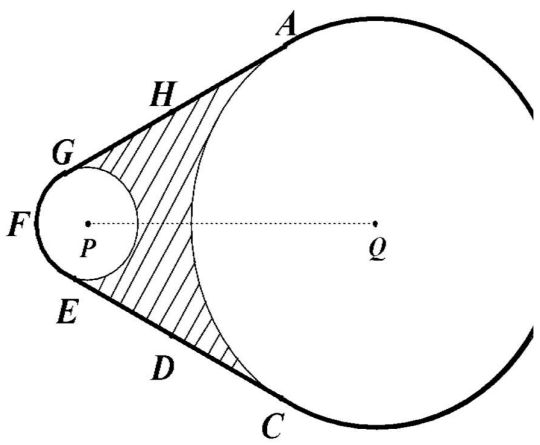
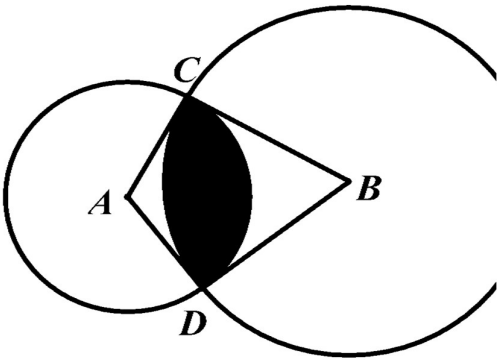
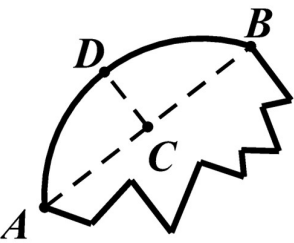
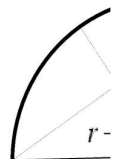


Topical Revision Worksheet 9

Topic: Circular Measure

Question 1 [2009 S4 EOY, Q21]	marks
A pulley system in a car comprises two circular discs with circumferences 28π cm and 8π cm respectively. The two discs are bound tightly by a belt $ABCDEFGHA$ and their centres P and Q are 20 cm apart.  (i) Show that angle AQP is $\frac{\pi}{3}$ radians. Solution: $\cos \angle AQP = \frac{10}{20} = \frac{1}{2}$ $\therefore \angle AQP = \frac{\pi}{3} \text{ (shown)}$	[2]
(ii) Find the length of the belt ABC in contact with the bigger disc. Solution: $AQ = \frac{28\pi}{2\pi} = 14 \text{ cm}$ $\angle AQC = 2\pi - 2\angle AQP = \frac{4\pi}{3}$ $\text{Arc } ABC = 14 \times \frac{4\pi}{3} = 58.6$	[3]
(iii) Find the area of the shaded region. Solution: $GA = \sqrt{20^2 - 10^2} = \sqrt{300}$ $\angle GPQ = \pi - \angle AQP = \frac{2\pi}{3}$ Shaded area = trapezium $AQPG$ – area of sectors $= 2 \left[\frac{1}{2} \sqrt{300} (4 + 14) - \frac{1}{2} (14)^2 \left(\frac{\pi}{3} \right) - \frac{1}{2} (4)^2 \left(\frac{2\pi}{3} \right) \right]$ $\approx 73.0 \text{ cm}^2 \text{ (3s.f.)}$	[4]

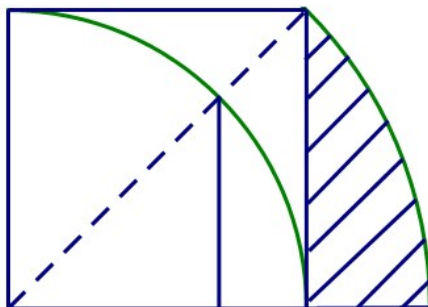
Question 2 [2010 S4 EOY, Q16]	marks
The diagram shows two circles with centres A and B, intersecting at points C and D. The radii of the circles are 8 cm and 12 cm respectively, and angle $ACB = \text{angle } ADB = \frac{\pi}{2}$ radians.  (i) Show that angle CBD is approximately 1.18 radians. Solution: (i) $\angle CBD = 2 \tan^{-1} \left(\frac{8}{12} \right) \approx 1.176 \approx 1.18 \text{ rad}$	[2]
The shaded region is bounded by minor arcs CD with centres A and B respectively. (ii) Find the perimeter of the shaded region. Solution: (ii) $\text{arc } CD \text{ centre } A + \text{arc } CD \text{ centre } B$ $= 8(\pi - 1.18) + 12(1.18)$ $\approx 29.9 \text{ cm}$	[3]
(iii) Find the area of the shaded region. Solution: (iii) Area of segment given by $\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta$. area of segment centre A $= \frac{1}{2}8^2 [(\pi - 1.18) - \sin(\pi - 1.18)] \approx 33.184$ [Accept 33.360 if students use 1.176] area of segment centre B $= \frac{1}{2}12^2 [1.18 - \sin 1.18] \approx 18.388$ Area of shaded region = 51.6 cm^2.	[5]

Question 3 [2011 S4 EOY, Q8]	marks
During a school field trip, Jane discovered a broken circular plate, as shown in the diagram below, from a deserted historical site. She measured the length of the chord AB as 14 cm and from C, the mid-point of AB, she measured the length CD as 1 cm where D is the point on the circumference nearest to C.  Find (i) the radius of the plate, Solution:  $r^2 = (r-1)^2 + 7^2$ $r = 25 \text{ A1}$	[3]
(ii) the area of the segment ADB. Solution: $\angle AOB = 2 \tan^{-1} \left(\frac{AC}{OC} \right) = 2 \tan^{-1} \left(\frac{7}{24} \right)$ Area of sector OAB $= \frac{1}{2} (25)^2 \left[2 \tan^{-1} \left(\frac{7}{24} \right) \right]$ Area of $\triangle OAB$ $= \frac{1}{2} (25)^2 \sin \left[2 \tan^{-1} \left(\frac{7}{24} \right) \right]$ Area of segment $= \text{Area of sector } OAB - \text{Area of } \triangle OAB$ $= \frac{1}{2} (25)^2 \left[2 \tan^{-1} \left(\frac{7}{24} \right) - \sin \left[2 \tan^{-1} \left(\frac{7}{24} \right) \right] \right]$ $\approx 9.37 \text{ cm}^2$	[4]

Question 4 [2012 S4 EOY, Q9]

marks

The diagram shows a quadrant inscribed in the square $ABCD$ of sides 6 cm. ACE is a sector centred at A . F is the point of intersection of the diagonal AC and the arc DB . FG is perpendicular to AE .



Find

- (i) the area of the shaded region BCE ,

Solution:

Area of shaded region BCE

$$= \frac{1}{2}(AC)^2 \left(\frac{\pi}{4}\right) - \frac{1}{2}(6)^2$$

$$= \frac{\pi}{8}(6^2 + 6^2) - 18$$

$$= 9\pi - 18 \approx 10.3 \text{ cm}^2$$

[3]

- (ii) the perimeter of $FGBECE$.

Solution:

Perimeter of $FGBECE$

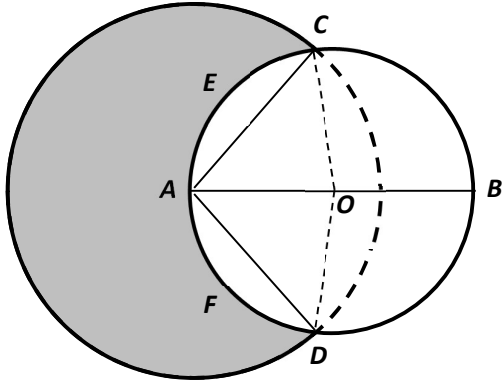
$$= GE + CE + FC + FG$$

$$= AC - AG + CE + FC + AG$$

$$= \sqrt{72} + \frac{\pi}{4}\sqrt{72} + \sqrt{72} - 6$$

$$\approx 17.6 \text{ cm}$$

[4]

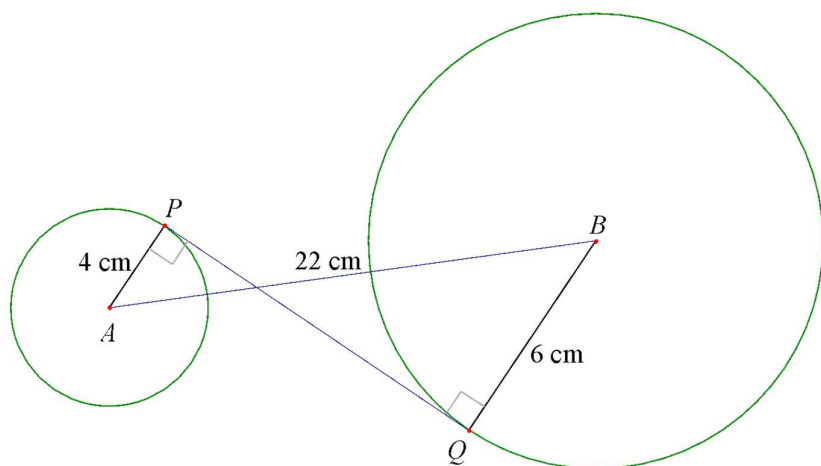
Question 5 [2013 S4 EOY, Q15]	marks
Two circles are such that A is the centre of the bigger circle and AB is the diameter of the smaller circle. The radius of the bigger circle is 7 cm and the diameter AB is of length 10 cm. Points C and D are the intersection of the two circles and O is the centre of the smaller circle.  (i) Show that $\cos \angle CAO$ is $\frac{7}{10}$. Solution: $AC = 7$ cm and $AO = 5$ cm Since triangle OAC is isosceles with $OA = OC$ $\cos \angle CAO = \frac{7}{2 \div 5} = \frac{7}{10}$	[2]
(ii) Find the area of the minor segment ACE. Solution: $\angle AOC = \cos^{-1} \frac{5^2 + 5^2 - 7^2}{2(5)(5)} = \cos^{-1} \frac{1}{50}$ Hence area of minor segment ACE $= \frac{1}{2}(5)^2 [\angle AOC - \sin \angle AOC] \text{ cm}^2$ $= 6.89 \text{ cm}^2 \text{ (3 d.p.)}$	[3]
(iii) Find the area of the shaded crescent. Solution: Area of sector ACD $= \frac{1}{2}(7)^2 2 \cos^{-1} \frac{7}{10} \text{ cm}^2$ $= 38.975 \text{ cm}^2 \text{ (3 d.p.)}$ Hence area of shaded crescent $= \pi(7)^2 - 2(6.887) - 38.975 \text{ cm}^2$ $= 101 \text{ cm}^2 \text{ (3 s.f.)}$	[3]

Question 6 [2014 S4 EOY, Q6]

marks

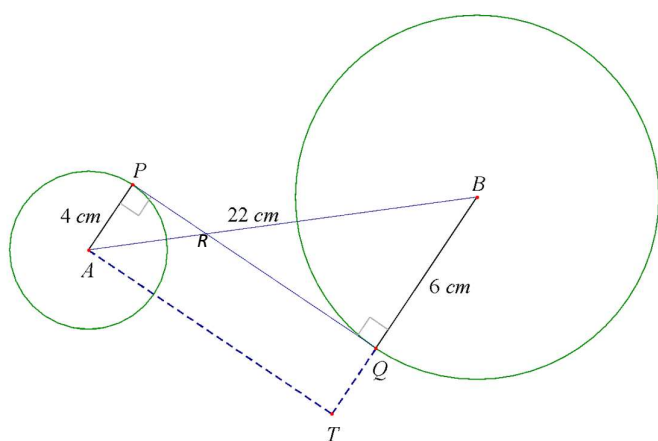
The diagram shows two circles, circle A , with radius $AP = 4$ cm and circle B , with radius $BQ = 6$ cm having a common internal tangent PQ . If the distance between A and B is 22 cm, find PQ .

[3]



Solution:

Extend line BQ . Construct line parallel to PQ passing through A to meet extended line BQ at T .



$$BT = 6 + 4 = 10 \text{ cm}$$

$$AT^2 + 10^2 = 22^2$$

$$AT = \sqrt{22^2 - 10^2}$$

$$= \sqrt{384}$$

$$= 19.6 \text{ cm (3s.f.)}$$

$$PQ = AT = 19.6 \text{ cm}$$

Using similar triangles,

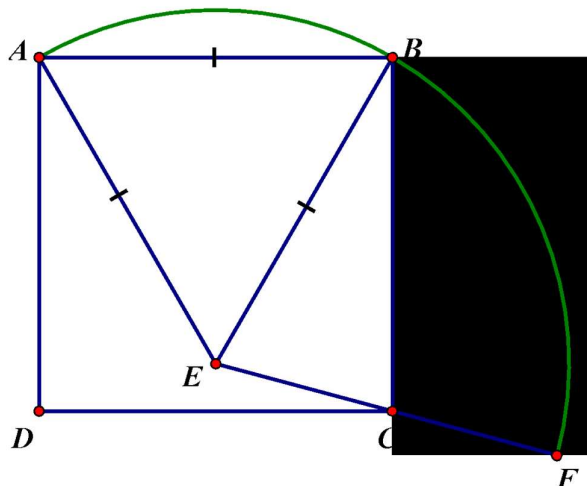
Let $AR = x$ and $RB = (22-x)$

$PQ =$

Question 7 [2015 S4 EOY, Q9]

marks

The diagram shows a square $ABCD$ of length 2 cm and an equilateral triangle ABE . An arc ABF is drawn with radius AE and centre at E . Points E , C and F lie on a straight line.



- (i) Show that $\angle BEC = \frac{5}{12}\pi$ radians.

Solution:

$$\angle EBC = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

Since $BE = BC$,

$$\angle BEC = \frac{1}{2} \left(\pi - \frac{\pi}{6} \right)$$

$$= \frac{5}{12}\pi \text{ radians (shown)}$$

[2]

- (ii) Find the length of line segment CE ,

Solution:

Using cosine rule,

$$CE = \sqrt{2^2 + 2^2 - 2(2)(2)\cos\frac{\pi}{6}} = 1.04 \text{ cm (3 s.f.)}$$

[2]

- (iii) Find the perimeter of the shaded region,

Solution:

$$\text{Length of arc } BF = \left(2\right)\left(\frac{5}{12}\pi\right) = \frac{5}{6}\pi \text{ cm}$$

Perimeter of shaded region

$$= \frac{5}{6}\pi + 2 + (2 - 1.035) = 5.58 \text{ cm (3 s.f.)}$$

[2]

- (iv) Find the area of the shaded region, leave your answer in the form $a\pi + b$, where a and b are constants to be determined.

[3]

Solution:

Area of shaded region

= Area of sector EBF – Area of triangle EBC

$$= \frac{1}{2}(2)^2 \left(\frac{5}{12} \pi \right) - \frac{1}{2}(2)(2) \sin \frac{\pi}{6}$$

$$= \left(\frac{5}{6} \pi - 1 \right) \text{ cm}^2$$