**3E AM EOY 2021 Marking Scheme**

| Qn | | Marks | Marker's Report |
|---|---|---|---|
| 1 | $\frac{53-5x-7x^2}{(3x-1)(x^2+10)}=\frac{A}{3x-1}+\frac{Bx+C}{x^2+10}$ | M1 (correct form) | Generally well done across the level.<br>Common mistakes:<br><br>1. Wrong form<br><br>2. $\frac{53-5x-7x^2}{(3x-1)(x^2+10)}=\frac{5}{3x-1}-\frac{4x-3}{x^2+10}$ |
| | $53-5x-7x^2=A(x^2+10)+(Bx+C)(3x-1)$ | M1 (DB) | |
| | $A=5, B=-4, C=-3$ | B1,B1,B1 | |
| | $\frac{53-5x-7x^2}{(3x-1)(x^2+10)}=\frac{5}{3x-1}-\frac{4x+3}{x^2+10}$ | A1 (conclusion) | |
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| 2(a) | $\frac{x^2}{y}=-\frac{by^2}{a}+\frac{1}{a}$ | B1 (linearization) | Badly done.<br>Candidates were unable to linearize the equation. Many mistook<br>$\frac{x^2}{y}$ as the horizontal axis (X)<br>$y^2$ as the vertical axis (Y) |
| | Vertical-intercept= $\frac{1}{a}$ , so a can be found | B1 (statement) | |
| | Gradient = $-\frac{b}{a}$ so b can now be found | B1 (statement) | |
| (b)(i) | Scale; Points; Line | B1, B1, B1 | Well done.<br>Only about 4 candidates (surprisingly from 3E1) did not follow the given scale. |
| (ii) | $-k$ = gradient using two points | M1 | Generally well done |
| | $k=0.0414$ | A1 | |
| (iii) | $\ln m$ = 4.3 to 4.5 | M1 | Well done |
| | $m=73.7$ to 90 | A1 | |
| (iv) | $\ln\frac{e^{4.4}}{2}=4.4-0.041379t$ | M1 | Badly done.<br>Many candidates computed the value of $t$ rather than using the graph to estimate the value of $t$.<br>Many did not answer the question by stating explicitly the year. |
| | $t=16.751\approx 17$ years | M1 | |
| | Year 2021 + 17 = 2038 | A1 | |

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| 3(a) | $T_{r+1}=\binom{9}{r}\left(2x^2\right)^{9-r}\left(-\frac{5}{x^3}\right)^r$ | M1 | Generally well done.<br>Candidates are able to simplify the general term to its simplest form. |
| | $=\binom{9}{r}(2)^{9-r}(-5)^r\left(x^{18-5r}\right)$ | M1<br>(simplify to simplest form) | |
| | For independent term, | | |
| | $18-5r=0$ | | |
| | $r=3.6$ | M1 | For the mathematical reasoning, many used the word "integer" rather than whole number. |
| | Since $r$ is not a whole number, it is not possible to get a term independent of $x$. | A1 | |
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| (b)(i) | $6561-8748x+5103x^2$ | B3 | Well done.<br>However, a handful of scripts from 3E2 left this as the final answer:<br>$9-12x+7x^2$ (candidates divided throughout by 729) |
| | | | |
| (ii) | $T_4=\binom{8}{3}(3)^5\left(\frac{-x}{2}\right)^3=-1701x^3$<br><br>$-8748(2)+5103(-7)=9(-1701)=-68526$ | M1<br><br>A1 | Badly done.<br>85% of the candidates were unaware that the $T_4$ contains the $x^3$ term. Hence, did not obtain the correct answer. |
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| 4(i) | $m_{AB}=-1$ | B1<br>(gradient of AB) | Well done. |
| | $m_{\perp}=1$ | B1<br>(gradient of bisector) | |
| | $M_{AB}=(8.5,13.5)$ | B1<br>(midpoint) | |

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| | $y-13.5=1(x-8.5)$ | M1 | |
| | $y=x+5$ | A1 | |
| (ii) | $Centre=(0,5)$ | B1 | Generally well done. |
| | Radius = 13 units | B1 | |
| | $x^2+(y-5)^2=169$ or $13^2$ or $x^2+y^2-10y-144=0$ | B1 | |
| | | | |
| (iii) | Distance of C from centre = 7.07 or $5\sqrt{2}$ or $\sqrt{50}$ units | M1 (DB dep on their centre) | Well done.<br>Interestingly, candidates failed to see that $\sqrt{169}=13$ |
| | Since distance of C from centre is <u>less than the radius</u>, the point C <u>lies inside the circle.</u> | A1 | |
| | | | |
| (iv) | Centre(-18,5) | B1 | Badly done.<br>Candidates were unable to figure out the centre of the reflected circle. |
| | $(x+18)^2+(y-5)^2=169$ or $13^2$ $x^2+y^2+36x-10y+11=0$ | B1 or B2<br>if centre not shown<br>If centre is wrong, max 1 out of 2 marks | |
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| 5(i) | Let $f(x)=2x^3+11x^2+22x+13$ | | Well done.<br>Quite a handful of candidates used long division. |
| | $f(-1)=2(-1)^3+11(-1)^2+22(-1)+13=0$<br>Therefore, $x+1$ is a factor. | B1 | |
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| (ii) | By long division, comparing coefficient or synthetic division, | M1 | Long division was the most commonly used method followed by Synthetic Division. |
| | $2x^3+11x^2+22x+13=(x+1)(2x^2+9x+13)$ | M1 | Candidates can write this step clearly in their working. |
| | To cut the *x*-axis, | | |
| | $(x+1)(2x^2+9x+13)=0$ | | |

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| | $x=-1$ or $2x^2+9x+13=0$ | | |
| | I do not agree, since $b^2-4ac=-23<0$, therefore there is <u>only 1 real root</u> so the graph only cuts the $x$-axis at one point. | DB1 (discriminant)<br>A1 (reasoning) | Careless mistake was seen in the value of the discriminant of -68. |
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| 6(a) | $x=-2y-1$ or $x+2y=-1$ o.e | M1 (Eqn 1) | Common error when forming equation<br>$5^x=\frac{1}{5(25)^y} \Rightarrow 5^x=5^{-3y} \therefore x=-3y$ |
| | $0.5x=2+1.5y$ or $x=4+3y$ o.e | M1 (Eqn 2) | Quite alright for most students |
| | $x=1, y=-1$ | A1,A1 | |
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| (b) | $BC=\frac{760}{(25-7\sqrt{5})(4\sqrt{5}-2)}$ Or $BC=\frac{380}{(25-7\sqrt{5})(2\sqrt{5}-1)}$ | M1 | There are a few students who wrote formula as area of triangle = $BC$ x height |
| | $=\frac{760}{114\sqrt{5}-190}$ Or $=\frac{380}{57\sqrt{5}-95}$ | M1 (Expansion & simplifying) | Mistakes in expansion commonly seen. |
| | $=\frac{760}{114\sqrt{5}-190}\times\frac{114\sqrt{5}+190}{114\sqrt{5}+190}$ Or $=\frac{380}{57\sqrt{5}-95}\times\frac{57\sqrt{5}+95}{57\sqrt{5}+95}$ | DB1 (rationalization) | |
| | $=3\sqrt{5}+5$ | A1 (simplest form) | |
| | Alternatively: | | |
| | $Area=\frac{380(25+7\sqrt{5})}{(25-7\sqrt{5})(25+7\sqrt{5})}=25+7\sqrt{5}$ | M1 (rationalization) | |
| | $BC=\frac{(25+7\sqrt{5})(2\sqrt{5}+1)}{(2\sqrt{5}-1)(2\sqrt{5}+1)}$ o.e | M1 (rationalization) | |
| | $BC=\frac{50\sqrt{5}+25+70+7\sqrt{5}}{19}$ o.e | M1(expansion) | |

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| | $BC = \frac{57\sqrt{5}+95}{19} = 3\sqrt{5}+5$ | A1(simplest form) | |
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| 7(a) | $3u^2 - 8u - 3 = 0$ | M1 | Majority obtained the right quadratic equation |
| | $(3u+1)(u-3) = 0$ | M1 | |
| | $3^x = -\frac{1}{3}(rejected)$ or $3^x = 3$ $x = 1$ | A1,A1 (deduct 1 if -3 is not rejected) | A number of students rejected the answer $3^x = -\frac{1}{3}$ after putting log on both sides |
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| (b) | $\log_4(x+4) - \frac{1}{\log_{2x-3} 4} = \log_4 \frac{2}{3} + 1$ | | |
| | $\log_4(x+4) - \frac{1}{\frac{\log_4 4}{\log_4(2x-3)}} = \log_4 \frac{2}{3} + 1$ Or $\log_4(x+4) - \frac{\log_4(2x-3)}{l \log_4 4} = \log_4 \frac{2}{3} + 1$ | M1(change of base) | Common error $\log_4(x+4) - \frac{1}{\log_{2x-3} 4} = \log_4 \frac{2}{3} + 1$ $\frac{1}{\log_{2x-3} 4} = \frac{\log_4 4}{\log_4 x - 3}$ |
| | $\log_4(x+4) - \log_4(2x-3) = \log_4 \frac{2}{3} + 1$ | M1 | A few students has this as first step - that is they did not show how the base is changed to 4. |
| | $\log_4 \frac{x+4}{2x-3} = \log_4 \frac{8}{3}$ OR $\log_4 \frac{3(x+4)}{2(2x-3)} = 1$ | M1 (quotient and product law) | |
| | $\frac{x+4}{2x-3} = \frac{8}{3}$ OR $\frac{x+4}{2x-3} = 4$ | M1 Remove log base 4 on both sides | |
| | $3(x+4) = 8(2x-3)$ OR $x+4 = 4(2x-3)$ | | |

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| | $x = \frac{36}{13}$ | A1 | |
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| 8(i) | $m_{AC} = 2.5$ | M1 | Careless mistakes were seen when solving simultaneous equation. Most of the students are able to tackle this question. |
| | Equation of AC: $y = 2.5x + 5.9$ | M1 | |
| | $2x + 5(2.5x + 5.9) = 15$ | DB1 (substitution/elimination) | |
| | $x = -1, y = 3.4$ | M1 ($x$ or $y$ values) | |
| | C(-1,3.4) | A1 (coordinate form) | |
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| (ii) | $M_{AC} = (1.5, 9.65)$ | | Badly done.<br>Common error – Students consider the mid point of *AB* = Mid point of *CD*. |
| | $B(7.5, 0)$ | M1 | |
| | $M_{BD} = \left(\frac{x+7.5}{2}, \frac{0+y}{2}\right) = (1.5, 9.65)$ | M1 (midpoint) | |
| | $D(-4.5, 19.3)$ | A1 | |
| | Alternatively : | | |
| | Equation of AD : $y = -\frac{2}{5}x + 17.5$ | M1 | Common error – Students consider solving simultaneous equation using eqn *AD* and *BD* |
| | Equation of CD : $y = -4\frac{19}{35}x - 1\frac{1}{7}$ | M1 | |
| | $D(-4.5, 19.3)$ | A1 | |
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| (iii) | Area = $\frac{1}{2}\left\lvert \begin{matrix} 7.5 & -1 & -5 & 7.5 \\ 0 & 3.4 & -9 & 0 \end{matrix} \right\rvert$ | DB1 (shoelace method) | Students are able to apply the shoelace method correctly. Students who did not obtain correct answer is due to wrong coordinates of C and/or B in earlier part. |

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| | $= 0.5((7.5)(3.4)+(-1)(-9))-(7.5(-9)+-5(3.4)) = 59.5$ units$^2$ | A1 | |
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| 9(i) | For minimum graph, $a > 0$ | B1 | A number of students gave incomplete set of 3 conditions.<br>A few scripts with this error: $(-3)^2 - 4ac < 0 \Rightarrow ac < \frac{9}{4}$ |
| | For graph to lie above the x axis, | | |
| | $(-3)^2 - 4ac < 0$ | M1 | |
| | $ac > \frac{9}{4}$ ; $4ac > 9$ ; $a > \frac{9}{4c}$ ; $c > \frac{9}{4a}$ | A1 | |
| (ii) | Any suitable pairs | B0 or B2 | Not so well done.<br>Students with incomplete set of conditions but able to give a correct pair of values were awarded the marks. |
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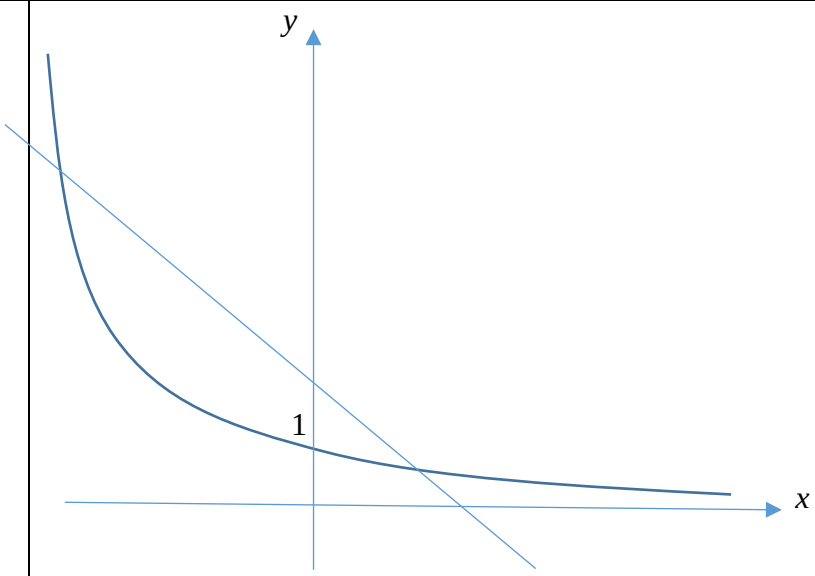
| 10(i) |  | Shape correct (B1);<br>Followed by y-intercept (B1) | Generally fine |
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| (ii) | $5^x = \frac{1}{2-3x}$ | M1 (index form) | Generally fine |
| | $2-3x = \frac{1}{(5^x)}$ | M1 | |
| | Equation of line: $y = 2-3x$ | A1 | |
| | No of solution = 2 | B1 | Badly done. Majority gave 1 solution. Mark is awarded only if the shape of curve and line are correct. |
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| 11(i) | $q+px = 4x^2+(3p-1)x+1+q$ | M1 (equate) | Careless mistakes for few $q-px = 4x^2+(3p-1)x+1+q$. Generally fine. |
| | $4x^2+(2p-1)x+1=0$ | M1 | |
| | $(2p-1)^2-4(4)(1)=0$ | DB1 | |
| | $(2p-1)^2=16$ | M1 | |
| | $p = 2.5(rej)$ or $p=-1.5$ | A1 (must indicate rejected) | |
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| (ii) | $q+px = 4x^2+(3p-1)x+1+q$ | | |
| | $y = 4x^2-5.5x+1+q$ | | |
| | $56.5 = 4(-3)^2-5.5(-3)+1+q$ | DB1 (substitution) | Careless mistakes were seen in substitution. Some left the parts blank. Unable to complete the paper. |
| | $q=3$ | A1 (value of q) | |
| | $4x^2-5.5x+1+3 = -1.5x+3$ | | |
| | $(2x-1)^2=0$<br>$x=\frac{1}{2}$ | | |
| | $A(\frac{1}{2}, 2\frac{1}{4})$ | A1 | |