02 FORCES AND MOMENTS

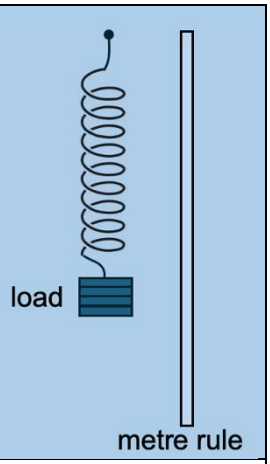
Reference notes (page 10)

Class exercise

The spring constant *k* of a spring may be determined by finding the extension of the spring and the load applied, using the apparatus shown. A student obtained the following readings.

reading on the rule for the lower end of the unextended spring	= 13.60 ± 0.05 cm
reading on the rule for the lower end of the extended spring	= 17.95 ± 0.05 cm
load	= 4.00 ± 0.02 N

Determine *k* and express it with its actual uncertainty.



Suggested solution

Extension of spring $x = 17.95 - 13.60 = 4.35$ cm

$\Delta x = 0.05 + 0.05 = 0.1$ cm

$$k = \frac{F}{x} = \frac{4}{0.0435} = 91.954 \text{ N m}^{-1}$$

$$\frac{\Delta k}{k} = \frac{\Delta F}{F} + \frac{\Delta x}{x}$$

$$\frac{\Delta k}{91.954} = \frac{0.02}{4} + \frac{0.1}{4.35}$$

$$\Delta k = 3 \text{ N m}^{-1}$$

$$\therefore k = (92 \pm 3) \text{ N m}^{-1}$$

Reference notes (page 11)

Combining springs

Springs can be combined in different ways, end-to-end (in series) and side-by-side (in parallel).

(a) Determine the equivalent force constant of two identical springs, each of force constant *k*, in series and two springs in parallel, as shown below.

springs in series

Hint for spring in series:

1. A same force equal in magnitude to the load acts on each spring.
2. The equivalent spring will have the same amount of extension as the total extension of the two springs.

springs in parallel

Hint for spring in parallel:

1. The two springs must have the same amount of extension.
2. The equivalent spring will have the same amount of extension as the two springs.

(b) Determine the equivalent force constant of three or more springs with difference force constants when they are combined in series and parallel.

Suggested solution

(a) Let *F* be load on the springs

[Series combination]
Extension of each spring $= \frac{F}{k}$
$k'x = F \Rightarrow k'\left(2 \times \frac{F}{k}\right) = F \Rightarrow k' = \frac{1}{2}k$

[Parallel combination]
Load on each spring $= \frac{F}{2}$
Extension of each spring $= \frac{F}{2k}$
$k'x = F \Rightarrow k'\left(\frac{F}{2k}\right) = F \Rightarrow k' = 2k$

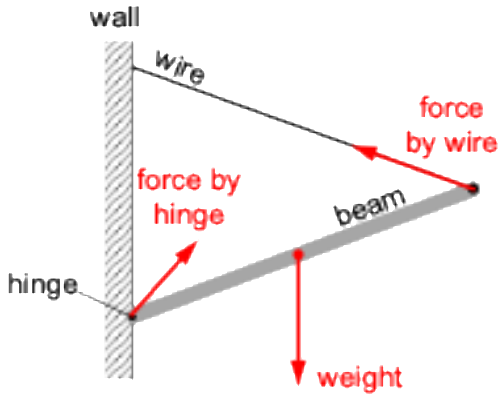
(b) [Series combination]
Extension of each spring is $\frac{F}{k_1}, \frac{F}{k_2}, \ldots, \frac{F}{k_N}$
Total extension of springs $x = \frac{F}{k_1} + \frac{F}{k_2} + \cdots + \frac{F}{k_N}$
$k'x = F \Rightarrow k'\left(\frac{F}{k_1} + \frac{F}{k_2} + \cdots + \frac{F}{k_N}\right) = F$
$\therefore k' = \left(\frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_N}\right)^{-1}$ (like resistors in parallel!)

[Parallel combination]
Let the equivalent spring have extension *x*.
Load $F = k_1x + k_2x + \cdots k_Nx$
$k'x = F \Rightarrow k'x = k_1x + k_2x + \cdots k_Nx$
$\therefore k' = k_1 + k_2 + \cdots k_N$ (like resistor in series!)

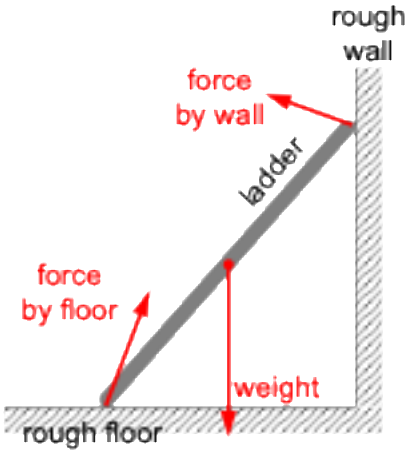
Reference notes (page 18)

Class exercise Draw the force diagrams for the following objects (underlined item).

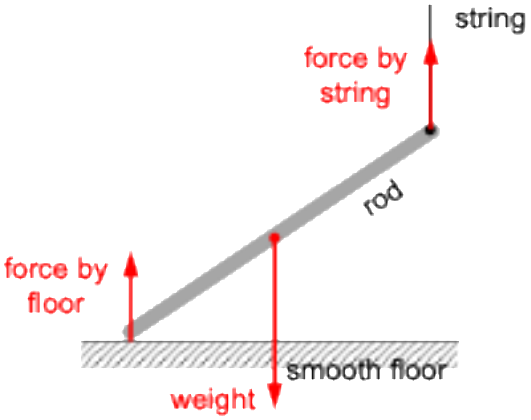
(a) A uniform <u>beam</u> hinged to a wall and supported by a wire.



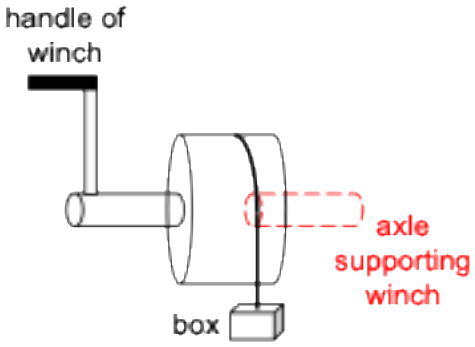
(b) A <u>ladder</u> resting on a rough wall and rough floor.



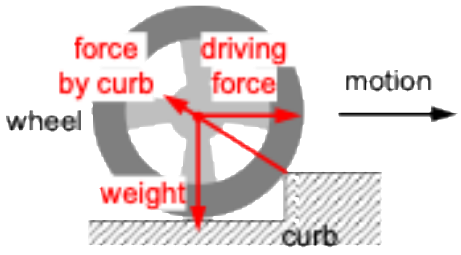
(c) A uniform <u>rod</u> supported by a vertical string on one end and the other end resting on a smooth floor.



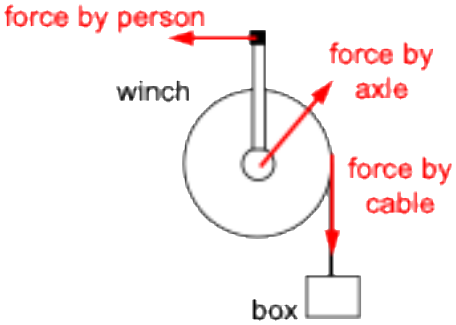
(d) A <u>light</u> <u>winch</u> holding up a heavy box. Draw the forces in the side view diagram.



(e) A <u>wheel</u> rolling forward with a forward driving force. It is just about to mount up a curb.



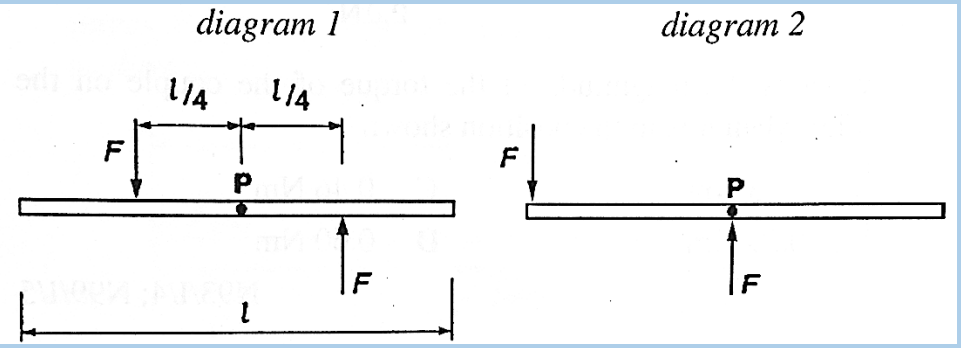
(side view)



Reference notes (page 25)

Class exercise

Diagram 1 shows two parallel forces *F* acting on a bar of length *l* pivoted at **P**. The forces give rise to a couple of torque *M*. Diagram 2 shows the lines of action of the forces moved $\frac{l}{4}$ to the left.



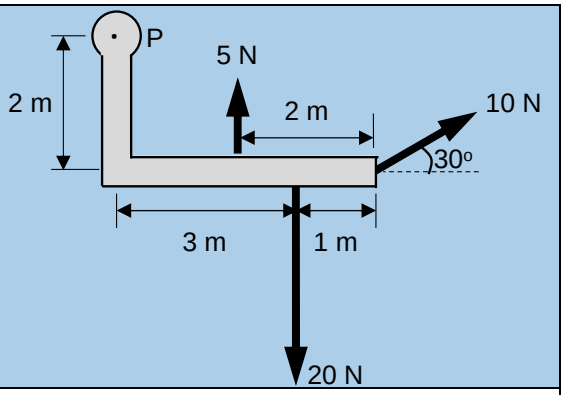
Determine the new torque of the couple in diagram 2 in terms of *M*.

Suggested solution

Perpendicular distance between couple remains the same, so torque remains the same as *M*.

Class exercise

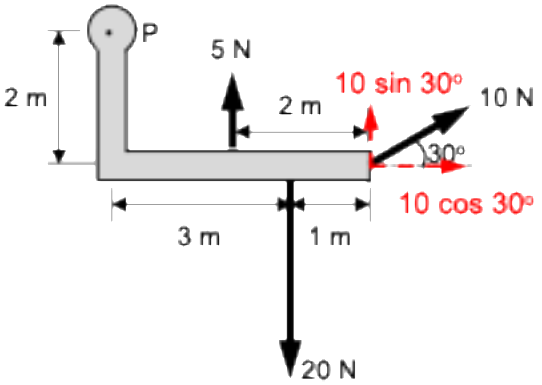
A L-shaped lever is pivoted at point P. The forces acting on the lever are as shown.



Determine the resultant moment about the point P.

Suggested solution

Resolve the 10 N force as shown.



5 N and the two components of 10 N give anticlockwise moment about P
= 5 × 2 + 10 sin 30° × 4 + 10 cos 30° × 2
= 47.32 N m

20 N gives clockwise moment about P
= 20 × 3
= 60 N m

Resultant moment about P = 60 – 47.32 = 12.7 N m (clockwise)

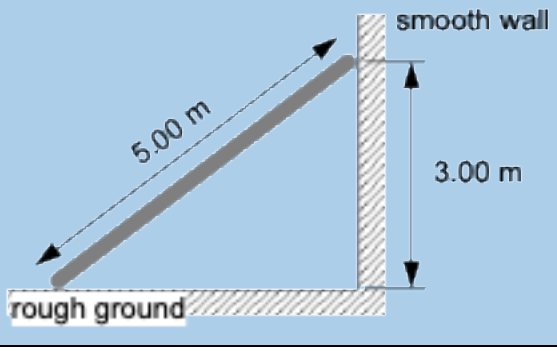
Reference notes (page 27)

Class exercise

A uniform ladder is of length 5.00 m and mass 2.00 kg rests against a wall. Its upper end is a height of 3.00 m above the ground.

The wall is smooth, and the ground is rough.

Determine the forces exerted by the wall and by the ground on the ladder.



Suggested solution

For rotational equilibrium, take moments about A,

$$2 \times 9.81 \times 2.00 = N \times 3.00$$
$$N = 13.08 = 13.1 \text{ N (3 s.f.)}$$

For translational equilibrium,

$$F_x = N = 13.08 \text{ N}$$
$$F_y = W = 19.62 \text{ N}$$

$$F = \sqrt{13.08^2 + 19.62^2} = 23.6 \text{ N}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{19.62}{13.08} = 56.3°$$