

Currents

Guiding Questions	Learning Objectives
1. How does the macroscopic phenomenon of current flow relate to the movement of microscopic charges?	(a) show an understanding that electric current is the rate of flow of charge and solve problems using $I = \frac{Q}{t}$. (b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity.
2. What happens to energy in an electrical circuit?	(c) recall and solve problems using the equation for potential difference in terms of electrical work done per unit charge, $V = \frac{W}{Q}$. (d) recall and solve problems using the equations $P = VI$, $P = I^2R$ and $P = \frac{V^2}{R}$. (e) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations.
3. How can we describe alternating current, e.g. mathematically and graphically?	(f) show an understanding and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage. (g) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$. (h) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current. (i) distinguish between r.m.s. and peak values, and recall and use $I_{r.m.s} = \frac{I_0}{\sqrt{2}}$ and $V_{r.m.s} = \frac{V_0}{\sqrt{2}}$ for the sinusoidal case. (j) explain the use of a single diode for the half-wave rectification of an alternating current.



Nature of Science Moment – Who invented Electricity?

Thales of Miletus – Greek philosopher Thales of Miletus discovered that rubbing amber (fossilized tree sap) with animal fur would attract objects like feathers. Without truly knowing it, he had noticed the effects of magnetism and static electricity.

William Gilbert – In his book “De Magnete”, English scientist William Gilbert coined the term ‘electricus’ in 1600, which means ‘amber-like’. Polymath Sir Thomas Browne later altered the word slightly, changing it to ‘electricity’ in 1646.

Stephen Gray – Stephen Gray discovered the difference between electrical insulators and conductors, finding that electricity would “flow along wires”.

Ewald Georg von Kleist and Pieter van Musschenbroek – In 1745, the two scientists invented the Leyden Jar. This was a key invention in the build-up of our understanding of electricity. The Leyden Jar was a glass jar or vial coated on the inside and outside with metal foil. The Leyden Jar was an early capacitor, or device for storing an electric charge. This device was able to store electricity. Scientists William Watson, Henry Cavendish and Charles A Coulomb all use the Leyden Jar in the key experiments with electricity, in the few years before Benjamin Franklin’s kite experiment.

Benjamin Franklin – In 1752, Franklin proved that lightning consisted of electricity by flying a kite during a thunderstorm. A metal key was tied to the string of the kite to conduct the electricity from lightning, and it worked (giving him a shock). This experiment is credited with sparking the idea of using electricity as a power source.

Luigi Galvani – Galvani discovered ‘animal electricity’ when in 1786 he discovered by accident that the spinal cords of a frog carried an electric charge. He compared the frog’s muscles to a Leyden Jar.

Alessandro Volta – Alessandro Volta was an Italian physicist who invented the first electric battery known as the ‘voltaic pile’ in 1800. This device produced a steady flow of electrical current and was a significant advancement in the field. Volta repeated and checked Galvani’s experiments and started to doubt that the conductions were caused by specific electricity intrinsic to the animal legs or other body parts. He believed that the contractions depended on the metal cable Galvani used to connect the nerves and muscles in his experiments.

Michael Faraday – Faraday, an English scientist, made groundbreaking discoveries in the field of electromagnetism. He formulated the laws of electromagnetic induction and demonstrated the generation of electricity through moving magnetic fields. Faraday’s work laid the foundation for the development of electric generators and transformers.



Nature of Science Moment – Who invented Electricity?



The Galvani-Volta Debate

Thomas Edison – Edison, an American inventor, is renowned for his contributions to the practical application of electricity. He developed the first successful practical electric light bulb and established the world's first electric power distribution system, which was crucial in bringing electricity into homes and businesses.

Nikola Tesla – Tesla, a Serbian-American inventor and engineer, made numerous advancements in the field of electrical engineering. He pioneered alternating current (AC) power transmission, which revolutionized the way electricity is generated, transmitted and distributed.

References:

BBC Science Focus – Who invented electricity

<https://www.sciencefocus.com/science/who-invented-electricity>

https://backyardbrains.com/experiments/Galvani_Volta

How does the macroscopic phenomenon of current flow relate to the movement of microscopic charges?

1 Relationship between current and charge

1.1 Electric current



The **current** is the charge per unit time (rate of charge) flowing through a cross-sectional area.

Average current I_{ave} is the amount of charge Q that flows through a cross-sectional area A in a time interval t .

$$I_{ave} = \frac{Q}{t}$$

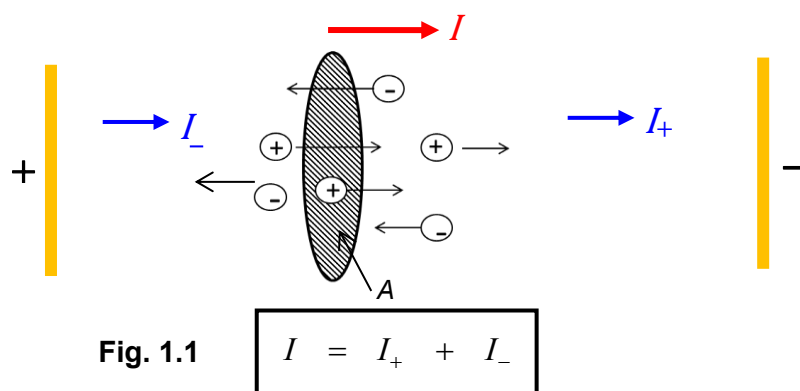
Instantaneous current I is the rate of charge flow through a cross-sectional area A (if rate of charge flows varies with time).

$$I = \frac{dQ}{dt}$$

The SI unit for electric current is ampere (A).

Examples of electric current:

1. Flow of electrons in a conductor such as a copper wire.
2. A beam of electrons in a television picture tube.
3. For gases and electrolytes (with both positive and negative charges), electric current is the sum of the flow of positive charges in one direction and the flow of negative charges in the opposite direction. Both positive and negative charges produce current in same direction (direction of flow of positive charges).



1.2 Electric charge and the coulomb

Electric charge is the physical property that causes it to experience a force when placed in an electromagnetic field. The charges of free-standing particles are integer multiples of the elementary charge e ; we say that electric charge is quantised.

There are two types of electric charges:

- positive (commonly carried by protons)
- negative (commonly carried by electrons).

An object with an absence of net charge is neutral. Coulomb is the SI unit of charge. It is the charge transferred by a steady current of one ampere in one second. ($1 \text{ C} = 1 \text{ A s}$).

If constant current I is flowing for a time t , the quantity of electric charge Q that passes a cross-section of the conductor is defined by the equation

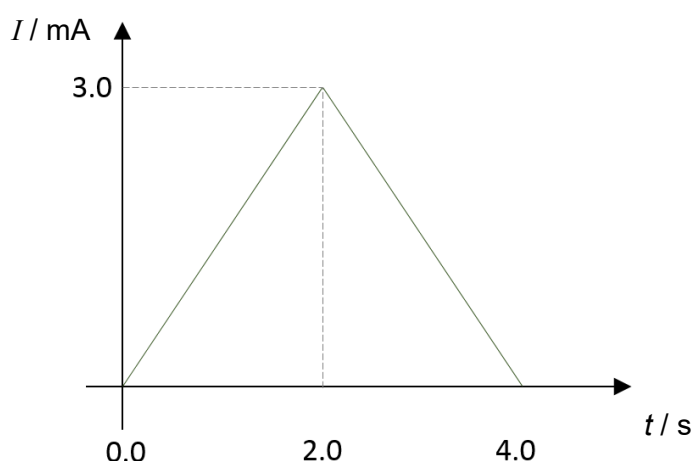
$$Q = I t$$

The SI unit of charge is coulomb (C).

One coulomb (C) is the quantity of electric charge passing through a cross section of a conductor in 1 s by an electric current of 1 A. Hence, $1 \text{ C} = 1 \text{ ampere-second (A s)}$. In electrical engineering, ampere-hour (A h) is commonly used.

Example 1.1

In the current-time graph below, find the net charge that flows in 6.0 s.



Source: SLS

Solution:

Charge is the area under the I - t graph.

$$Q = \frac{1}{2}(3 \times 10^{-3})(4.0)$$

$$Q = 6.0 \times 10^{-3} \text{ C}$$

Example 1.2

A high potential is applied between the electrodes of a hydrogen discharge tube so that the gas is ionised. Electrons then move towards the positive electrode and protons towards the negative electrode. In each second, 5×10^{18} electrons and 2×10^{18} protons pass a cross-section of the tube.

Determine the current flowing in the discharge tube.

(Note: An electron has a charge of $-e$ and a proton has a charge of $+e$, where $e = 1.60 \times 10^{-19} \text{ C}$)



Source: http://upload.wikimedia.org/wikipedia/commons/b/be/Hydrogen_discharge.jpg

Fig. 1.2

Solution:

Total current is the sum of current by positive charges flowing in one direction, and current by negative charges flowing in opposite direction.

$$\begin{aligned} I &= I_+ + I_- \\ I &= (2 \times 10^{18} \times 1.6 \times 10^{-19}) + (5 \times 10^{18} \times 1.6 \times 10^{-19}) \\ &= 1.12 \text{ A} \end{aligned}$$

1.3 Transport charge equation

- At $T > 0$ K, electrons in a wire move randomly about respective fixed positions with zero net displacement.
- When an e.m.f. source is applied, at $T > 0$ K, electric force on electrons cause them to accelerate to gain speed (kinetic energy). As electrons move (current flows), they collide continuously with the lattice atoms to lose speed (lose kinetic energy to lattice atoms) or rebound in different directions. Hence, the electrons in the wire drift along the wire at a very low drift speed v .

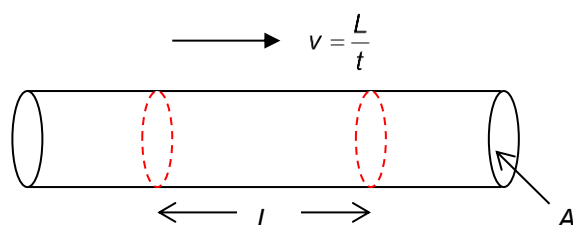


Fig. 1.3

Current is due to the number of electrons per unit time flowing through a cross-sectional area A .

$$\begin{aligned}
 I &= \frac{Q}{t} & \text{where } \frac{N}{t} &= \text{number of electrons per unit time} \\
 &= \frac{N}{t} e & \text{where } n &= \text{number density (no. of electrons per unit volume, } n = \frac{N}{V}) \\
 &= \frac{nV}{t} e \\
 &= n \frac{AL}{t} e & \text{where } V &= AL, \text{ and } v = \frac{L}{t} \\
 &= n e A v
 \end{aligned}$$

In general, $I = n q v A$ where q – charge of charge carriers

- For metals, $n = 10^{29}$ electrons per m^3
- For semiconductors, $n = 10^{18}$ electrons per m^3

**What happens to energy
in an electrical circuit?**

2 Energy conversion in the circuit

2.1 Power

An e.m.f source supplies the electrical energy W for a charge Q to flow between the 2 points a circuit with a potential difference V ,

$$W = Q V \quad \dots\dots\dots (1)$$

A device of power, P operating for time t converts an amount of energy W given by

$$W = P t \quad \dots\dots\dots (2)$$

From (1) and (2)

$$P t = Q V$$

$$P = \frac{Q}{t} V$$

Hence, when there is a potential difference V across and a current I flows through a device, the **electrical power** P converted by a device is given by

$$P = I V \text{ OR } P = I^2 R \text{ OR } P = \frac{V^2}{R}$$

The SI unit of power is watt (W).

Example 2.1

The same potential difference is applied to two bulbs arranged in parallel to each other. One bulb is 60 W while the other is 30 W.

Which one of the following statements is true?

- A** The 30 W bulb carries the greater current and has the higher resistance.
- B** The 30 W bulb carries the greater current, but the 60W bulb has the higher resistance.
- C** The 30 W bulb has the higher resistance, but the 60 W carries the greater current.
- D** The 60 W bulb carries the greater current and has the higher resistance.

Solution:

$$P = IV = \frac{V^2}{R}$$

(C)

Example 2.2

An electric kettle is rated at 2.0 kW when operating on a 240 V supply.

Determine the power that the kettle will dissipate if the supply is reduced to 220 V.

Solution:

The resistance of the heating element remains constant (if temperature stays constant). Therefore, when V changes, I will change correspondingly. Hence, we cannot use $P = IV$ or $P = I^2 R$, can only apply $P = \frac{V^2}{R}$.

$$P \propto V^2$$

$$\frac{P_{220}}{P_{240}} = \frac{V_{220}^2}{V_{240}^2}$$

$$P_{220} = \frac{(220)^2}{(240)^2} (2 \times 10^3)$$

$$P_{220} = 1.68 \text{ kW}$$

Example 2.3

The power output at a power station is 500 MW at a p.d. of 132 kV. If the total resistance of the complete power line is 4.0Ω , calculate

- (a) the current flowing in the power line,
- (b) the voltage drop caused by the power line, and
- (c) the percentage loss of power in transmission.

Solution:

(a) *Current from power station,*

$$P = IV$$

$$I = \frac{P}{V} = \frac{500 \times 10^6}{132 \times 10^3} = 3.79 \text{ kA}$$

(b) *Voltage drop along the power line,*

$$V = IR$$

$$V = (3.79 \times 10^3)(4) = 15.2 \text{ kV}$$

(c) *% loss of power*

$$P_{\text{loss}} = \frac{V^2}{R} = \frac{(15.2 \times 10^3)^2}{4} = 57.8 \text{ MW}$$

$$\%P_{\text{loss}} = \frac{57.8 \text{ MW}}{500 \text{ MW}} \times 100 = 11.6\%$$

2.2 Distinguish between the electromotive force of a cell and the potential difference between its terminals

A source (battery) sets up an electric field along the connecting wires. Thus, each charge in the wire has an electric potential energy. When negative (positive) charges move from lower (higher) potential to higher (lower) potential, they lose electric potential energy. Flow of charges produces electric current.



Fig. 2.1



The electric **potential difference** V between two points in a circuit is the amount of electrical energy converted to other forms of energy per unit charge passing through them.

$$V = \frac{W}{Q}$$

The SI unit for potential difference is volt (V).

The *volt* is defined as the potential difference between two points in a circuit where 1 J of electrical energy is converted to other forms of energy when 1 C of charge passes from one point to the other. ($1 \text{ V} = 1 \text{ J C}^{-1}$)

Electromotive force (e.m.f.) E of a cell	Potential difference (p.d.) V between terminals of a cell
e.m.f. refers only to source	p.d. refers to any two points in a circuit
e.m.f. is the amount of non-electrical energy converted into electrical energy per unit charge passing through the terminals of the cell (source).	p.d. is the amount of electrical energy converted to other forms of energy per unit charge passing from one point to the other.
E is a source of energy. E exist with/without current in the circuit	$E > V$ V exist only when current flows in the circuit.

Fig. 2.2



Nature of Science Moment – War of the Currents: AC vs DC!

In the late 1880s and early 1890s, there was intense rivalry between two companies, Edison Electric Light Company and Westinghouse Electric Company. The former company supported distributing electricity using Direct Current (DC), while the latter supported Alternating Current (AC). Thomas Edison (best known for his invention of the light bulb) had patents that made him invested in the widespread adoption of DC.

However, distributing power using DC meant distributing electricity at low voltages (required by household appliances), at very high currents. This resulted in low efficiency as a lot of energy was wasted as heat during transmission. Despite this, Edison was adamant about the DC system he was invested in. Edison even resorted to electrocuting animals with AC to prove how dangerous AC is! For a period of time, many municipalities were convinced by Edison and adopted the DC system.

However, due to lower cost and higher efficiency of AC systems, the AC mode of distribution won and eventually became the dominant power source.

Comments

This is an example of the subjective nature of Science. Edison was personally invested in DC technology, and hence wanted widespread adoption of this technology. Scientists and inventors are humans and they come with certain biasness and subjectivity. Hence even great inventors might at some point in time allow their own biasness to cloud their judgement.

This is also an example of how social factors can affect the progress of science and technology. Edison tried shaped public opinion on DC to delay the progress AC, but to no avail, as AC still won in the end.

References:

WHAT'S THE DIFFERENCE BETWEEN AC AND DC? MIT (2013)

<http://engineering.mit.edu/ask/what%E2%80%99s-difference-between-ac-and-dc>

How Electricity Works, How Stuff Works, 2004

<http://science.howstuffworks.com/electricity8.htm>

**How can we describe alternating current,
e.g. mathematically and graphically?**

3 Description of alternating currents

3.1 Characteristics of alternating currents (a.c.) & direct currents (d.c.)

3.1.1 Characteristics of d.c.



In d.c. circuits, the current and electromotive force is typically assumed to remain constant with time

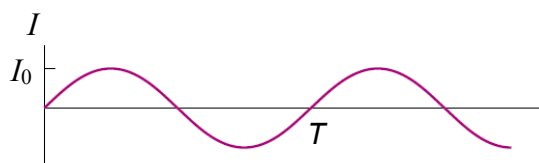
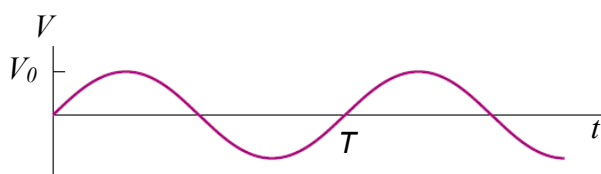
$$I = I_0$$

$$V = V_0$$

Instantaneous, peak and average values are the same values

3.1.2 Characteristics of a.c.

An alternating current is an electric current that periodically reverses its direction in a circuit with a frequency, f independent of the constants of the circuit.



[Diagram source: Serway 'Physics for Scientists & Engineers']

The time-variation of a sinusoidal current or voltage can be represented by the equation

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin \omega t$$

where V_0 or I_0 = peak (or maximum) value

and ω = angular frequency of the supply (in rad s^{-1})

Here are some terms associated with an alternating current:

Frequency, f , for an a.c. is defined as the number of times the current passes through its zero value in the same direction per unit time (where $2\pi f = \omega$). It is the frequency of the a.c. power supply.

For local mains, $f = 50$ Hz

Period, T , for an a.c. is the time taken by the a.c. to make one complete alternation. $(T = \frac{1}{f})$

Peak value of an a.c., I_o , (or amplitude) of alternating current is the maximum value of the current in a periodic cycle.

Instantaneous current of an a.c., I , is the current flowing at a particular instant in time.

Root mean square (r.m.s) of an alternating current I_{rms} is defined as the value of a.c. that is equal to the steady direct current which would dissipate heat at the same average rate in a given resistor.

Now, what this means can be illustrated as shown:

For a direct current I passing through a resistor R :

$$P_{ave} = I^2 R$$

For alternating current with I_{rms} passing through a resistor R :

$$P_{ave} = I_{rms}^2 R$$

The magnitude of an alternating current is constantly changing with time, hence the term I_{rms} is used as the value of a.c.

Both P_{ave} dissipated in the resistor R for the two cases are the same. This is the basis of the definition of I_{rms} .

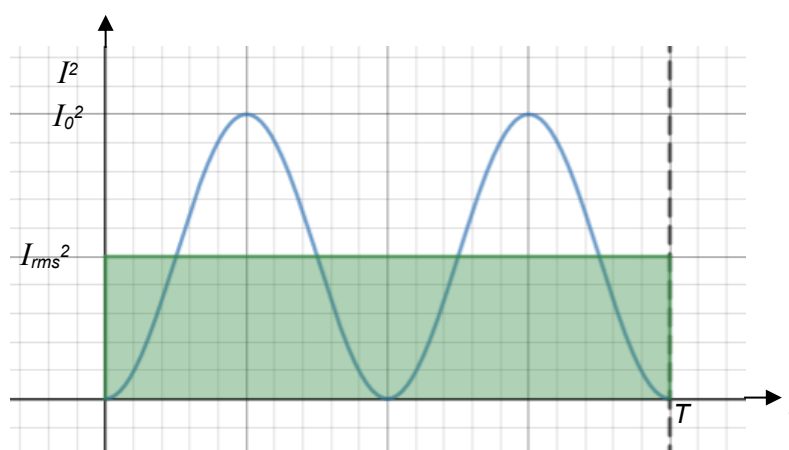
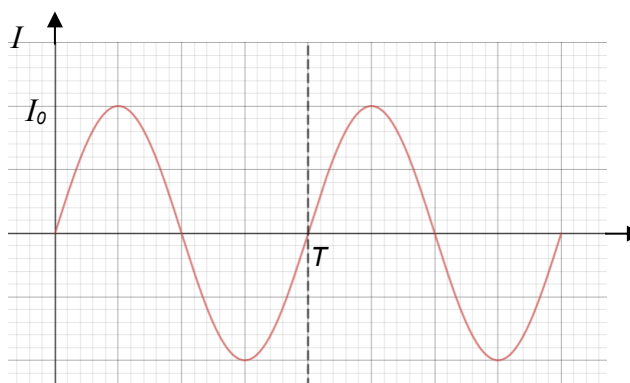
3.2 Root-mean-square current and mean power

3.2.1 Deriving root-mean-square current, I_{rms} for sinusoidal alternating current

For a sinusoidal alternating current shown on the right, we have

$$I = I_0 \sin \omega t$$

where I_0 = peak (maximum) current
and $\omega = 2\pi f = 2\pi / T$



The aim is to find an I_{rms} value where the area under the above graphs are identical.

To find I_{rms} , we carry out the following steps:

Step 1: Square the alternating current.

Step 2: Take the mean (or average) value of the squared alternating current over a duration of 1 period.

From the graph of I^2 against t (shown above), values of I^2 varies from 0 to I_0^2 .

The mean of I^2 over duration of 1 period = $\langle I^2 \rangle = \frac{1}{2} I_0^2$

Step 3: Square root the value of the mean-square alternating current (Step 2)

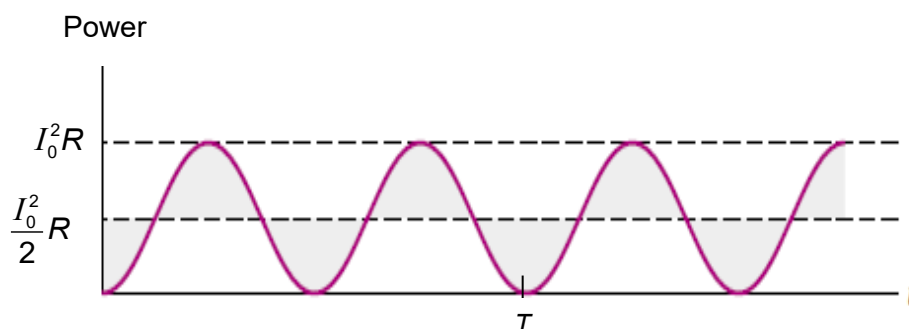
$$\text{Root-mean-square value, } I_{rms} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$$

** $I_{rms} = \frac{I_0}{\sqrt{2}}$ applies only for sinusoidal alternating current

Note: The same steps apply for sinusoidal alternating voltage (graph of p.d. against time)

Refer to Appendix A for mathematical proof on the above.

3.2.2 Relating mean power to I_{rms}



[Diagram source: Halliday ' Fundamentals of Physics']

$$\text{Average power, } \langle P \rangle = \langle I^2 \rangle R = \frac{I_0^2}{2} R = \frac{1}{2} P_{\max}$$

$$\text{Also, } \langle P \rangle = \langle I^2 \rangle R = \frac{I_0^2}{2} R = \left(\frac{I_0}{\sqrt{2}} \right)^2 R = I_{rms}^2 R$$

Summary

(for sinusoidal alternating current and voltage only)

	current	voltage	power
peak values	I_0	V_0	P_0
instantaneous values	$I = I_0 \sin \omega t$	$V = V_0 \sin \omega t$	$P = I_0 V_0 \sin^2 \omega t$ or $I_0^2 R \sin^2 \omega t$ or $\frac{V_0^2}{R} \sin^2 \omega t$
r.m.s. and mean values (only for sinusoidal)	$I_{rms} = \frac{I_0}{\sqrt{2}}$	$V_{rms} = \frac{V_0}{\sqrt{2}}$	$P_{mean} = \frac{P_0}{2}$ or $I_{rms} V_{rms}$ or $I_{rms}^2 R$ or V_{rms}^2 / R

Example 3.1

An electric kettle is marked 240 V, 1440 W. Although not stated, these values may be taken to be r.m.s. values.

Determine

(a) the r.m.s. current

$$\begin{aligned} P_{\text{mean}} &= I_{\text{rms}} V_{\text{rms}} \\ 1440 &= I_{\text{rms}} (240) \\ I_{\text{rms}} &= 6.00 \text{ A} \end{aligned}$$

(b) the peak current

$$\begin{aligned} I_{\text{rms}} &= \frac{I_0}{\sqrt{2}} \\ I_0 &= 8.49 \text{ A} \end{aligned}$$

(c) the peak potential difference

$$\begin{aligned} V_{\text{rms}} &= \frac{V_0}{\sqrt{2}} \\ V_0 &= 339 \text{ V} \end{aligned}$$

(d) the peak power

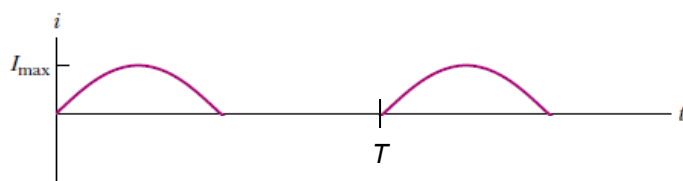
$$\begin{aligned} P_{\text{mean}} &= \frac{P_0}{2} \\ P_0 &= 2880 \text{ W} \end{aligned}$$

(e) the average power

$$P_{\text{mean}} = 1440 \text{ W}$$

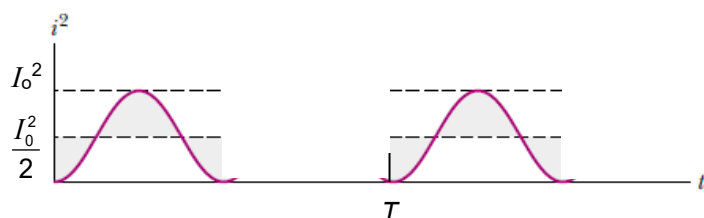
Example 3.2

Determine the value of the root-mean-square current for the following half-wave rectified alternating current.



[Diagram source: Serway 'Physics for Scientists & Engineers' modified]

Step 1: Square the alternating current.



Step 2: Take the mean (or average) value of the squared alternating current over a duration of 1 period. From the graph of I^2 against t (shown above), values of I^2 varies from 0 to I_0^2 .

The mean of I^2 over duration of $\frac{1}{2}$ period = $\frac{1}{2} I_0^2$

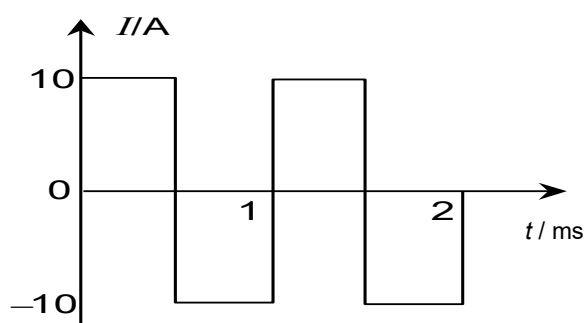
However, the mean of I^2 over duration of 1 period = $\langle I^2 \rangle = \frac{1}{4} I_0^2$

Step 3: Square root the value of the mean-square alternating current.

$$\text{Root-mean-square value, } I_{rms} = \sqrt{\frac{I_0^2}{4}} = \frac{I_0}{2}$$

3.2.3 Deriving root-mean-square current, I_{rms} for non-sinusoidal alternating current

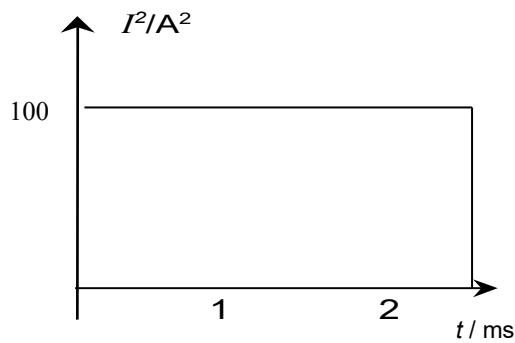
3.2.3.1 Graphical method



The alternating current signal shown on the left is not sinusoidal.

Determine the root-mean-square current over a 1 period duration.

Step 1: Square the alternating current.



Step 2: Take the mean (or average) value of the squared alternating current over a 1 period duration.

The mean of I^2 over duration of 1 period

$$= \langle I^2 \rangle$$

$$= I_0^2$$

Step 3: Square root the value of the mean-square alternating current.

Root-mean-square value, $I_{rms} = I_0$

3.2.3.2 Calculation method

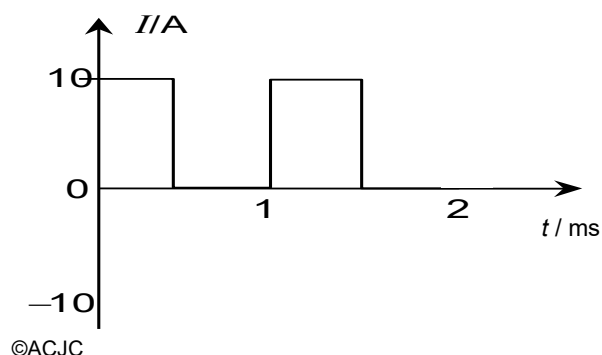
Over a 1 period duration:

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T I^2 dt}$$

Substituting in the values from the current-time graph in Section 3.2.3.1,

$$I_{rms} = \sqrt{\frac{[10^2(0.5 \times 10^{-3}) + (-10)^2(0.5 \times 10^{-3})]}{1.0 \times 10^{-3}}} = 10\text{A}$$

Example 3.3



Find the r.m.s. current of the following signal over the following periods

- (a) 1 ms
(b) 1.5 ms

Solution:

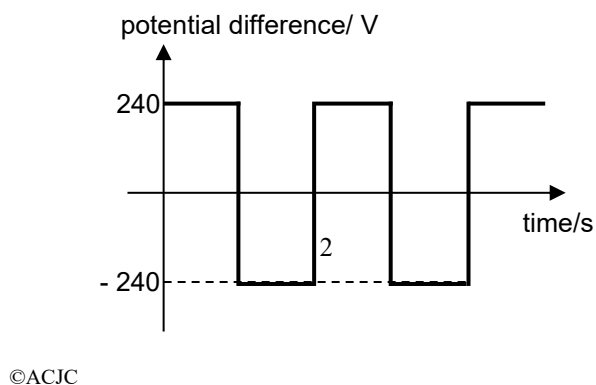
$$(a) \quad I_{rms} = \sqrt{\frac{10^2(0.5 \times 10^{-3}) + 0^2(0.5 \times 10^{-3})}{1.0 \times 10^{-3}}}$$

$$= 7.1 \text{ A}$$

$$(b) \quad I_{rms} = \sqrt{\frac{10^2(0.5 \times 10^{-3}) + 0^2(0.5 \times 10^{-3}) + 10^2(0.5 \times 10^{-3})}{1.5 \times 10^{-3}}}$$

$$= 8.2 \text{ A}$$

Example 3.4



The potential difference across an electrical device of resistance $100 \text{ k}\Omega$ takes the form of a square wave as shown by the graph on the left.

What is the average power dissipated in the electrical device?

Solution:

$$\text{Mean power} = \frac{V_{rms}^2}{R}$$

$$V_{rms} = \sqrt{\frac{240^2 + (-240)^2}{2}}$$

$$P_{\text{mean}} = \frac{240^2}{100 \times 10^3} = 0.576 \text{ W}$$

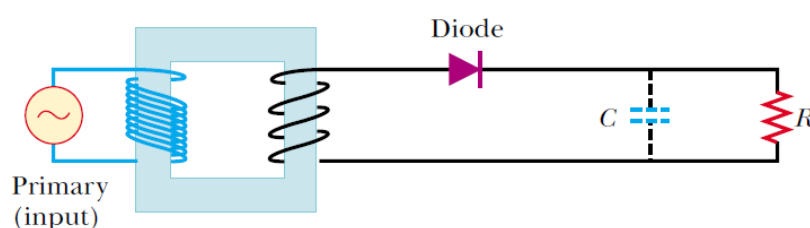
3.3 Half-wave rectification

Rectification is the process of converting alternating current to a unidirectional current.

A rectifier is a circuit component or device or a combination of components in a circuit.

In a half-wave rectifier, current flows through the resistor only during one half of every generator's voltage cycle. A single diode is used for half-wave rectification.

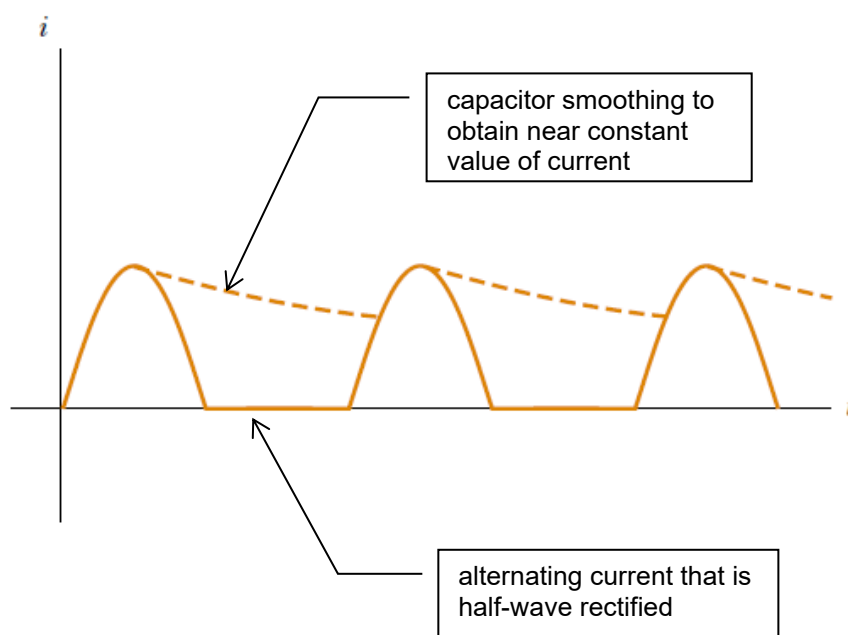
Diodes are unidirectional devices, hence they are used in rectifier circuits to convert an ac voltage into a dc voltage. Ideally, it allows current to flow in one direction but allows no current to flow in the other direction.



[Diagram source: Serway, Physics for scientists and engineers]

Smoothing effect

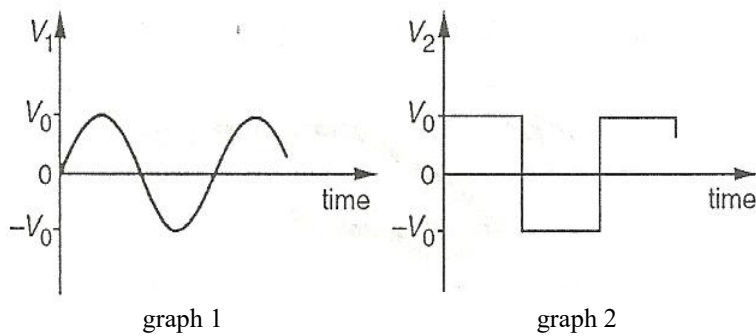
In a rectified circuit, a capacitor connected parallel to the load can smoothen the half-wave rectified voltages (to obtain close to a constant voltage value). The larger the capacitance, the better the smoothing effect.



[Diagram source: Serway, Physics for scientists and engineers]

Example 3.5 [J82/P2/Q18]

A sinusoidal potential difference V_1 shown in graph 1 produces heat at a mean rate W when applied across a resistor R .



What is the mean rate of production of heat when the square-wave potential difference V_2 shown in graph 2 is applied across the resistor?

- A** $\frac{W}{2}$
 B $\frac{W}{\sqrt{2}}$
 C $\sqrt{2}W$
 D $2W$

Solution:

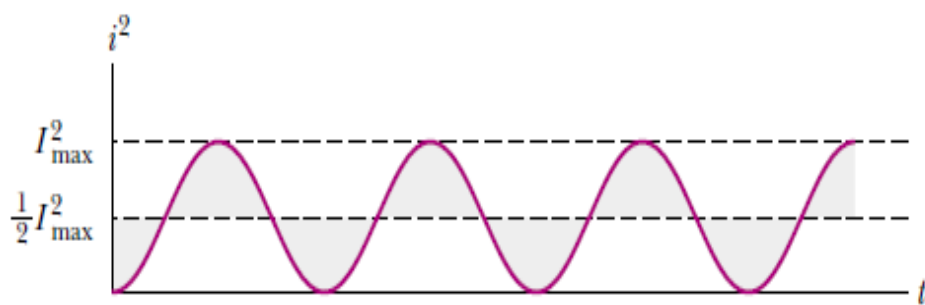
For a sinusoidal alternating p.d.,

$$\langle P \rangle = \frac{P_o}{2} = \frac{V_o^2 / R}{2} = W$$

For the square-wave,

$$\langle P \rangle = \frac{V_o^2}{R} = 2W$$

Appendix A



$$\begin{aligned}
 I_{rms}^2 &= \frac{1}{T} \int_0^T I^2 dt \\
 &= \frac{1}{T} \int_0^T I_0^2 \sin^2(\omega t) dt \\
 &= \frac{I_0^2}{T} \int_0^T \frac{1}{2} [1 - \cos(2\omega t)] dt \\
 &= \frac{I_0^2}{2T} \left[t - \frac{1}{2\omega} \sin(2\omega t) \right]_0^T \\
 &= \frac{I_0^2}{2T} T \\
 &= \frac{I_0^2}{2} \\
 I_{rms} &= \frac{I_0}{\sqrt{2}}
 \end{aligned}$$