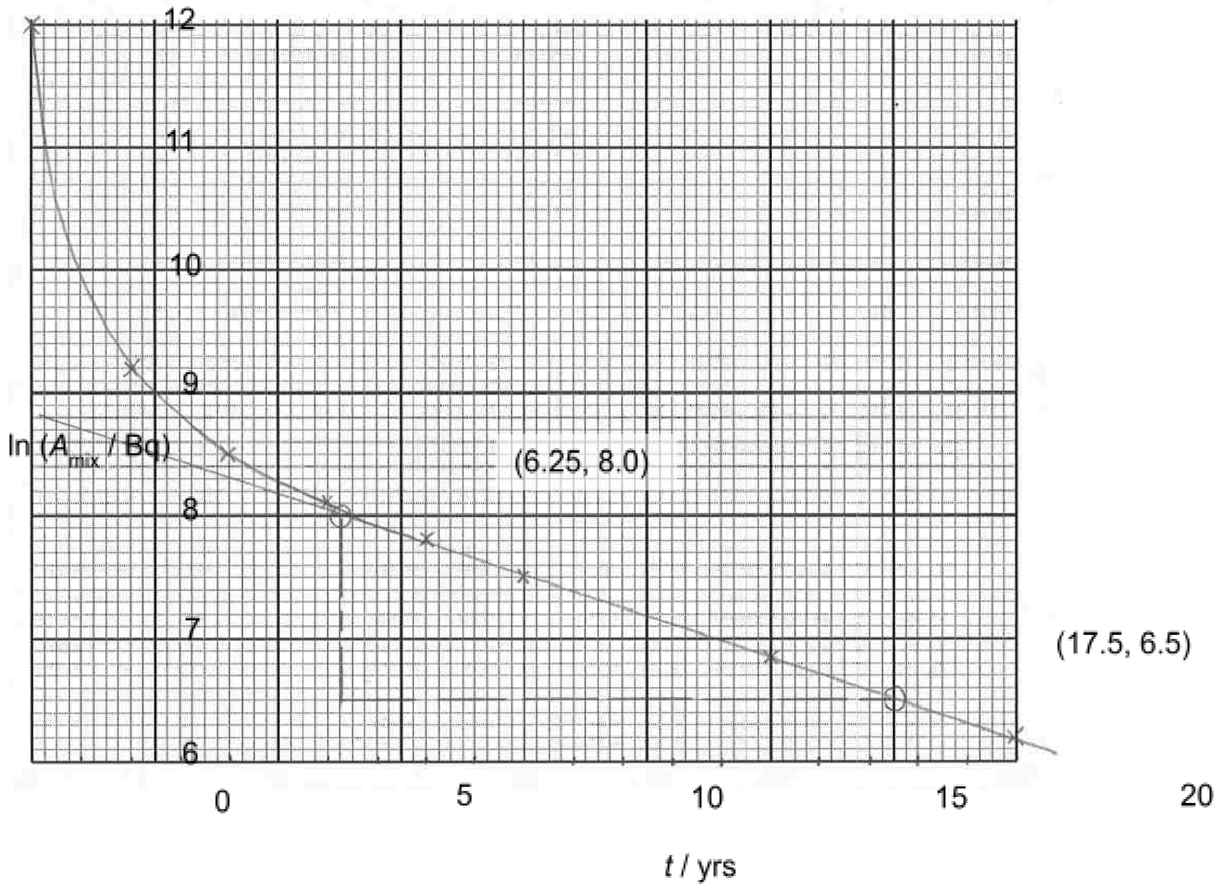
(c) The uncertainty in position $\Delta r = (\Delta v_y)(t)$

$$= \Delta v_y\left(\frac{x}{v_x}\right)$$

$$= 1.455\left(\frac{0.300}{7.26\times10^{7}}\right)$$

$$= \; = 6.01\times10^{-9}\,\text{m}$$

7(a)



(b) Explanation:

After a sufficiently long time, the nuclides in the mixture that have shorter half-lives would have decayed away, leaving the nuclide with the longest half-life. The tail end of the graph of ln A_{mix} versus t will then be a straight line. This straight line is the graph of ln $A_{\text{longest-lived nuclide}}$ versus t for the nuclide with the longest half-life in the mixture.

The gradient of this line will be equal to $-\lambda = -\frac{\ln 2}{\tau}$, where λ and τ are the decay constant and half-life of the longest-lived nuclide respectively.

Calculations: From the straight line at the end of the graph drawn in Fig. 7.2, and using gradient co-ordinates (6.25, 8.0) and (17.5, 6.5),

$$\text{gradient} = \frac{6.5 - 8.0}{17.5 - 6.25} = -\frac{1.5}{11.25}$$

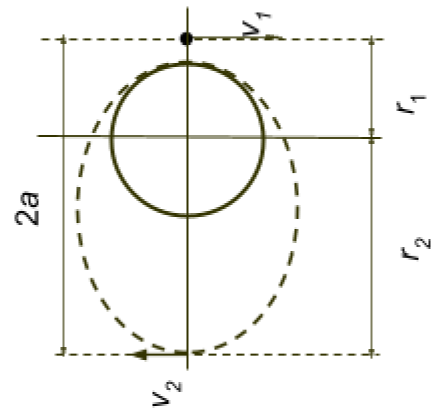
$$\text{gradient} = -\lambda = -\frac{\ln 2}{\tau}, \qquad \therefore \tau = \frac{\ln 2}{1.5} \times 11.25 = 5.199 \approx 5.2 \text{ years}$$

(c)

- Extrapolate the straight line at the end of the curve drawn in Fig. 7.2 all the way back to t = 0 years.
- Read off the values of ln $A_{\text{longest-lived nuclide}}$ at corresponding times t, then obtain the values of $A_{\text{longest-lived nuclide}}$.
- Subtract $A_{\text{longest-lived nuclide}}$ from the total activity A_{mix} to get the activity $A_{\text{remaining nuclides}}$ of the remaining nuclides.

- Plot the graph of ln $A_{\text{remaining nuclides}}$ versus time t and as before, obtain the half-life of the next longest-lived nuclide from the straight line at the tail end of the graph.
- Repeat this procedure until the final graph of ln A_{mix} versus t is a straight line corresponding to the last radioactive nuclide in the mixture.

8 **(a)** **(i)**



Let $v_1 = 8.0 \times 10^3$ m s^{-1}, and $r_1 = 6.7 \times 10^3$ m

Total energy is conserved and is equal to the sum of KE and PE.

$$KE + PE = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}, \text{where } m \text{ is the mass of the satellite}$$

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

$$\therefore\ v_1^2 - \frac{2GM}{r_1} = v_2^2 - \frac{2GM}{r_2} \quad ----(1)$$

Since angular momentum is conserved, $mv_1r_1 = mv_2r_2$

$$\therefore r_2 = \frac{v_1r_1}{v_2} \quad ----(2)$$

Substitute eqn (2) into eqn (1), we will get

$$v_1^2 - \frac{2GM}{r_1} = v_2^2 - \frac{2GMv_2}{v_1r_1} \quad ----(3)$$

Plugging in values of known quantities, we will get a quadratic eqn in v_2

$v_2^2 - 1.4932 \times 10^4 v_2 + 5.546 \times 10^7 = 0$ (use GC to solve)

$\therefore v_2 = 8.004 \times 10^3$ m s^{-1} or $6.929 \times 10^3 \approx 6.9 \times 10^3$ m s^{-1}

The first value is actually v_1, hence v_2 is $= 6.9 \times 10^3$ m s^{-1}

(ii)

$$r_2 = \frac{v_1r_1}{v_2} = \frac{8.0 \times 10^3 \left(6.7 \times 10^6\right)}{6.929 \times 10^3} = 7.736 \times 10^6 \approx 7.7 \times 10^6 \text{ m}$$

(b) $2a = r_1 + r_2 = 6.7 \times 10^6 + 7.736 \times 10^6$

$\therefore a = 7.218 \times 10^6 \approx 7.2 \times 10^6$ m

(c)

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3 = \frac{4\pi^2\left(7.218\times10^6\right)^3}{6.67\times10^{-11}\times6.0\times10^{24}}$$

$$\therefore\ T = 6091.3\text{ s} = 6.1\times10^3\text{ s}$$

9 **(b)** **(i)** From conservation of angular momentum for the system of two discs:

$$\left(I_1 + I_2\right)\omega_f = I_1\omega_1$$

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_1$$

(ii) Kinetic energies of the discs before and after are

$$K_i = \tfrac{1}{2}I_1\omega_1^2 \text{ and } K_f = \tfrac{1}{2}\left(I_1 + I_2\right)\omega_f^2$$

So

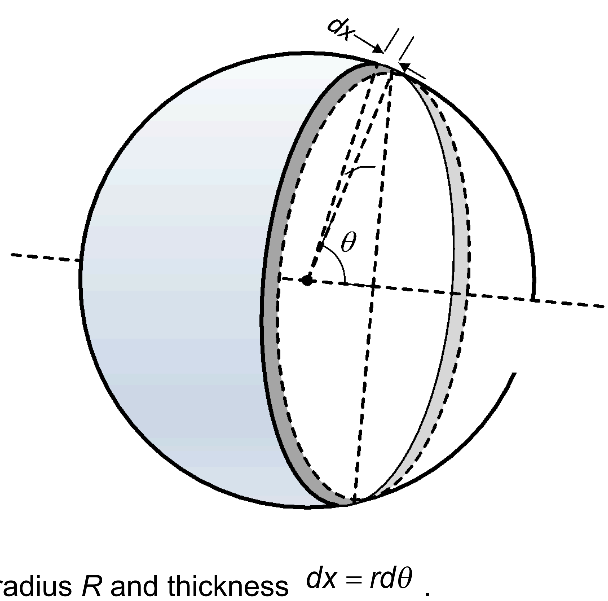
$$E_{loss} = \tfrac{1}{2}I_1\omega_1^2 - \tfrac{1}{2}\left(I_1 + I_2\right)\omega_f^2$$

$$= \tfrac{1}{2}I_1\omega_1^2 - \tfrac{1}{2}\left(I_1 + I_2\right)\left(\frac{I_1}{I_1 + I_2}\right)^2\omega_1^2$$

$$= \tfrac{1}{2}\cdot\frac{I_1 I_2}{I_1 + I_2}\omega_1^2$$

(iii) There is **<u>frictional force</u>** between the two discs as the discs are **<u>rotating with different angular speeds</u>**.
Due to slipping between the discs (before they attained the same angular speed), **<u>work is done by friction</u>** which results in energy loss.

(b)



Consider a ring of radius R and thickness $dx = rd\theta$.

The mass of this ring is

$$dm = \frac{2\pi R dx}{4\pi r^2} \times m = \tfrac{1}{2} m \sin\theta d\theta$$

with moment of inertia about axis of rotation

$$dI = R^2 dm = \tfrac{1}{2} mr^2 \sin^3\theta d\theta$$

So moment of inertia of hollow sphere

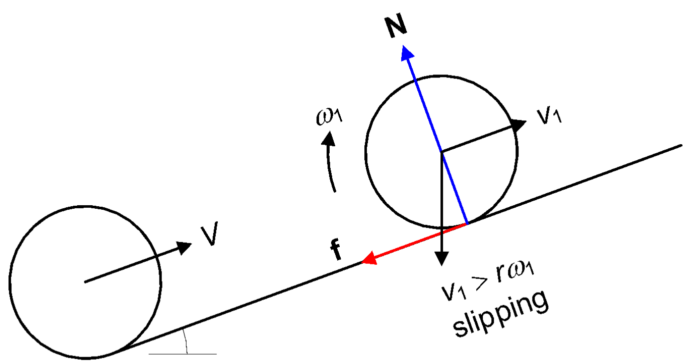
$$I = 2\int_0^{\pi/2} \tfrac{1}{2} mr^2 \sin^3\theta d\theta$$

$$= mr^2 \int_0^{\pi/2} \left(\cos^2\theta - 1\right) d\left(\cos\theta\right)$$

$$= mr^2 \left[\tfrac{1}{3}\cos^3\theta - \cos\theta\right]_0^{\pi/2} = \tfrac{2}{3} mr^2$$

(c) **(i)** $v_{CM} = r\omega$

(ii)



As the sphere rolls and slides up the slope, friction and weight slow down its translational motion but speed up its rotation. This continues until $v_{CM} = r\omega$. We will let the duration of this motion be *t*.

Considering the forces along the plane,

$$f + mg\sin\alpha = ma$$

$$\Rightarrow \quad a = \frac{mg\sin\alpha + \tfrac{1}{8} mg\sin\alpha}{m} = \tfrac{9}{8} g\sin\alpha \qquad ❶$$

This acceleration is directed down the plane.

Using $v = u + at$,

$$v_{CM} = V - \tfrac{9}{8} g \cdot \sin\alpha \cdot t \qquad ❷$$

The angular acceleration is given by

$$\tfrac{2}{3} mr^2 \ddot{\theta} = \tfrac{1}{8} mg\sin\alpha \cdot r$$

$$\Rightarrow \quad \ddot{\theta} = \frac{3g\sin\alpha}{16r}$$

At the point where friction vanishes,

$$v_{\text{CM}} = r\omega = r\ddot{\theta}t = \tfrac{3}{16}gt\sin\alpha$$

Substituting into ❷,

$$t = \frac{16V}{21g\sin\alpha}$$

(iii) Thereafter, the translational motion continues to slow down, so the friction should act upward to reduce the angular velocity of the sphere.
Acceleration of the sphere down the slope

$$a = \tfrac{7}{8}g\sin\alpha$$

Angular acceleration of the sphere

$$\ddot{\theta} = \frac{3g\sin\alpha}{16r}$$

So $r\ddot{\theta} < a$, i.e., $r\omega > v_{\text{CM}}$.
Hence slipping occurs.

10 (a) (i) Consider a thin spherical shell of radius x and thickness dx centred about the centre of the solid sphere. The charge it contains is

$$\begin{aligned} dq &= 4\pi x^2 dx \cdot \rho(x) \\ &= 4\pi\rho_0 x^2\left(1 - \frac{x}{R}\right)dx \\ &= 4\pi\rho_0\left(x^2 - \frac{x^3}{R}\right)dx \end{aligned}$$

$$\begin{aligned} q(r) &= \int_0^r 4\pi\rho_0\left(x^2 - \frac{x^3}{R}\right)dx \\ &= 4\pi\rho_0\left[\frac{x^3}{3} - \frac{x^4}{4R}\right]_0^r \\ &= 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \end{aligned}$$

(ii) **1.** <u>$r \le R$:</u>

Apply Gauss's Law to a spherical surface of radius r :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q(r)}{\varepsilon_0} = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

Since $\vec{E}$ is parallel to $d\vec{A}$ and $|\vec{E}|$ is constant over the spherical surface,

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

$$\Rightarrow \quad E \cdot 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

$$\Rightarrow \quad E = \frac{\rho_0}{3\varepsilon_0} r \left(1 - \frac{3r}{4R}\right)$$

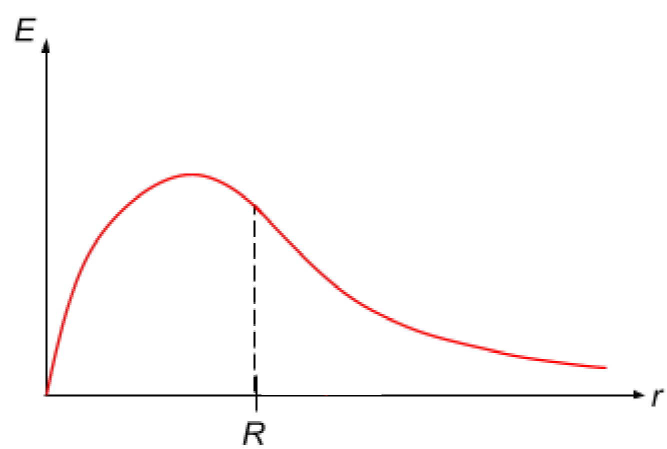
2. $\underline{r \le R}$:

For $r > R$, $$q = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^3}{4}\right) = \frac{\pi\rho_0 R^3}{3}$$

$$\Rightarrow \quad E \cdot 4\pi r^2 = \frac{\pi\rho_0 R^3}{3\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$$

(iii)



(iv) For $r \le R$,

$$\frac{dE}{dr} = \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{2R}\right)$$

$$\frac{dE}{dr} = 0 \Rightarrow r = \frac{2R}{3}$$

And
$$E_{\max} = \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3}\left(\frac{2R}{3}\right) - \frac{1}{4R}\left(\frac{2R}{3}\right)^2\right)$$

$$= \frac{\rho_0 R}{9\varepsilon_0}$$

10 **(b)** **(i)** **1.** increase flux linkage (with secondary coil) / to reduce flux loss

2. constant / direct voltage gives constant flux / field and e.m.f. (induced only) when flux (in core/coil) is changing

(ii) **1.**

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$$N_s = \left(\frac{V_s}{V_p}\right)N_p$$

$$= \left(\frac{52}{150}\right)(1200)$$

$$= 416$$

2. 0 ms or 7.5 ms or 15.0 ms or 22.5 ms

(iii) **1.**

$$\bar{P} = \frac{V_{\text{r.m.s}}{}^2}{R}$$

$$= \frac{V_0^2}{2R} \quad \left(\because V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}\right)$$

$$R = \frac{V_0^2}{2\bar{P}}$$

$$= \frac{52^2}{2(1.2)}$$

$$= 1100\ \Omega$$

2. sinusoidal shape with troughs at zero power
only 3 ‘cycles’
each ‘cycle’ is 2.4 W high and zero power at correct times

11 **(a)** **(i)** Using Kirchhoff’s voltage law,

$$E - V_R - V_C = 0$$

$$E - IR - \frac{Q}{C} = 0$$

Applying $I = \frac{dQ}{dt}$ and re-arranging,

$$\frac{dQ}{dt} + \frac{1}{RC}Q - \frac{E}{R} = 0$$

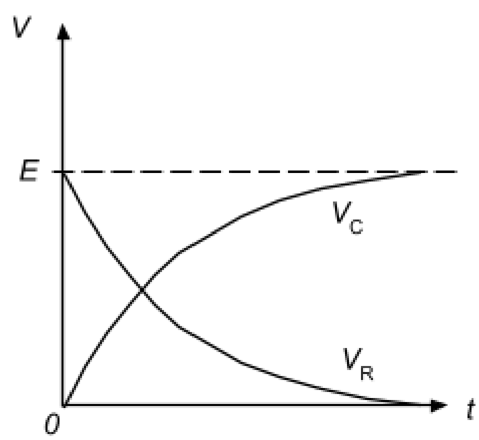
Solving the above 1st order differential equation gives

$$Q = CE\left(1 - e^{-\frac{t}{RC}}\right)$$

Differentiating the above solution gives

$$I = \frac{dQ}{dt} = \frac{CE}{RC}e^{-\frac{t}{RC}} = \frac{E}{R}e^{--\frac{t}{RC}}$$ (shown)

(ii)



(iii) $P_R = I^2 R$

$$= \left(\frac{E}{R}e^{--\frac{t}{RC}}\right)^2 R$$

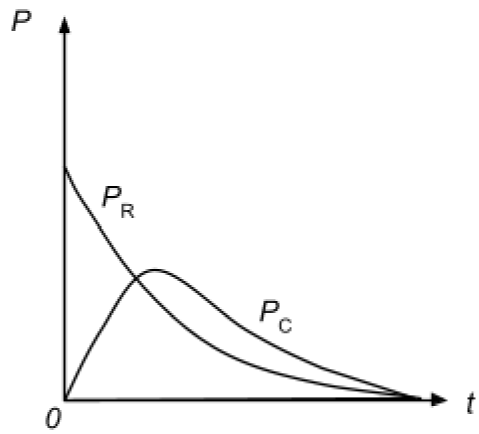
$$= \frac{E^2}{R}e^{--\frac{2t}{RC}}$$ (shown)

(iv) $U_C = \frac{1}{2}\frac{Q^2}{C}$

(v) Using the expression in (a)(iv),

$$P_C = \frac{dU_C}{dt}$$
$$= \frac{Q}{C}\frac{dQ}{dt}$$
$$= \frac{CE}{C}\left(1-e^{-\frac{t}{RC}}\right).\frac{CE}{RC}e^{-\frac{t}{RC}} \qquad \left(\text{using } Q = CE\left(1-e^{-\frac{t}{RC}}\right) \text{ from (a)(i)}\right)$$
$$= \frac{E^2}{R}\left(1-e^{-\frac{t}{RC}}\right)e^{-\frac{t}{RC}}$$

(vi)



Note: power supplied P_E by battery has same shape as P_C, but just above it.

$$P_E = P_R + P_C$$
$$= \frac{E^2}{R}e^{-\frac{t}{RC}} + \frac{E^2}{R}\left(1-e^{-\frac{t}{RC}}\right)e^{-\frac{t}{RC}}$$
$$= \frac{E^2}{R}\left(2-e^{-\frac{t}{RC}}\right)e^{-\frac{t}{RC}}$$

(b) (i) Initial energy stored in the fully charged capacitor is

$$U_i = \frac{1}{2}CE^2$$

(ii) Initial and final charge stored in capacitors is
$Q_{eff} = CE$.

Effective capacitance of both capacitors in parallel is
$$C_{eff} = C + C = 2C$$

Final energy stored in both capacitors is

$$U_f = \frac{1}{2}\frac{{Q_{eff}}^2}{C_{eff}} = \frac{1}{2}\frac{(CE)^2}{(2C)} = \frac{1}{4}CE^2$$

(iii) The <u>total energy in the system decreases</u> (to half the initial amount) as <u>electrical energy is transformed to light and thermal energy in the sparks produced</u> (or as electromagnetic radiation when the charges accelerate in the circuit) when switch is moved to position z.

(c) **(i)**

$$U_{total} = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

(ii) Since total energy is constant, the rate of change of U_{total} with time t is zero.

$$\frac{dU}{dt} = 0$$

$$\frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \qquad \left(\text{since } I = \frac{dQ}{dt}\right)$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

This expression is similar to that of an SHM, where $a = -\omega^2 x$. Hence,

$$\omega^2 = \frac{1}{LC}$$

$$2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}} \quad \text{(shown)}$$