Jurong Pioneer Junior College

H2 Mathematics **JC2-2025**
Preliminary Exam Paper 1 Solution

Q1

(i)

$y=\dfrac{1}{4x-x^2}=\left(4x-x^2\right)^{-1}$

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\left(4x-x^2\right)^{-2}\left(4-2x\right)=\dfrac{2x-4}{\left(4x-x^2\right)^2}$

At turning point,

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\left(4x-x^2\right)^{-2}\left(4-2x\right)=0\Rightarrow x=2$

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\left[\left(4x-x^2\right)^{-2}(-2)-2\left(4x-x^2\right)^{-3}\left(4-2x\right)^2\right]$

When $x=2$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\left[\left(4(2)-(2)^2\right)^{-2}(-2)\right]=\dfrac{1}{8}>0$

Or

Using first derivative test,

x	1.9	2	2.1
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	–0.0126	0	0.0126
tangent	\	–	/

The turning point at $x=2$ is a minimum point.

(ii)

Area

$=\displaystyle\int_1^2 \frac{1}{4x-x^2}\,\mathrm{d}x$

$=\displaystyle\int_1^2 \frac{1}{-\left(x^2-4x+2^2-2^2\right)}\,\mathrm{d}x$

$=\displaystyle\int_1^2 \frac{1}{2^2-\left(x-2\right)^2}\,\mathrm{d}x$

$=\left[\dfrac{1}{2(2)}\ln\left|\dfrac{2+\left(x-2\right)}{2-\left(x-2\right)}\right|\right]_1^2$

$=\left[\dfrac{1}{4}\ln\left|\dfrac{x}{4-x}\right|\right]_1^2$

$=-\dfrac{1}{4}\ln\dfrac{1}{3}$

$=\dfrac{1}{4}\ln 3$

or

$=\displaystyle\int_1^2 \frac{1}{x\left(4-x\right)}\,\mathrm{d}x$

$=\dfrac{1}{4}\displaystyle\int_1^2 \frac{1}{x}+\frac{1}{4-x}\,\mathrm{d}x$

$=\dfrac{1}{4}\Big[\ln|x|-\ln|4-x|\Big]_1^2$

$=\dfrac{1}{4}\left[\ln\left|\dfrac{x}{4-x}\right|\right]_1^2$

$=-\dfrac{1}{4}\ln\dfrac{1}{3}$

$=\dfrac{1}{4}\ln 3$

Q2

(i)

$$\frac{1}{(x-a)^2}=|x-a|$$

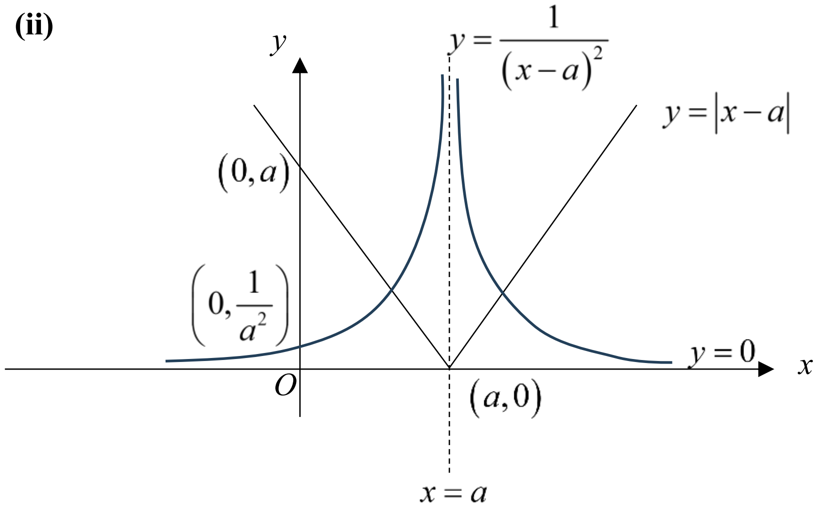
$$\frac{1}{(x-a)^2}=x-a \quad \text{or} \quad \frac{1}{(x-a)^2}=-(x-a)$$

$$(x-a)^3=1 \qquad\qquad (x-a)^3=-1$$

$$x-a=1 \qquad\qquad x-a=-1$$

$$x=a+1 \qquad\qquad x=a-1$$

(ii)



For $\frac{1}{(x-a)^2}>|x-a|$,

$a-1<x<a+1, \qquad x\neq a$

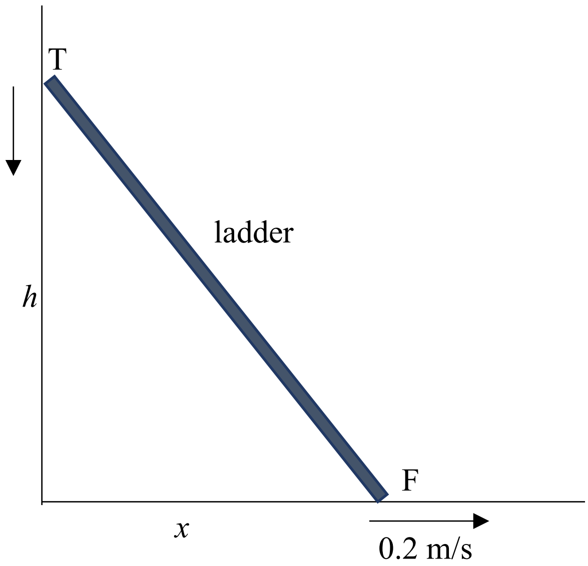
Or

$a-1<x<a \quad \text{or} \quad a<x<a+1$

Q3

Let h m be the vertical distance between the top of the ladder and the floor
Let x m be the horizontal distance between the foot of the ladder and the corner of the wall

Method 1



By Pythagoras' theorem,

$$h^2 + x^2 = 3.12^2$$

By implicit differentiation w.r.t. t,

$$2h\frac{dh}{dt} + 2x\frac{dx}{dt} = 0$$

When $h = 1.2$, $\frac{dx}{dt} = 0.2$ and

$$x^2 = 3.12^2 - 1.2^2$$
$$x = 2.88 \quad \text{(Since } x > 0\text{)}$$

Hence,

$$2(1.2)\frac{dh}{dt} + 2(2.88)(0.2) = 0$$
$$\frac{dh}{dt} = -0.48$$

Hence, the top of ladder is sliding down at a rate of 0.48 m/s.

Method 2

By Pythagoras' theorem,

$$h^2 + x^2 = 3.12^2$$

By implicit differentiation w.r.t. x, or By implicit differentiation w.r.t. h

$$2h\frac{dh}{dx} + 2x = 0 \Rightarrow \frac{dh}{dx} = -\frac{x}{h}$$

When $h = 1.2$, $\frac{dx}{dt} = 0.2$ and

$$x^2 = 3.12^2 - 1.2^2$$
$$x = 2.88 \quad \text{(Since } x > 0\text{)}$$

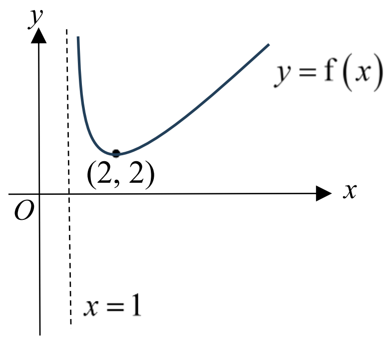
Hence, $\frac{dh}{dt} = \frac{dh}{dx} \times \frac{dx}{dt}$

$$= -\frac{2.88}{1.2} \times 0.2$$
$$= -0.48$$

Hence, the top of ladder is sliding down at a rate of 0.48 m/s.

Q4

(i)



Smallest value of $a = 2$.
(For f^{-1} to exist, f must be a one-one function.)

(ii)

Let $y = 4 + 3x - x^2$

$$y = -\left[x^2 - 3x - 4\right]$$
$$= -\left[\left(x - \frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 - 4\right]$$
$$= -\left[\left(x - \frac{3}{2}\right)^2 - \frac{25}{4}\right]$$
$$y = -\left(x - \frac{3}{2}\right)^2 + \frac{25}{4}$$

or

$$x^2 - 3x - 4 + y = 0$$
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-4+y)}}{2}$$
$$x = \frac{3 \pm \sqrt{25 - 4y}}{2}$$
$$x = \frac{3}{2} \pm \sqrt{\frac{25}{4} - y}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{25}{4} - y$$

$$x - \frac{3}{2} = \pm\sqrt{\frac{25}{4} - y}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{25}{4} - y}$$

since $x \le \frac{3}{2}$, $x = \frac{3}{2} - \sqrt{\frac{25}{4} - y}$

Thus, $g^{-1} : x \mapsto \frac{3}{2} - \sqrt{\frac{25}{4} - x}, \ x \le \frac{25}{4}$

$D_{g^{-1}} = R_g = \left(-\infty, \frac{25}{4}\right]$

(iii)

$R_{f^{-1}} = D_f = [2, \infty)$

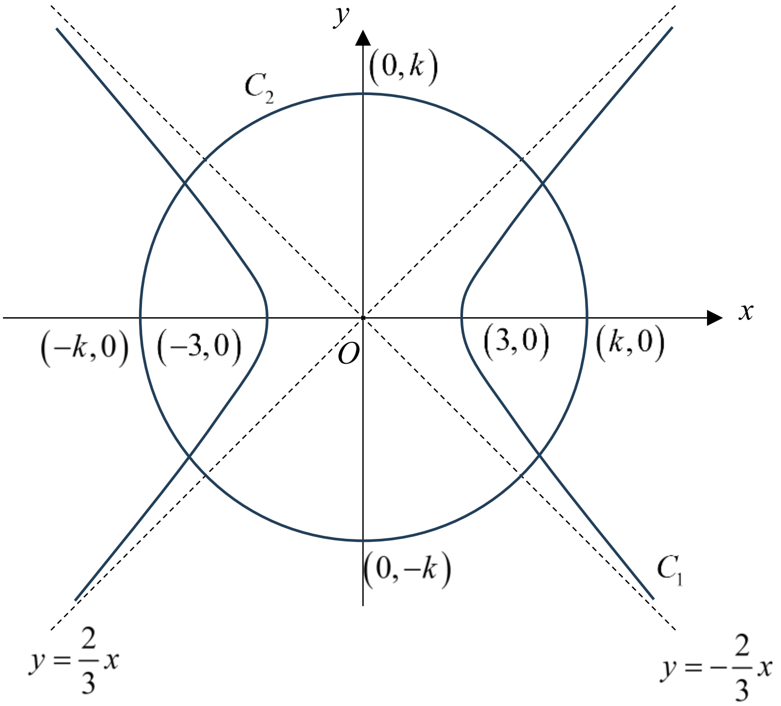
$D_{g^{-1}} = R_g = \left(-\infty, \frac{25}{4}\right]$

Since $R_{f^{-1}} \not\subset D_{g^{-1}}$, $g^{-1}f^{-1}$ does not exist.

Q5

(a)

(i)



(ii)

For C_1 and C_2 to intersect, $k \geq 3$ (since $k > 0$).

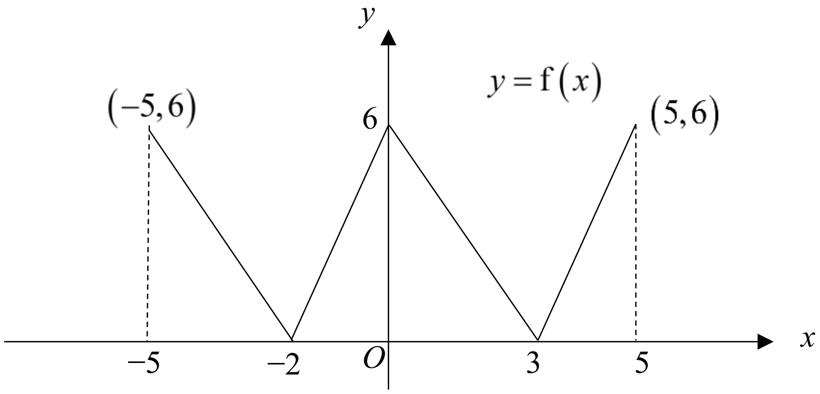
(iii)

Common lines of symmetry for both C_1 and C_2 are $x = 0$ and $y = 0$.

(b)
(i)

$\mathrm{f}(46) = \mathrm{f}(41) = \mathrm{f}(36) = \ldots\ldots = \mathrm{f}(1) = -2(1) + 6 = 4$

(ii)



(iii)

For $-5 \leq x \leq 5$. the roots of $\mathrm{f}(-x) = 0$ are -3 and 2.

Q6

(i)

$$\int x^2 e^{-2x}\,dx$$
$$= -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\,dx$$
$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$
$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$
$$= -\frac{1}{4}e^{-2x}\left[2x^2 + 2x + 1\right] + C$$

$u = x^2$ $\quad \frac{dv}{dx} = e^{-2x}$

$\frac{du}{dx} = 2x$ $\quad v = -\frac{1}{2}e^{-2x}$

$u = x$ $\quad \frac{dv}{dx} = e^{-2x}$

$\frac{du}{dx} = 1$ $\quad v = -\frac{1}{2}e^{-2x}$

(ii)

Volume

$$= \pi\int_0^1 \left(xe^{-x}\right)^2 - \left(\frac{1}{e}x\right)^2\,dx \qquad \text{or} \qquad \pi\int_0^1 \left(xe^{-x}\right)^2\,dx - \frac{1}{3}\pi\left(\frac{1}{e}\right)^2(1)$$
$$= \pi\int_0^1 x^2e^{-2x} - \frac{1}{e^2}x^2\,dx$$
$$= \pi\left[-\frac{1}{4}e^{-2x}\left(2x^2+2x+1\right)\right]_0^1 - \frac{\pi}{3e^2}\left[x^3\right]_0^1$$
$$= \pi\left[-\frac{1}{4}e^{-2}\left(2(1)^2+2(1)+1\right)\right] + \frac{\pi}{4} - \frac{\pi}{3e^2}(1)^3$$
$$= -\frac{5\pi}{4e^2} + \frac{\pi}{4} - \frac{\pi}{3e^2}$$
$$= \frac{\pi}{4} - \frac{19\pi}{12e^2}$$
$$= \frac{\pi}{12}\left(3 - 19e^{-2}\right)$$

Q7

(a)(i)

$$\int \frac{x}{\sqrt{25-x^2}}\,dx$$
$$= -\frac{1}{2}\int -2x\left(25-x^2\right)^{-\frac{1}{2}}\,dx$$
$$= -\sqrt{25-x^2} + C$$

(a)(ii)

$$\int_\alpha^4 \left|\frac{x}{\sqrt{25-x^2}}\right| \mathrm{d}x = 3$$

$$-\int_\alpha^0 \frac{x}{\sqrt{25-x^2}}\,\mathrm{d}x + \int_0^4 \frac{x}{\sqrt{25-x^2}}\,\mathrm{d}x = 3$$

$$-\left[-\sqrt{25-x^2}\right]_\alpha^0 + \left[-\sqrt{25-x^2}\right]_0^4 = 3$$

$$\left[\sqrt{25-x^2}\right]_\alpha^0 - \left[\sqrt{25-x^2}\right]_0^4 = 3$$

$$5-\sqrt{25-\alpha^2} - [3-5] = 3$$

$$\sqrt{25-\alpha^2} = 4$$

$$\alpha^2 = 9$$

$$\alpha = \pm 3$$

Since $\alpha < 0$, $\alpha = -3$

(b)

$$x = 4\tan\theta$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sec^2\theta$$

When $x = 4$, $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

When $x = 0$, $\tan\theta = 0 \Rightarrow \theta = 0$

$$\int_0^4 \sqrt{\frac{x^2}{16+x^2}}\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{16\tan^2\theta}{16+16\tan^2\theta}}\ 4\sec^2\theta\ \mathrm{d}\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\ 4\sec^2\theta\ \mathrm{d}\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \frac{\tan\theta}{\sec\theta}\sec^2\theta\ \mathrm{d}\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \tan\theta\sec\theta\ \mathrm{d}\theta$$

$$= 4[\sec\theta]_0^{\frac{\pi}{4}}$$

$$= 4\left[\sec\frac{\pi}{4} - \sec 0\right]$$

$$= 4\left(\sqrt{2}-1\right)$$

or

$$= 4\int_0^{\frac{\pi}{4}} \sin\theta\sec^2\theta\ \mathrm{d}\theta$$

$$= -4\int_0^{\frac{\pi}{4}} -\sin\theta[\cos\theta]^{-2}\ \mathrm{d}\theta$$

$$= -4\left[\frac{(\cos\theta)^{-1}}{-1}\right]_0^{\frac{\pi}{4}}$$

$$= 4\left[\left(\cos\frac{\pi}{4}\right)^{-1} - (\cos 0)^{-1}\right]$$

$$= 4\left(\sqrt{2}-1\right)$$

Q8

(a)

$u_n = an^2 + bn + c$

$u_1 = a + b + c = 9 ------(1)$

$u_2 = 4a + 2b + c = 27 ---(2)$

$u_3 = 9a + 3b + c = 55 ---(3)$

Using GC, $a = 5, b = 3, c = 1$

$u_n = 5n^2 + 3n + 1$

(b)(i)

$$\sum_{r=1}^{n}\left(2r^3+5\right) = 2\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} 5$$

$$= 2\left(\frac{n^2\left(n+1\right)^2}{4}\right) + 5n$$

$$= \frac{n^2\left(n+1\right)^2}{2} + 5n$$

(b)(ii)

Method 1

$$\sum_{r=2}^{n}\left(2\left(r+2\right)^3+5\right) = \left(2\left(2+2\right)^3+5\right) + \ldots + \left(2\left(n+2\right)^3+5\right)$$

$$= \sum_{r=4}^{n+2}\left(2r^3+5\right)$$

$$= \sum_{r=1}^{n+2}\left(2r^3+5\right) - \sum_{r=1}^{3}\left(2r^3+5\right)$$

$$= \frac{\left(n+2\right)^2\left(n+3\right)^2}{2} + 5\left(n+2\right) - \left[\left(\frac{3^2\left(4\right)^2}{2}\right) + 5\left(3\right)\right]$$

$$= \frac{\left(n+2\right)^2\left(n+3\right)^2}{2} + 5n - 77$$

<u>Method 2</u>

$$\sum_{r=2}^{n}\left(2(r+2)^3+5\right)=\left(2(2+2)^3+5\right)+...+\left(2(n+2)^3+5\right)$$

$$=\sum_{r=4}^{n+2}\left(2r^3+5\right)$$

$$=2\left[\sum_{r=1}^{n+2}r^3-\sum_{r=1}^{3}r^3\right]+\sum_{r=4}^{n+2}5$$

$$=2\left[\frac{(n+2)^2(n+3)^2}{4}-\frac{3^2(4)^2}{4}\right]+5(n-1)$$

$$=\frac{(n+2)^2(n+3)^2}{2}+5n-77$$

(b)(iii)

From **(i)** $\sum_{r=1}^{n}u_r=\frac{n^2(n+1)^2}{2}+5n$

As $n\to\infty,\ \frac{n^2(n+1)^2}{2}\to\infty,\quad 5n\to\infty$

Hence, $\sum_{r=1}^{n}u_r=\frac{n^2(n+1)^2}{2}+5n\to\infty$, series $\sum_{r=1}^{\infty}u_r$ does not converge.

Q9

(i)

$$\int \frac{1}{4+9x^2}\,dx = \int \frac{1}{9\left(\frac{4}{9}+x^2\right)}\,dx = \frac{1}{6}\tan^{-1}\frac{3x}{2}+K$$

(ii)

$$f(x) = (4+9x^2)^{-1} = \frac{1}{4}\left(1+\frac{9x^2}{4}\right)^{-1}$$

$$= \frac{1}{4}\left[1+(-1)\left(\frac{9x^2}{4}\right)+\frac{-1(-2)}{2}\left(\frac{9x^2}{4}\right)^2+...\right]$$

$$= \frac{1}{4}\left(1-\frac{9x^2}{4}+\frac{81}{16}x^4+...\right)$$

$$\approx \frac{1}{4}-\frac{9}{16}x^2+\frac{81}{64}x^4$$

(iii)

From **(i)** $\tan^{-1}\frac{3x}{2} = 6\int \frac{1}{4+9x^2}\,dx + C,$ where $C = -6K$

From **(ii)** $\tan^{-1}\frac{3x}{2} = 6\int\left(\frac{1}{4}-\frac{9}{16}x^2+\frac{81}{64}x^4\right)dx + C$

$$= 6\left(\frac{1}{4}x-\frac{3}{16}x^3+\frac{81}{320}x^5\right)+D$$

$$= \frac{3}{2}x-\frac{9}{8}x^3+\frac{243}{160}x^5+D$$

$x = 0, \tan^{-1}0 = 0 \Rightarrow D = 0$

$$\therefore \tan^{-1}\frac{3x}{2} = \frac{3}{2}x-\frac{9}{8}x^3+\frac{243}{160}x^5$$

(iv)

$\int_0^{0.5} \tan^{-1}\frac{3x}{2}\,dx = \int_0^{0.5}\left(\frac{3}{2}x-\frac{9}{8}x^3+\frac{243}{160}x^5\right)dx = 0.174$ (to 3 dp) (Using GC)

(v)

From GC, $\int_0^{0.5} \tan^{-1}\frac{3x}{2}\,dx = 0.173$ (to 3 dp)

(vi)

The estimate in **(iv)** is accurate up to 2 decimal places but not to 3 decimal places.

To improve the estimate, we can include higher-order terms in the Maclaurin series expansion of $\tan^{-1}\frac{3x}{2}$.

Q10

(a)

$$y = \ln\left(\frac{e^2}{3x}\right) = \ln e^2 - \ln 3x = 2 - \ln 3x$$

$$y = \ln x \xrightarrow{\text{A}} y = \ln 3x \xrightarrow{\text{B}} y = -\ln 3x \xrightarrow{\text{C}} y = 2 - \ln 3x$$

A: A scaling parallel to the x-axis by a factor of $\frac{1}{3}$.

B: Reflection in the x-axis.

C: A translation of 2 units in the positive direction of the y-axis.

(b)(i)

$$g(x) = 2f(x-1)$$

$(a, b) \rightarrow (a+1, b) \rightarrow (a+1, 2b)$

Hence the corresponding point R is $(a+1, 2b)$.

$g'(x) = 2f'(x-1)$

Given $f'(a) = 5$

$f'(a+1) = 5$ [gradient remains the same after translation]

At $(a+1, 2b)$,

$g'(x) = 2f'(a+1) = 2(5) = 10$

<u>Alternative</u>

When $x = a+1$

$g'(x) = 2f'(a+1-1) = 2f'(a)$

Given $f'(a) = 5$,

$g'(x) = 2(5) = 10$

(b)(ii)

$$g(x) = \frac{1}{f(x)}$$

The corresponding point R is $\left(a, \frac{1}{b}\right)$.

$$g'(x) = -\frac{f'(x)}{[f(x)]^2}$$

Given $f'(a) = 5$, at $\left(a, \frac{1}{b}\right)$,

$$g'(x) = -\frac{f'(a)}{[f(a)]^2} = -\frac{5}{b^2}$$

Q11

(i)

$u_{n+1} = (1+k)u_n$

$\dfrac{u_{n+1}}{u_n} = 1 + k =$ constant independent of n (since k is a constant),

$\therefore \{u_n\}$ is a geometric progression.

(ii)

<u>Method 1</u>

$u_1 = 311 \qquad r = 1+k$

$S_3 = \dfrac{311((1+k)^3 - 1)}{1+k-1} = 4043$ or Using GC Table

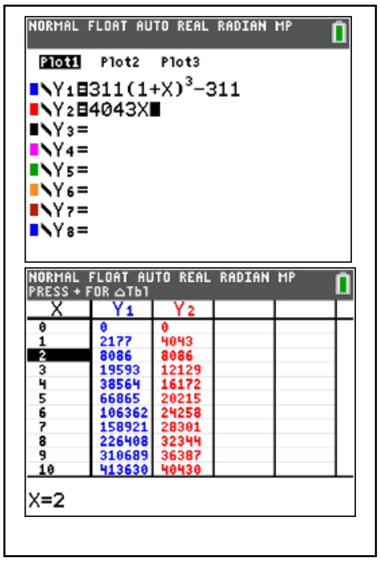
$311(1+k)^3 - 311 = 4043k$

$(1+k)^3 - 13k - 1 = 0$

$k^3 + 3k^2 - 10k = 0$

Using GC, $k = -5, \quad 0, \quad 2$

Since $k > 0, \quad k = 2$



<u>Method 2</u>

$u_1 = 311 \qquad u_2 = (1+k)311 \qquad u_3 = (1+k)^2 311$

$311 + (1+k)311 + (1+k)^2 311 = 4043$

$1 + 1 + k + (1+k)^2 = 13$

$k^2 + 3k - 10 = 0$

$k = 2$ or -5 (NA since $k > 0$)

(iii)

Since common ratio $r = 3 > 1$, the series does not converge as $n \to \infty$. Sum to infinity does not exist and hence there is no limit to total number of daily views in the long run.

(iv)

r	v_r
1	$v_1 = u_1 = 311$
2	$v_2 = u_2 = (3)311$
3	$v_3 = u_3 = (3^2)311$
4	$v_4 = u_4 = (3^3)311$
5	$v_5 = v_4 + 80$
6	$v_6 = v_5 + 80$
$\vdots$	

(rows 4 onwards:) AP with 1st term $(3^3)311 = 8397$, common difference 80

From v_4 to v_r, no. of terms $= r - 4 + 1 = r - 3$

$\therefore\ S_r = S_3 + \frac{r-3}{2}\left[2(8397)+(r-4)(80)\right]$

$= 4043 + 8397r - 25191 + 40(r-3)(r-4)$

$= 40r^2 + 8117r - 20668$ where $t = 20668$ (shown)

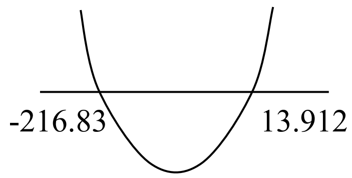
(v)

$40r^2 + 8117r - 20668 > 100000$

$40r^2 + 8117r - 120668 > 0$

$r < -216.83$ (NA since $r \geq 1$) or $r > 13.912$

Least number of days = 14



Or using GC Table

r	$40r^2 + 8117r - 120668$
13	-8387 < 0
14	810 > 0
15	10087 > 0

Least number of days = 14

Q12

(i)

Plane contains $\overrightarrow{OA}=\begin{pmatrix}0\\0\\3\end{pmatrix}$ and is parallel to $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}4\\5\\1\end{pmatrix}$.

Normal of plane, $\mathbf{n}=\begin{pmatrix}0\\1\\0\end{pmatrix}\times\begin{pmatrix}4\\5\\1\end{pmatrix}=\begin{pmatrix}1\\0\\-4\end{pmatrix}$

Equation of plane,

$$\mathbf{r}\cdot\begin{pmatrix}1\\0\\-4\end{pmatrix}=\begin{pmatrix}0\\0\\3\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-4\end{pmatrix}$$

$$\mathbf{r}\cdot\begin{pmatrix}1\\0\\-4\end{pmatrix}=-12$$

$$x-4z=-12 \qquad \text{(Shown)}$$

(ii)

Normal of wall, $\mathbf{n}_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and normal of roof, $\mathbf{n}_2=\begin{pmatrix}1\\0\\-4\end{pmatrix}$

Let θ be acute angle between wall and roof.

$$\cos\theta=\frac{\left|\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-4\end{pmatrix}\right|}{(1)\left(\sqrt{17}\right)}$$

$$=\frac{1}{\sqrt{17}}$$

$$\theta=75.9638^\circ$$

Hence, obtuse angle between wall and roof $=180^\circ-75.9638^\circ$

$$=104.0362$$

$$\approx 104^\circ \qquad \text{(nearest degree)}$$

(iii)

<u>Method 1</u>

$\overrightarrow{AP} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ -3 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}$

Minimum length required = $\left|\overrightarrow{AP} \cdot \mathbf{n}_2\right|$ or $\left|\overrightarrow{BP} \cdot \mathbf{n}_2\right|$

$= \dfrac{\left|\begin{pmatrix} 4 \\ 3 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}\right|}{\sqrt{17}} = \dfrac{16}{\sqrt{17}}$ or $= \dfrac{\left|\begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}\right|}{\sqrt{17}} = \dfrac{16}{\sqrt{17}}$

<u>Method 2: (Finding foot of perpendicular from *P* to roof)</u>

Eq of roof: $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} = -12$ and Eq of wooden strut: $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

Then, $\begin{pmatrix} 4+\lambda \\ 3 \\ -4\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} = -12$

$$4 + \lambda + 16\lambda = -12$$
$$17\lambda = -16$$
$$\lambda = -\frac{16}{17}$$

Length of wooden strut = $\left|\overrightarrow{PF}\right| = \left|\overrightarrow{OF} - \overrightarrow{OP}\right|$

$$= \left|\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} - \frac{16}{17}\begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}\right|$$
$$= \left|\begin{pmatrix} -\frac{16}{17} \\ 0 \\ \frac{64}{17} \end{pmatrix}\right|$$
$$= \sqrt{\left(-\tfrac{16}{17}\right)^2 + \left(\tfrac{64}{17}\right)^2}$$
$$= \frac{16}{\sqrt{17}}$$

(iv)

Equation of wooden strut: $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

Equation of light ray: $\mathbf{r} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix}, \quad \mu \in \mathbb{R}$

Assuming they intersect at X,

$$\overrightarrow{OX} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix}, \quad \text{for some } \lambda, \mu \in \mathbb{R}$$

$$4 + \lambda = 0 + 7\mu \quad -(1) \Rightarrow \quad \lambda - 7\mu = -4 \quad -(4)$$

$$3 = 5 - 4\mu \quad -(2) \Rightarrow \quad \mu = \frac{1}{2}$$

$$-4\lambda = 4\mu \quad -(3) \Rightarrow \quad \lambda = -\mu = -\frac{1}{2}$$

Substitute $\lambda = -\frac{1}{2}$ and $\mu = \frac{1}{2}$ into (4): LHS = $\lambda - 7\mu = -\frac{1}{2} - 7\left(\frac{1}{2}\right) = -4$ = RHS

Hence, $\lambda = -\frac{1}{2}$ and $\mu = \frac{1}{2}$ satisfy all 3 equations.

The light ray meets the strut.

And $\overrightarrow{OX} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \left(-\frac{1}{2}\right) \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 3 \\ 2 \end{pmatrix}$ or $\overrightarrow{OX} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} + \left(\frac{1}{2}\right) \begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 3 \\ 2 \end{pmatrix}$

The light ray <u>meets</u> the wooden strut at $(3.5, 3, 2)$.