

Electromagnetic Induction

Guiding Questions	Learning Objectives
1. If an electric current produces a magnetic field, can a magnetic field produce an electric current?	(a) define magnetic flux as the product of magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density (b) show an understanding of and use the concept of magnetic flux linkage (c) recall and solve problems using $\Phi = BA$ and $N\Phi = NBA$ to solve problems, where N is the number of turns
2. Can we achieve perpetual motion by invoking the magic of electromagnetism?	(d) infer from appropriate experiments on electromagnetic induction: i. that a changing magnetic flux can induce an e.m.f. ii. that the direction of the induced e.m.f. opposes the change producing it iii. that factors affecting the magnitude of the induced e.m.f. (e) recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law (f) explain simple applications of electromagnetic induction (g) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s} \text{ for an ideal transformer}$



Nature of Science Moment – Electromagnetism before Faraday

(Source: *The birth of the electric machines: a commentary on Faraday (1832) 'Experimental researches in electricity by Jim Al-Khalili'*)

The early nineteenth century was an exciting time for experimental physics. It was also a time of great confusion about the nature of electricity. The work of two Italians, Luigi Galvani and Alessandro Volta on the characteristics of bioelectricity had led Volta to invent the battery in 1799. His 'voltaic pile' suddenly gave natural philosophers (the term 'scientists' was not coined until 1834¹) a reliable and far more useful source of electricity than the Leyden jars or ever-more sophisticated electrostatic machines and turned the subject from an intellectual curiosity into a proper science. In a real sense, it *galvanised* science, which is of course the origin of that everyday word.

In particular, it was the confusing relationship between electricity and magnetism that so fascinated many scientists. Indeed, it was argued by some that there was no connection at all between the two phenomena, although it had been known since the mid-eighteenth century that, for example, thunderbolts created certain magnetic effects.

Then, in 1820, the Danish scientist Hans Christian Ørsted carried out an experiment for which he is credited with the discovery of electromagnetism. On 21 April 1820, he observed, while preparing for a lecture, that when he ran an electric current through a wire, a nearby compass needle was temporarily deflected from its stable position of pointing towards the magnetic north. This happened at the moment the current from a battery was switched on, and then again when it was switched off, thus confirming a direct relationship between electricity and magnetism, namely that the change in an electric current (from none to flowing, and vice versa) produced a temporary magnetic effect in its vicinity.

Ørsted's original interpretation was that the magnetic effects produced by the current through the wire radiated outwards in the same way that heat or light does. But after further experimentation he showed that in fact the produced magnetic field circled around the wire (although of course no one was yet thinking in terms of fields).

Within months of Ørsted's discovery, the French physicist and mathematician André-Marie Ampère had shown that two current carrying wires placed in parallel close to each other each generated magnetic lines of force that caused the wires to be attracted or repelled from each other depending on whether the currents were flowing in the same or in opposite directions. Ampère would go on to help found the field of classical electromagnetism and has the SI unit of electric current named after him.

Ampère and Ørsted had shown that, somehow, electricity could be converted into magnetism, but they and others had failed to do the reverse: to create electricity *from* magnetism.



An equally famous (at the time, though less so today) French physicist by the name of François Arago then carried out an experiment that completely baffled most scientists at the time and served as one of the main motivations for Faraday's great work. In 1824, Arago demonstrated that a spinning copper disc caused a magnetic needle suspended above it to rotate. This result was remarkable for two reasons. Firstly, there was no external electric current being applied to the copper disc and, secondly, although copper is a conductor, it is not magnetic. Yet here was a magnetic field seemingly being produced just by the rotation of this disc that was influencing the compass needle. It would take the brilliance and inventiveness of Michael Faraday to describe what was going on and, in a series of carefully and clearly described experiments between August and November 1831, he would change the face of science in ways that have an impact on all our lives to this day.

The article describes early practitioner of science as natural philosopher before the term "scientist" was coined in 1834. In fact, as the name suggests, science was philosophical, a way of thinking about and explaining natural phenomena.

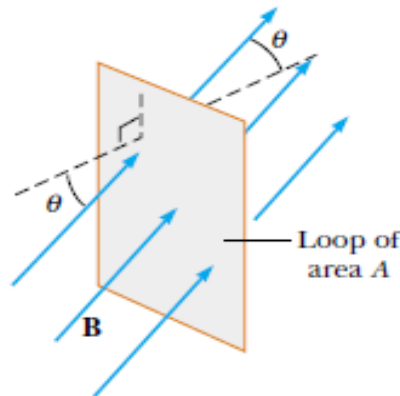
One of the tenets of the nature of science showed by the article is that science relies heavily, but not entirely on observation, experimental evidence, rational arguments and scepticism. It was the confusing relationship between electricity and magnetism that fascinated scientists. Some argued that there was no connection at all between the two phenomena. Only in 1820, it was Hans Christian Ørsted who observed the movement of a compass needle in the vicinity of a current-carrying wire that led to the discovery of electromagnetism.

Science has a tentative character; it is both evolutionary and revolutionary. French physicist, François Arago built on Ørsted and Ampère's discovery and in 1824 demonstrated that a spinning copper disc caused a magnetic needle suspended above it to rotate. This set the stage for Michael Faraday's great work on the concept of Electromagnetic Induction.

If an electric current produces a magnetic field, can a magnetic field produce an electric current?

1. Magnetic flux and weber

1.1 Magnetic Flux $\phi = \vec{B} \cdot \vec{A}$



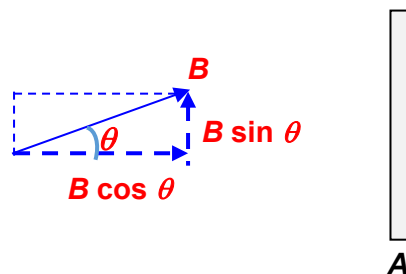
(source: Serway Physics for Scientists & Engineers)



The **magnetic flux**, ϕ is defined as the product of magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density.

If the angle between B and A is θ , the magnetic flux, ϕ through the surface area, A is thus

$$\phi = BA \cos \theta$$



Magnetic flux, ϕ is a scalar quantity.

SI unit of ϕ : weber (Wb)

1.2 The Weber

The weber is the flux of a uniform magnetic field of flux density 1 tesla , through a plane surface area 1 m^2 , placed *normal* to the magnetic flux density.

1.3 Magnetic Flux Linkage

The magnetic flux linkage, Φ through a whole coil of N turns is the sum of the magnetic flux through the individual turns where each turn the magnetic flux through it is ϕ .

$$\Phi = N\phi$$

$$\Phi = NBA \cos \theta$$

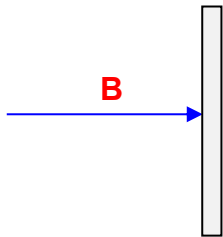
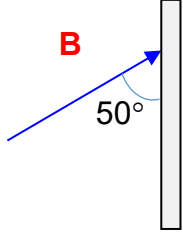
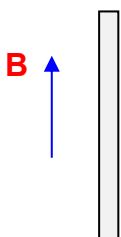
Example 1.1

The plane area of a square loop of wire is 1.0 m^2 . It is placed in a magnetic field of uniform magnetic flux density of 0.40 T . Determine the magnetic flux through the conducting loop if the magnetic flux density is directed

- (a) perpendicular to the loop;
- (b) at 50° to the plane of the loop; and
- (c) parallel to the loop.

Solution

Apply $\phi = BA \cos \theta$ to the following:

 (a) $\theta = 0^\circ$	 (b) $\theta = 40^\circ$	 (c) $\theta = 90^\circ$
$\phi = (0.4)(1) \cos 0^\circ$ $\phi = 0.4 \text{ Wb}$	$\phi = (0.4)(1) \cos 40^\circ$ $\phi = 0.306 \text{ Wb}$	$\phi = (0.4)(1) \cos 90^\circ$ $\phi = 0 \text{ Wb}$

Can we achieve perpetual motion by invoking the magic of electromagnetism

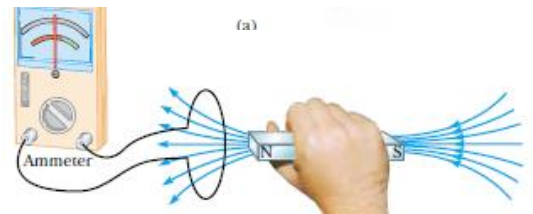
2. Electromagnetic Induction



Electromagnetic induction can be demonstrated in a lab using a bar magnet and a metal coil attached to an ammeter.

When there is no relative motion between the coil and the bar magnet,

- ammeter shows zero or no reading,
- indicating no current flowing in the coil, and
- as the circuit is closed, no e.m.f. is induced in the coil



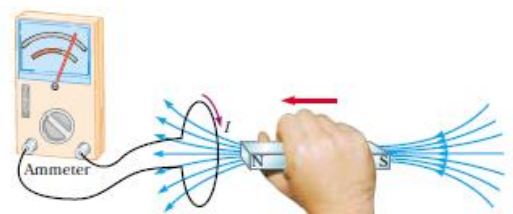
[diagram source: Serway 'Physics for Scientists & Engineers']

When the magnet moves toward the coil,

- ammeter shows a deflection
- indicating a current is flowing in the coil.
- Hence an e.m.f. must have been induced in the coil (to give rise to the current)

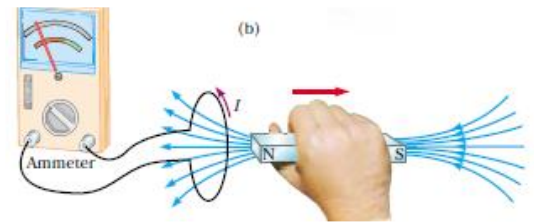
Also, as the magnet moves faster toward the coil,

- rate of change of flux increases, and
- the needle deflects at a larger angle
- indicating a larger current flows in the coil
- as a larger e.m.f. is induced in the coil



When magnet moves away from the coil,

- ammeter shows deflection in the opposite direction,
- indicating current is flowing in the opposite direction.
- as e.m.f. is induced in the coil, but of opposite polarity.



This simple experiment shows that

- a change of magnetic flux linkage Φ produces an induced e.m.f. ε ,
- the rate of change in magnetic flux linkage Φ affects the magnitude of the induced e.m.f. ε , and
- an increase in magnetic flux linkage Φ induced current in one direction, and a decrease would produce an induced current in the opposite direction.

3. Laws of Electromagnetic Induction



Faraday's law states that the magnitude of the induced e.m.f. is directly proportional to the rate of change of magnetic flux linkage.

In mathematical form,

$$\varepsilon \propto \frac{d\Phi}{dt}$$



Lenz's law states that the direction of the induced e.m.f. is such as to cause effects to oppose the change producing it.

Combining and expressing the above laws in SI units,

$$\varepsilon = -\frac{d\Phi}{dt}$$

the negative sign expresses Lenz's Law, indicating that the direction of the induced e.m.f. oppose the change in magnetic flux linkage causing it.

where $\Phi = N\phi$, and N is number of turns in a coil.

Note:

- According to Faraday's law of electromagnetic induction, an induced e.m.f. across a conductor will be present when there is a change in magnetic flux linkage.
- If the circuit is closed, the induced e.m.f. drives an induced current through the circuit.
- The induced e.m.f. will produce an induced current which in turn produces a magnetic field to oppose the change in magnetic flux linkage. The direction of the induced e.m.f. is determined by Lenz's law.

3.1 Steps in Determining Induced e.m.f.

instantaneous induced emf	average induced emf
$\varepsilon_{\text{inst}} = -\frac{d}{dt}(NBA \cos \theta)$	$\varepsilon_{\text{ave}} = -N \left(\frac{\phi_{\text{final}} - \phi_{\text{initial}}}{t} \right)$

Identify the changing variable(s). Example if B is changing:

instantaneous induced emf	average induced emf
$\varepsilon_{\text{inst}} = -\frac{d\Phi}{dt}$ $= -NA \cos \theta \frac{dB}{dt}$	$\varepsilon_{\text{ave}} = -N \frac{\phi_{\text{final}} - \phi_{\text{initial}}}{t}$ $= -NA \frac{B_{\text{final}} - B_{\text{initial}}}{t}$ <i>Note for this discussion: $\phi = BA \cos \theta$ and θ is zero since B is perpendicular to loop</i>

3.1.1 Factors affecting magnitude of induced e.m.f

The magnitude of the induced e.m.f. is dependent on:

- the strength (magnetic flux density) of the magnet
- the number of coils
- the area of the coil
- the speed at which the magnet / coil is moving at

In summary the magnitude of the induced e.m.f. is dependent on the rate of change of magnetic flux linkage in a conducting loop.

3.1.2 Factors affecting direction of induced e.m.f.

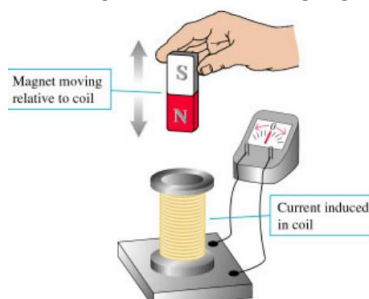
The direction of the induced e.m.f. (which leads to induced current for closed circuits) is due to

- direction of magnetic field lines
- whether there is an increase or decrease in magnetic flux linkage

3.1.3 Factors affecting change in magnetic flux linkage

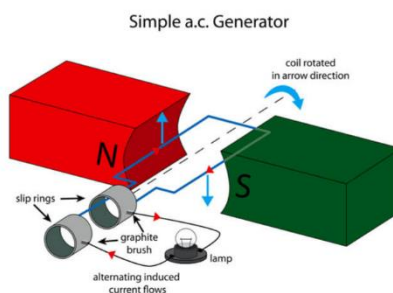
In most of the problems you will encounter, a change in magnetic flux linkage in a conducting loop is usually caused by the following situations.

- constant plane area of conducting loop but changing magnetic flux density



Source: www.phys.hawaii.edu

b) constant magnetic flux density but changing plane area of conducting loop

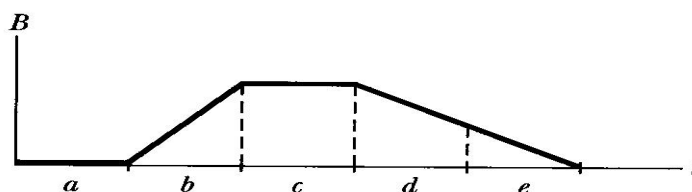


Source: www.powerplus.com/

Example 3.1

The graph below shows how the magnitude of the magnetic flux density B of a magnetic field that exists through a conducting loop, perpendicular to the plane of the loop vary with time.

Rank the five regions of the graph according to the magnitude of the e.m.f. induced in the loop, greatest to lowest.



Solution

Ranking: b, d & e, a & c

Example 3.2

A flat circular coil of 120 turns, each of area 0.070 m^2 , is placed with its axis parallel to a uniform magnetic field. The flux density of the field is changed steadily from 80 mT to 20 mT over a period of 4.0 s.

Determine the magnitude of e.m.f. induced in the coil during this time.

Solution

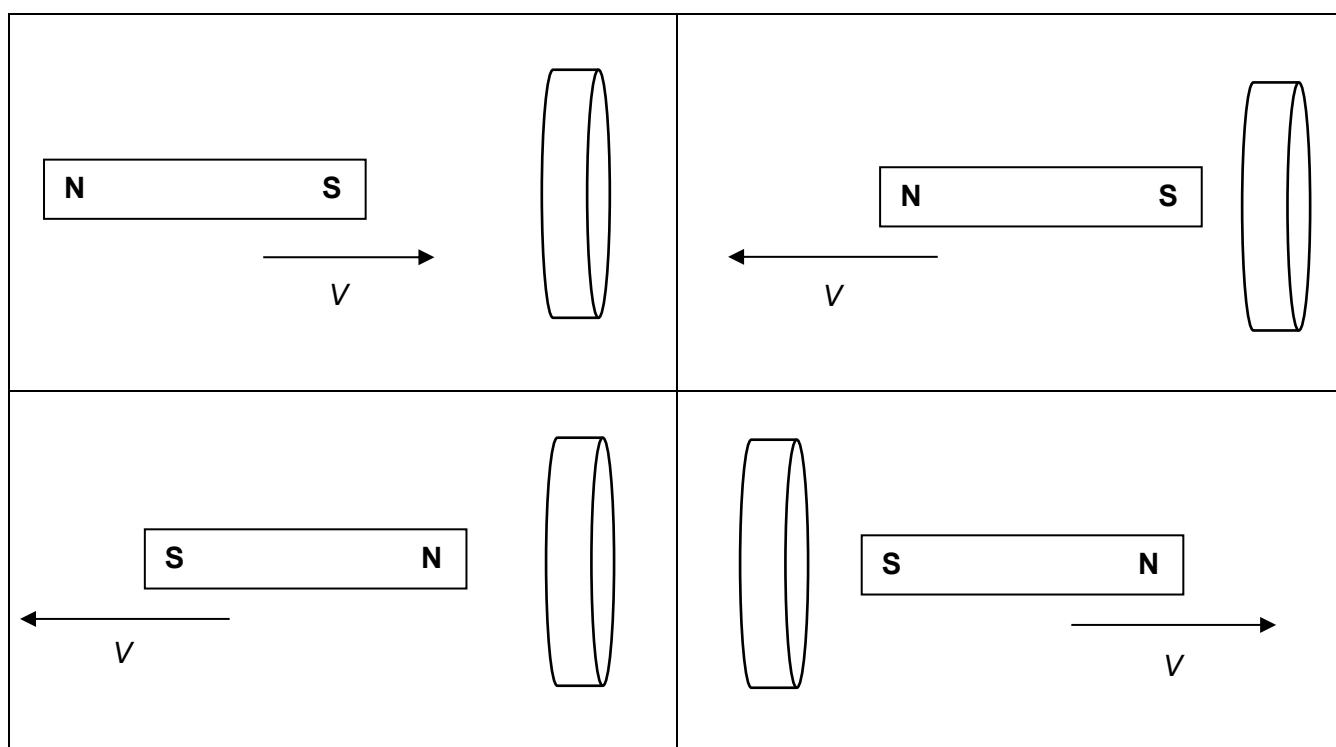
$$\begin{aligned}
 |\varepsilon| &= \left| -\frac{d\Phi}{dt} \right| = \left| -\frac{dNBA \cos \theta}{dt} \right| = \left| -NA \frac{dB}{dt} \right| \\
 &= \left| -(120)(0.070) \left[\frac{(20 - 80)(10^{-3})}{4.0} \right] \right| \\
 &= 0.126 \text{ V}
 \end{aligned}$$

3.2 General Steps in Determining Direction of Induced e.m.f.

How to determine polarity of the induced e.m.f. and direction of induced current in a system which experiences a change in magnetic flux linkage?

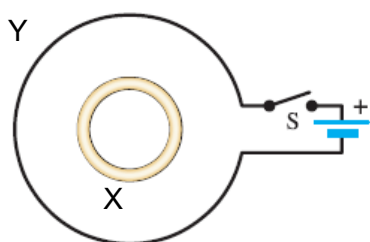
1. Identify System 1: External System that sets up external magnetic field B_{ext} .
2. Identify System 2: Conducting System that experiences a change in magnetic flux linkage Φ , and where the e.m.f. is induced $\mathcal{E}_{\text{ind}}$ and induced current I_{ind} flows.
3. Is the magnetic flux linkage experienced by System 2 increasing (or decreasing)?
4. If magnetic flux linkage is increasing (decreasing), then the direction of the induced current I_{ind} will flow to produce an induced magnetic flux density B_{ind} in the opposite (same) direction to B_{ext} .
5. The direction of the induced current can be found from the direction of induced magnetic field B_{ind} (using *right hand grip rule*).
6. As the coil is an e.m.f. source, and within the source, the e.m.f. induced current I_{ind} flows from the lower potential to the higher potential. Hence, the polarity of the induced e.m.f. $\mathcal{E}_{\text{ind}}$ can be determined.

Quiz: The following diagrams show a magnet with an initial velocity V and a conducting coil. Deduce the direction of current in the coil in each of the cases.



Example 3.3

A small circular wire loop X lies in the plane of a larger loop Y and is connected to a battery as shown. State the direction of the induced current in X



- (a) at the moment when the switch S is closed,
- (b) at the moment when S is opened,
- (c) when S is closed, and loop X is raised parallel to itself out of the plane of the figure.

Explain your answer.

Solution

- (a) At the moment the switch S is closed, the current through the circuit increases from zero to a steady value in the anticlockwise direction in loop Y, generating an increasing magnetic flux density that it pointing out of the paper.

Loop X experiences an increasing in magnetic flux linkage (and by Faraday's Law of Electromagnetic Induction) and an e.m.f. is induced. Since X is a closed loop, a current will flow in the loop

To oppose the increase magnetic flux density that it pointing out of the paper, by Lenz's Law, the induced current in loop X must flow in the clockwise direction to produce a magnetic field that points into the paper.

- (b) At the moment the switch S is opened, the current through the circuit decreases from a steady value to zero in the anticlockwise direction in loop Y, generating an decreasing magnetic flux density that it pointing out of the paper.

Loop X experiences a decreasing in magnetic flux linkage (and by Faraday's Law of Electromagnetic Induction) and an e.m.f. is induced. Since X is a closed loop, a current will flow in the loop

To oppose the decrease magnetic flux density that it pointing out of the paper, by Lenz's Law, the induced current in loop X must flow in the anticlockwise direction to produce a magnetic field that points out the paper.

- (c) When the switch S is closed, the current through the circuit flows in the anticlockwise direction in loop Y, generating a constant magnetic flux density that it pointing out of the paper.

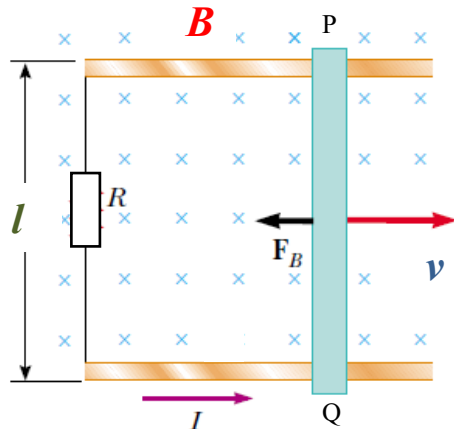
When Loop X is raised out of the page, it experiences a decreasing in magnetic flux linkage (and by Faraday's Law of Electromagnetic Induction) and an e.m.f. is induced. Since X is a closed loop, a current will flow in the loop

To oppose the decrease magnetic flux density that it pointing out of the paper, by Lenz's Law, the induced current in loop X must flow in the anticlockwise direction to produce a magnetic field that points out the paper.

3.3 Moving Conductor in a Magnetic Field

3.3.1 Straight Moving Conductor (*Slide-wire generator*)

[diagram source: Serway 'Physics for Scientists & Engineers']



Consider a conductor of length l moving with uniform speed v in a uniform transverse magnetic flux density B . It moves through a distance x in time t .

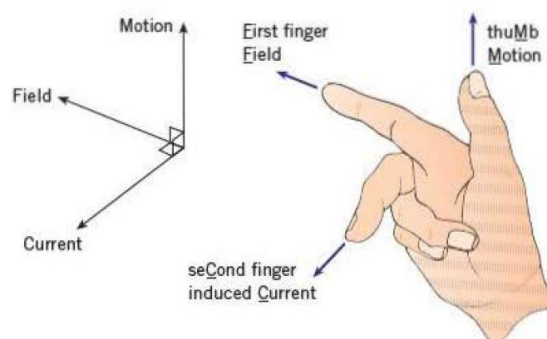
rate of change in magnetic flux linkage ,

$$\begin{aligned}\frac{d\Phi}{dt} &= \frac{d}{dt}(NB.A) \\ &= B \frac{dA}{dt} \\ &= B \frac{lx}{t} \\ &= Blv\end{aligned}$$

From Faraday's laws of electromagnetic induction,

$$\boxed{\mathcal{E} = Blv}$$

As the conductor moves to the right, it experienced a change in magnetic flux linkage. An e.m.f is induced across the conductor. The direction of the induced e.m.f. is dependent on the direction in which the magnetic flux lines are being cut. The direction of the induced e.m.f. or current in a wire moving through a magnetic field at right angles to the field may be determined by using Fleming's right-hand rule. This is illustrated in the figure below:

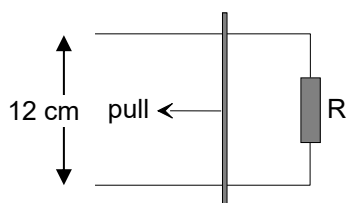


(Source: Cambridge International AS and A Level Physics)

Applying the Fleming's right-hand rule to this example, the current flows from Q to P, through the resistor in the closed circuit. An e.m.f. is induced across conductor PQ, with P at a higher potential than Q.

Example 3.4 [Young 'College Physics' modified]

A U-shaped conductor lies perpendicular to a uniform magnetic field B of 0.750 T that is directed out of the page. A resistor R , $8.00 \, \Omega$ is connected. A conducting rod is laid on the conductor and moves to the left with a constant velocity of $3.60 \, \text{m s}^{-1}$ as shown.

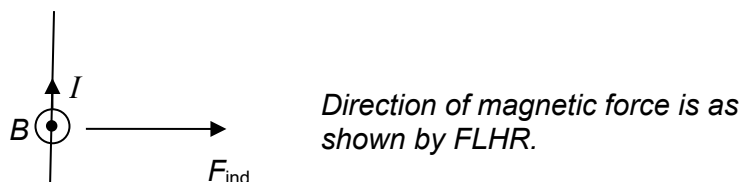


- State and explain the direction of the induced current and magnetic force on the bar.
- Determine the power dissipated in the resistor.
- State the rate at which the pulling force does work on the bar. Explain your answer.

Solution

- (a) As the bar is moved away from resistor R , the area enclosed and hence, the magnetic flux within it which is directed out of the page, increases. An e.m.f is induced across the conductor and the induced current must flow in a direction to produce a magnetic flux that is directed into the page to oppose the external magnetic field. Direction of induced current in the bar must be from bottom to top using the right hand grip rule.

Using Fleming's left hand rule, the force on the conductor can be determined to be pointing towards the resistor R .



- (b) Magnitude of induced e.m.f.

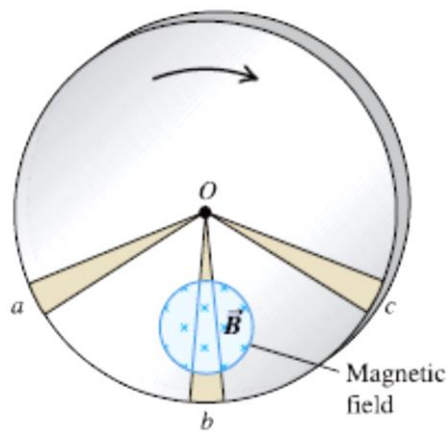
$$\begin{aligned}\varepsilon &= B l v \\ &= (0.750)(12.0 \times 10^{-2})(3.60) = 0.324 \, \text{V}\end{aligned}$$

$$\begin{aligned}\text{Power dissipated in resistor, } P_R &= I^2 R = \frac{V^2}{R} \\ &= (0.324)^2 / (8.00) = 13.1 \, \text{mW}\end{aligned}$$

- (c) The rate of work done by the pulling force = 13.1 mW

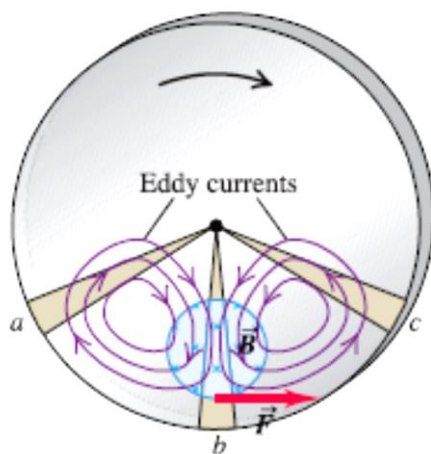
Power is dissipated in the resistor as a result of the induced current flowing through it. The current is caused by the induced e.m.f. which is caused by the external force acting on the conductor. Therefore, the rate of work done by the external force will be equal to the power dissipated in the resistor.

3.3.2 Rotating Conductor (Disc Brakes)



Consider a conducting disc rotating clockwise at a constant angular velocity. A uniform external magnetic field B_{ext} pointing into the page is incident normally at the bottom of the disc.

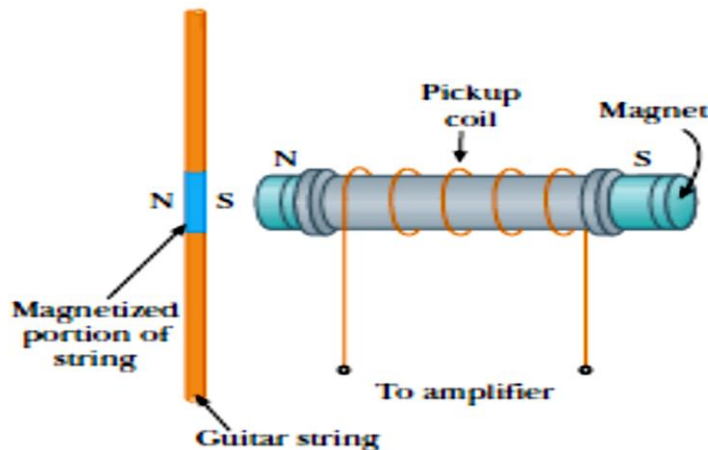
In order to determine the polarity of the induced e.m.f, we will consider a segment of the disc Oc . As the disc rotates, the segment moves from Oc to Ob to Oa . The segment experience a change in magnetic flux link linkage and an e.m.f is induced across the segment. Using Fleming left hand rule, we can deduce that the induced current will flow from O to b as this will produce a magnetic force to the right to oppose the motion of the disc causing the braking to occur (that is damping effect).



(source : Young 'College Physics')

4. Some Applications

4.1 Electric Guitar Pick-up



[diagram source: Serway 'Physics for Scientists & Engineers']

System 1: Guitar String

The guitar string is magnetised by the permanent magnet.

System 2: Pickup Coil

The pickup coils are at different locations under the guitar string.

- When the string is plucked, it oscillates sinusoidally,
- due to a change in distance from the coils, the magnetic flux linkage Φ experienced by the coils varies sinusoidally (at the same frequency)
- By Faraday's laws of electromagnetic induction, when the string is deflected away (nearer) from the coil,
- the magnetic flux linkage Φ decreases (increase),
- Hence the induced magnetic field B_{ind} due to the coils will be in the same (opposite) direction as the magnetic field B_{ext} of the string.
- The direction of the induced current I_{ind} can be determined using right hand grip rule.
- Hence the polarity of e.m.f. induced $\mathcal{E}_{\text{ind}}$ can be deduced.

Note:

1. All the parameters will vary sinusoidally.
2. In every half cycle, the string changes direction, and so all the other quantities will also have a change in direction, including the sound wave that is produced eventually by the speakers.
3. When the guitar string is plucked, different harmonics (stationary waves at different frequencies) are produced.
4. Each pickup coil is sensitive to a different harmonic (frequency) produced by the vibrating guitar string.

4.2 Use of Search Coil (to determine B_{ext})

System 1 : External magnetic field

System 2 : Search coil

- A search coil is made up of a few turns of coil which is then connected in series to a galvanometer.
- In the galvanometer,

$\text{angle of deflection of pointer} \propto \text{amount of charge (due to induced current) passing through it}$

$$\theta \propto Q$$

- The search coil is placed where its plane is normal to the direction of the field lines. Hence, magnetic flux linkage through the coil, $\Phi_i = BA$.
- Then reduce the magnetic flux linkage through the coil to zero, $\Phi_f = 0$
(This can be done by either
 - rotating it about a suitable axis so that the plane is parallel to the direction of B
 - OR removing the coil completely out of the field)
- By Faraday's laws of electromagnetic induction,

$$\begin{aligned}\varepsilon &= -\frac{\Delta\Phi}{\Delta t} \\ IR &= -A \frac{\Delta B}{\Delta t} \\ \frac{Q}{\Delta t} R &= -A \frac{0 - B}{\Delta t} \\ Q &\propto B\end{aligned}$$

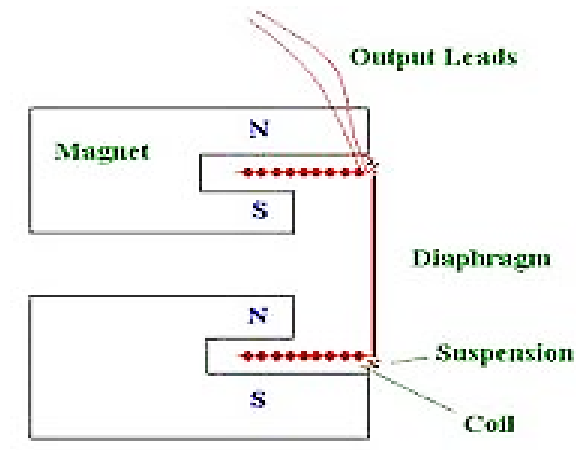
$\Delta\Phi = A \Delta B$
 where A : area of coil
 B : maximum magnetic field strength experience by the coil
 Q : charge that moves around the circuit
 R : resistance of the coil
 I : induced current, $I = \frac{Q}{\Delta t}$

Since $\theta \propto Q$, $\theta \propto B$.

Note: Deflection of galvanometer pointer can be pre-calibrated to give values of B .

4.3 Microphone

[diagram source : google image]



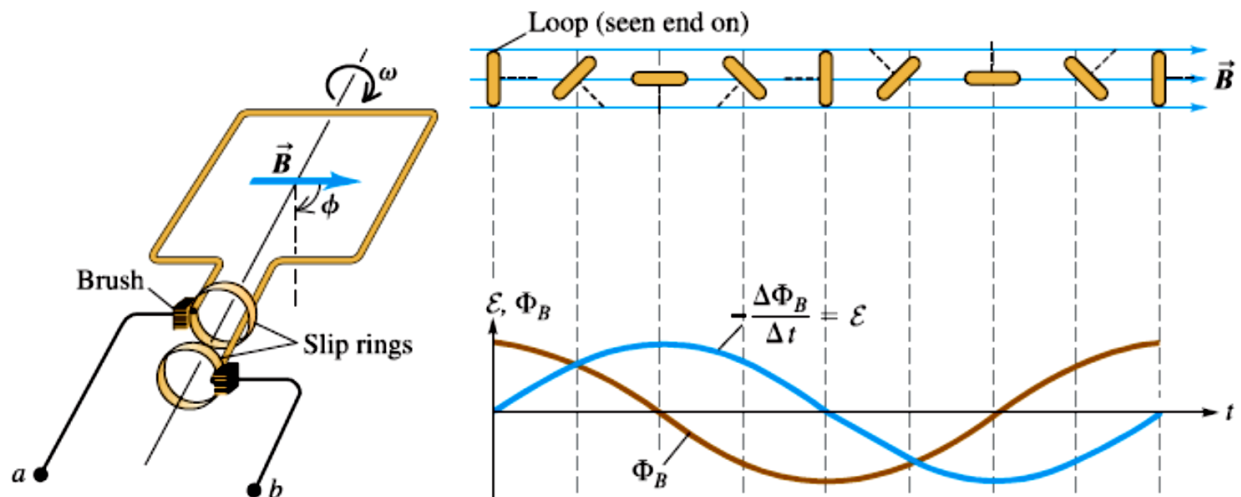
System 1 : Magnet

System 2 : Coil of wire

- A small coil attached to a membrane is suspended close to a small permanent magnet.
- When sound waves strike the membrane, causing it to vibrate, the small coil is made to move in the magnetic field.
- The movement of the coil in the magnetic field changes the magnetic flux linkage experienced by the coil, and an e.m.f. is induced in the coil.
- This varying induced e.m.f. is amplified before sending to a tape recorder or loudspeaker.
- The frequency of the e.m.f. induced in the coil depends on the frequency of vibration of the coil, which in turn depends on the frequency of the sound waves striking the membrane.

4.4 A.C. Generator

[diagram source: Young 'College Physics']



Ends of the coil are connected to a pair of slip-rings.

Contact with the rings is made by brushes (small blocks of conductor such as carbon, supported on springs)

System 1 : Magnet

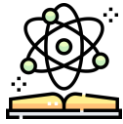
System 2 : Coil

- As the coil rotates, the magnetic flux linkage changes sinusoidally.
- By Faraday's laws of electromagnetism,
- an induced e.m.f., hence induced current and induced magnetic field will vary sinusoidally.
The induced current will flow in the direction to produce an induced magnetic field to oppose the change in magnetic flux linkage.

Note:

1. The direction of the induced e.m.f. reverses every time the plane of the coil passes the vertical position.
2. As the direction of induced current and the polarity of the induced e.m.f. change, we have to consider the position of the coil relative to the external magnetic field at different times to find their respective directions.

5. Power Transformer



Nature of Science Moment – War of the Currents: AC vs DC

The transformer is what makes AC superior to DC. Being able to transmit electricity at high voltages meant reducing transmission loss greatly. The high efficiency and cost savings of AC led to its eventual widespread adoption as the prime choice for power distribution!

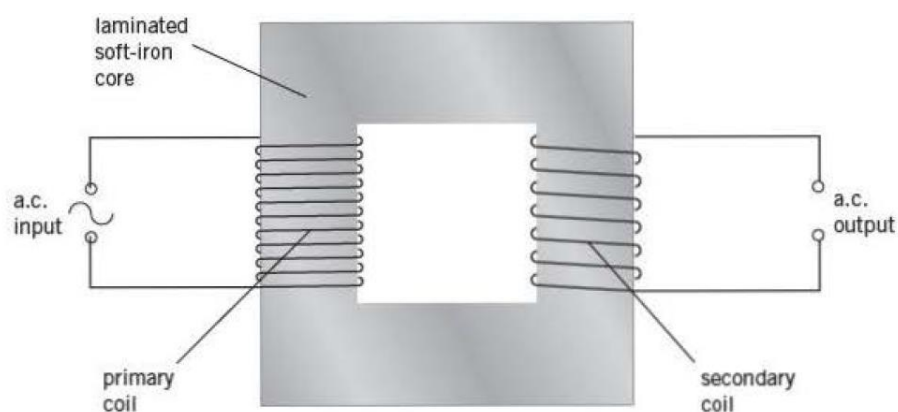


The War of the Currents

5.1 Basic Structure

A transformer consists of

- a primary coil
- a secondary coil
- both coils are wound on to a laminated soft iron core



(Source: Cambridge International AS and A Level Physics)

An a.c. input across the primary coil will give rise to an alternating magnetic flux in the core due to the alternating current in the primary coil. The magnetic flux is linked to the secondary coil via the iron core. As the magnetic flux is changing, by Faraday's law, there will be an induced e.m.f. across the secondary coil. This induced e.m.f. will be alternating at the same frequency as the current in the primary coil.

5.2 Ideal Transformer and Turns Ratio

Consider a transformer with N_P turns on the primary coil and N_S turns on the secondary coils. The currents in the coils are I_P and I_S in the primary and secondary coils respectively when the e.m.f.s across the coils are V_P and V_S .

As the current changes in the primary coil, the magnetic flux density (hence magnetic flux) due to this current also changes and thus the magnetic flux through the core of the transformer changes at the same rate. This magnetic flux ϕ is the same in each turn of the primary and secondary coils.

The rate of change of magnetic flux through the iron core is $\frac{d\phi}{dt}$.

The induced emf in the secondary coil is $N_S \frac{d\phi}{dt}$.

$$\text{Hence,} \quad \frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \dots\dots\dots (1)$$

$\frac{N_S}{N_P}$ is known as the turns ratio.

If $N_S > N_P$, then $V_S > V_P$, it is a step-up transformer (eg. used in power-generating stations).

If $N_S < N_P$, then $V_S < V_P$ and we have a step-down transformer (eg. used in handphone chargers).

Assume transformer is 100% efficient (ideal transformer), then

$$\begin{array}{ccc} \text{power input} & & \text{power output} \\ \text{at the primary coil} & = & \text{at the secondary coil} \\ \\ P_P & = & P_S \\ I_P V_P & = & I_S V_S \quad \dots\dots\dots (2) \end{array}$$

Combining (1) and (2), we have

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Note: In practice, the transformer is not ideal as there is some power loss (see section 5.4).

5.3 Uses of Transformers

Electrical energy from power generating stations is transmitted in cables at very high voltage. For this electrical energy (from the cables) to be transmitted to the residential areas, the high voltage has to be stepped down first using step-down transformers.

Similarly, for the electrical energy (from the power generating stations) to be transmitted to the cables, the voltage needs to be increased using step-up transformers.

Example 5.1

An ideal small capacity transformer steps down potential difference of 240 V to 6 V.

Determine the turns ratio of the secondary to primary coils of the transformer.

Solution

The potential difference values stated in the question are understandably root mean square values for an alternating p.d. since a transformer is involved which works only on alternating p.d.

Using

$$\begin{aligned}\frac{N_s}{N_p} &= \frac{V_s}{V_p} \\ &= \frac{6}{240} \\ &= 0.025\end{aligned}$$

Since turns ratio of secondary coil to primary coil is less than 1, it is a step-down transformer.

Example 5.2 (Mike Crundell 'AS2 Physics')

A transformer is to be used with an alternating supply of 240 V to power a lamp rated as 12 V, 3 A.

The secondary coil of the transformer has 80 turns. Assuming that the transformer is ideal, calculate, for the primary coil:

- the number of turns,
- the current.

Solution

(a)

$$\begin{aligned}\frac{N_s}{N_p} &= \frac{V_s}{V_p} \\ \frac{80}{N_p} &= \frac{12}{240} \\ N_p &= 1600\end{aligned}$$

(b)

$$\begin{aligned}\frac{I_p}{I_s} &= \frac{V_s}{V_p} \\ \frac{I_p}{3} &= \frac{12}{240} \\ I_p &= 0.15 \text{ A}\end{aligned}$$

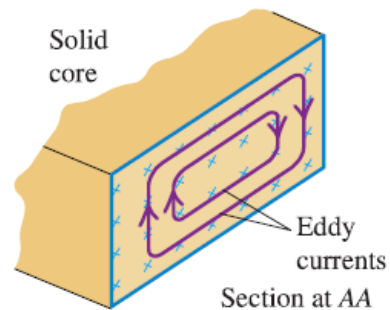
5.4 Sources of Power Losses in Practical Non-ideal Transformers

5.4.1 Eddy Currents

In the iron core of the transformer are induced currents as a consequence of the alternating magnetic flux. Eddy currents waste energy through electrical heating of the core.

In a solid core

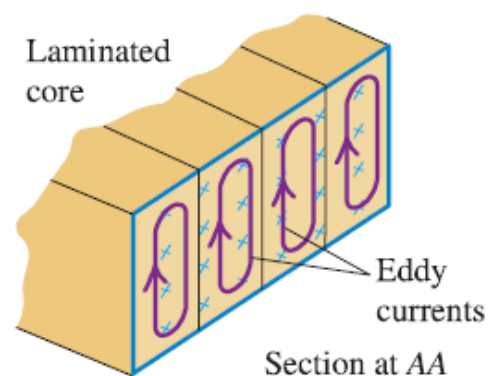
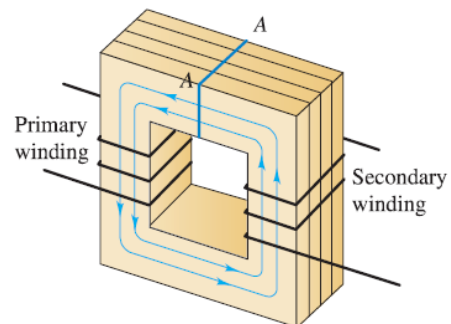
Cross-section of solid core which shows large eddy current flowing with little resistance



In a laminated solid core

Eddy current could be reduced by

- the use of a laminated iron core. Lamination confines eddy currents to individual plates of the core. Eddy current paths are narrower, the induced emf in each path is smaller and eddy current is greatly reduced. Hence power losses is reduced.
- the use of ferromagnetic materials (ferrites, complex oxides of iron and other metals) that have much greater resistivity than pure iron to minimize the heat loss due to flow of eddy current.



Appreciably smaller eddy current induced in the core

[Diagrams source: Young College Physics]

5.4.2 Other Sources of Power Losses

Apart from the eddy current in the iron core, we need to consider the following:

1. The windings of a transformer (primary and secondary coils) are usually made of copper which has some resistance. There is joule heating in the wire (I^2R losses) where R is the resistance of the copper and I is the current passing through it.
2. The core undergoes a continuous process of magnetisation, demagnetisation and re-magnetisation (continuous changing of alignment of domains in the iron) many times per unit time. This requires some power. However, the use of soft iron reduces this power loss.
3. Loss of magnetic flux between the primary and secondary coils. Design of the core and use of soft iron reduces this power loss.

Note: Modern transformers are well-designed with low power loss and the largest one can be very efficient with power losses as low as 1%. However, when dealing with large power, the power loss can still be quite considerably large such that the transformers need to be cooled.

5.5 Transmission of Electrical Power

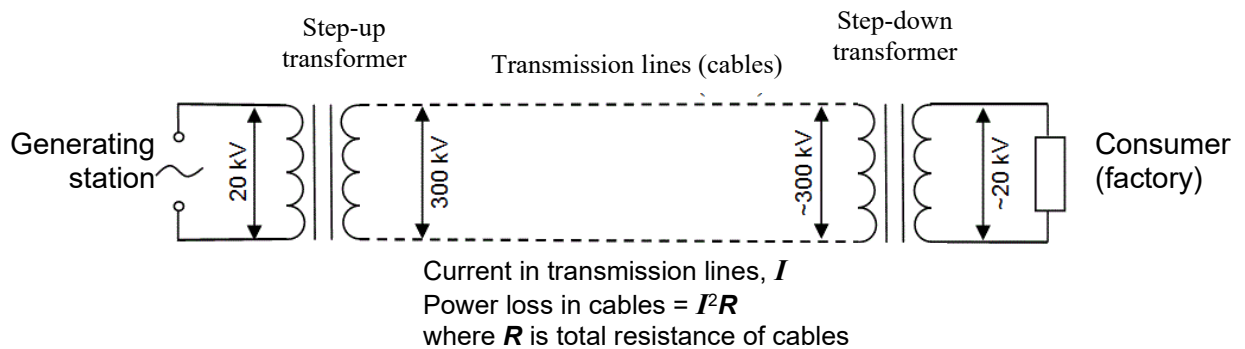
This section discusses the benefits of using alternating current instead of direct current.

When electrical power is transmitted over large distance through transmission lines, it is economical to use high transmission voltage V and hence low current so that less power is dissipated as heat in the transmission lines ($P = I^2R$).

This leads to lower cost of electricity for the consumer (factory/household) since the economic cost of power loss in the transmission cables is kept low. Also, since power generating stations are powered by fossil fuel, reduction of power loss in cables also reduces wastage of valuable non-renewable resources, hence more environmentally friendly.

In order to achieve this, we need to use a step-up transformer at the power generation station and transformers only work using alternating current.

Example 5.3



Given:

Resistance of transmission lines = 100Ω

Power from generating station = 10 MW

Potential difference across primary coil of step-up transformer = 20 kV

Determine the percentage of power loss during transmission of electricity.

Assuming transformers are ideal, power from primary coil of step-up transformer
= power at secondary coil of step-up transformer

Root mean square current in the transmission line

= power at secondary coil / p.d. across secondary coil = $10 \times 10^6 / 300 \times 10^3 = 33.3 \text{ A}$

Power loss in transmission lines

= $I_{rms}^2 R = (33.3)^2 (100) = 111 \text{ kW}$

Percentage of power loss during transmission of electricity = $111 \text{ kW} / 10 \times 10^6 \text{ W} = 1.11 \%$

6 Review Questions

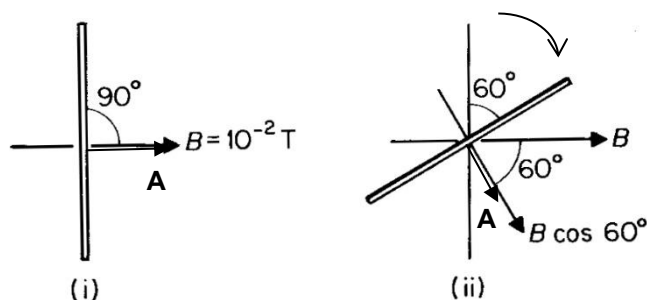
Question 1 (ACJC)

A narrow coil of 10 turns and an area of $4.0 \times 10^{-2} \text{ m}^2$ is placed in a uniform magnetic field of flux density B of $1.0 \times 10^{-2} \text{ T}$ so that the flux links the turns normally. If the coil is now rotated about an axis through its middle so that it turns through 60.0° clockwise in 0.20 s in the field B , calculate the average induced e.m.f. in the coil. $[1.0 \times 10^{-2} \text{ V}]$

Solution:

Note that in this case as the coil is rotated, the flux linking the coil changes from $\phi = NBA \cos 0^\circ$

Draw a diagram showing the relative position of the coil to the magnetic field.



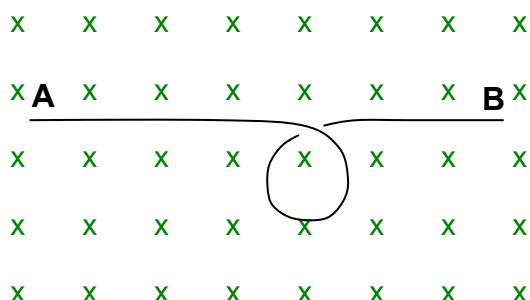
$$\Phi_i = NBA$$

$$\Phi_f = NBA \cos 60^\circ$$

$$\begin{aligned} \varepsilon &= \left| -\frac{\Phi_f - \Phi_i}{\Delta t} \right| \\ &= \left| -\left[\frac{NBA(\cos 60^\circ - 1)}{0.2} \right] \right| \\ &= \frac{(10)(4 \times 10^{-2})(1.0 \times 10^{-2})(0.5)}{0.2} \\ &= 1.0 \times 10^{-2} \text{ V} \end{aligned}$$

Question 2 (ACJC)

The magnetic field shown in the figure has a uniform magnitude of 25.0 mT and is directed into the paper. The initial diameter of the kink is 2.00 cm.



- (a) The wire is quickly pulled taut, such that the diameter of the kink becomes zero in 50.0 ms. Determine the average induced e.m.f. between endpoints A and B. Specify the endpoint at the higher potential.

[1.57×10^{-4} V, B is at a higher potential]

- (b) Suppose the kink is undisturbed, but the magnetic field increases to 100 mT in 4.00×10^{-3} s. Determine the average induced e.m.f. between endpoints A and B, and state the endpoint with the higher potential.

[5.89×10^{-3} V]

Solution:

(a)

In this case, the B-field through the coil remains the same but the area decreases, leading to a decrease in magnetic flux linkage.

$$|\varepsilon| = \left| -\frac{d\Phi}{dt} \right| = \left| -NB \frac{dA}{dt} \right|$$

$$|\varepsilon| = \left| -(1)(25 \times 10^{-3}) \left(\pi \frac{0 - (0.01)^2}{(50.0 \times 10^{-3})} \right) \right|$$

$$= 1.57 \times 10^{-4} \text{ V}$$

No current flows if circuit is opened.

Assume if an external resistor is connected across AB,

by **Lenz's law**, the induced current produces an induced magnetic field into the page to strengthen the weakening external magnetic field.

Using the **right hand grip rule**, the induced current flows in the clockwise direction in the loop.

As the wire is an e.m.f. source, B is at a higher potential than at A.

(b)

In this case, the area through the coil remains the same but the B-field increases, leading to an increase in magnetic flux linkage.

$$|\varepsilon| = \left| -\frac{d\Phi}{dt} \right| = \left| -NA \frac{dB}{dt} \right|$$

$$|\varepsilon| = \left| -(1) \left[\pi (0.01)^2 \right] \left(\frac{[(100 \times 10^{-3}) - (25 \times 10^{-3})]}{(4.00 \times 10^{-3})} \right) \right|$$

$$= 5.89 \times 10^{-3} \text{ V}$$

No current flows if circuit is opened.

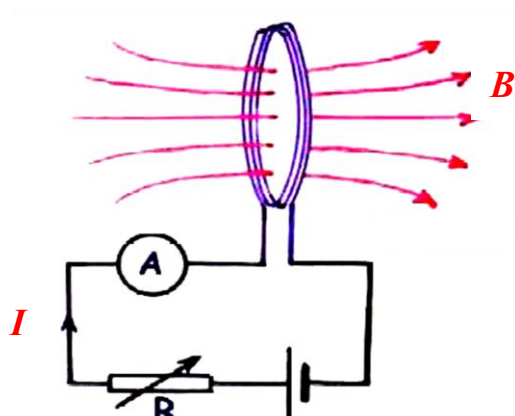
If an external resistor is connected across AB,

by **Lenz's law**, the induced current produces an induced magnetic field out of the page to weaken the strengthening external magnetic field.

Using the **right hand grip rule**, the induced current flows in the anti-clockwise direction in the loop.

As the wire is an e.m.f. source, A is at a higher potential than at B.

Question 3 (ACJC)



A conducting coil is connected to an electrical circuit as shown.

State and explain the direction of the induced current in the coil as

- (a) R is increased, and
- (b) R is decreased.

The initial direction of B produced due to the current flowing in the coil is shown in the diagram.

Solution:

- (a) When R is **increased**, I decreases, hence Φ decreases.

By Lenz's Law, the direction of the induced current must be in such a direction as to create an induced B in the same direction as the initial B to strengthen Φ .

Using the RH grip rule, the direction of the induced current is anticlockwise looking from the right side of the coil.

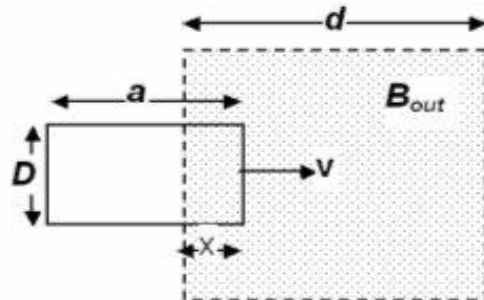
- (b) When R is **decreased**, I increases, hence Φ increases.

By Lenz's Law, the direction of the induced current must be in such a direction as to create an induced B in the opposite direction as the initial B to weaken the Φ .

Using the RH grip rule, the direction of the induced current is clockwise looking from the right side of the coil.

Question 4 (Serway 'Physics for Scientists and Engineers' modified)

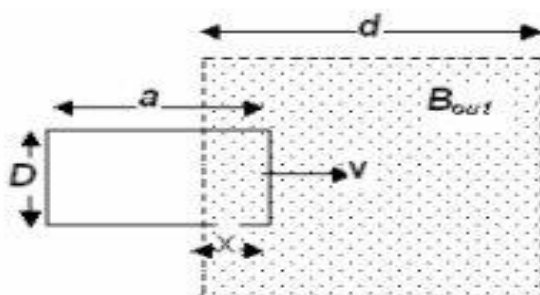
Diagram below shows a rectangular loop of resistance $16\ \Omega$, and length a being pulled at a constant speed $v = 1.0\ \text{m s}^{-1}$ through a region of uniform magnetic flux density B_{out} of $2.0\ \text{T}$. The field is directed out of the paper.



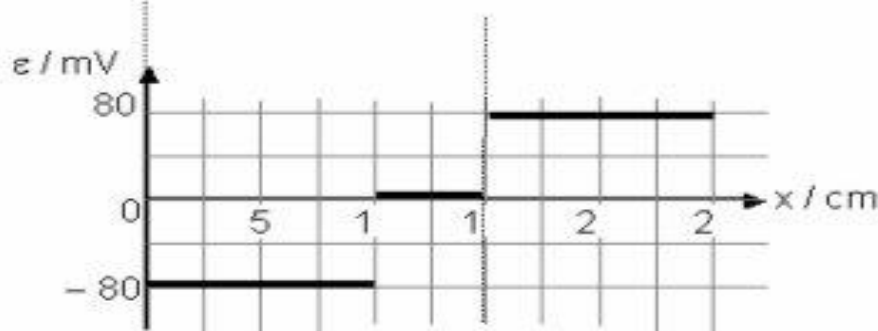
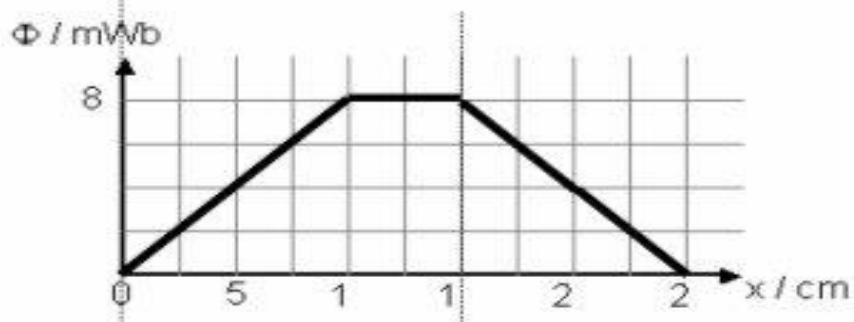
Given that $D = 4.0\ \text{cm}$, $a = 10.0\ \text{cm}$, $d = 15.0\ \text{cm}$, as functions of the position x (from 0 to 25 cm) of the right-hand edge of the loop, plot

- the magnetic flux Φ through the loop, and
- the induced e.m.f. $\mathcal{E}$.

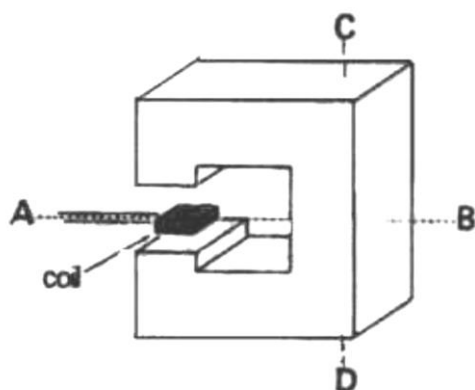
Solution:



- Time taken for loop to fully enter magnetic field $= \frac{10}{1} = 10\ \text{s}$
- $\Phi_{\text{max}} = BA = 2(0.04)(0.1) = 8.0 \times 10^{-3}\ \text{Wb}$
- $\mathcal{E} = Bv = 2(0.04)(1) = 8.0 \times 10^{-2}\ \text{V}$



Question 5



A small square coil has its plane set at right angles to the uniform magnetic field between the pole pieces of a horseshoe magnet as shown in the figure.

- (a) The magnet is rotated at constant angular velocity about the axis of AB.

Sketch graphs showing the variation with time of

- (i) the magnetic flux through the coil,
- (ii) the e.m.f. induced in the coil.

Explain clearly how your graphs are obtained.

- (b) The magnet is now rotated at constant angular velocity about the axis CD.

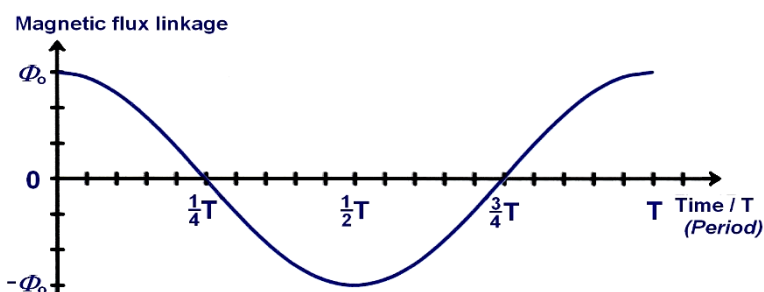
Sketch graphs again as in (a)(i) and (a)(ii). Explain how your new graphs are obtained.

(Hint: Draw diagrams showing relative position of coil and magnet for one cycle at suitable time intervals.)

Solution:

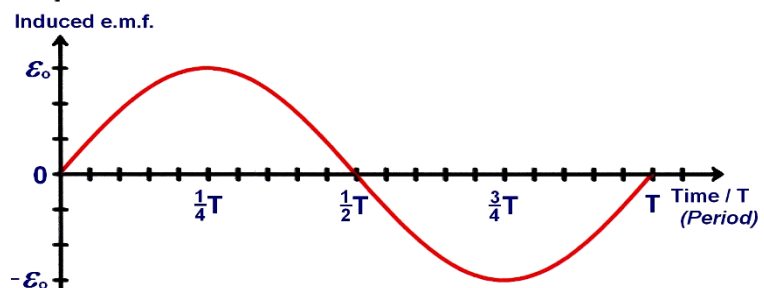
(a)(i)

{Graph should be sketch over 2 cycles}



(a)(ii)

{Graph should be sketch over 2 cycles}



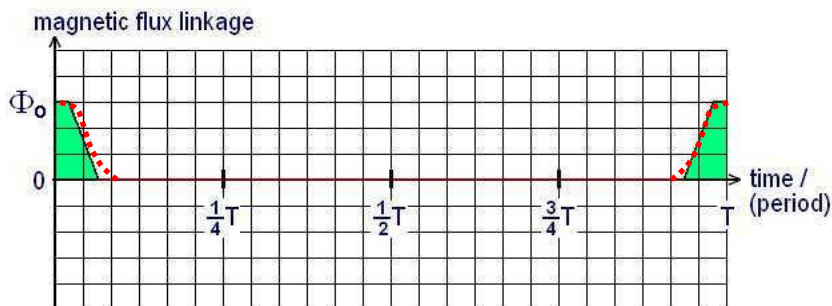
When the magnet is rotated about axis AB at constant angular velocity, the direction of B is initially perpendicular to the plane of the coil, hence the magnetic flux density is maximum and decreases to zero when B is parallel to the plane of the coil.

The magnetic flux linkage ($\Phi = BA \cos \theta$) decreases as a cosine function of time from a maximum value to zero within the first quarter of the period {Fig.(a)(i)}. The e.m.f. induced will also be sinusoidal and its polarity is such as to give an effect that opposes the decreasing magnetic flux linkage {Fig.(a)(ii)}.

Note: e.m.f is proportional to the rate of change of flux linkage, hence both graphs are sinusoidal.

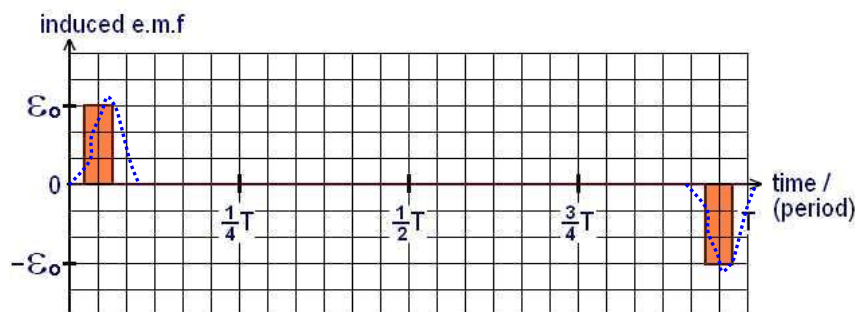
(b)(i)

{Graph should be sketch over 2 cycles}



(b)(ii)

{Graph should be sketch over 2 cycles}



Note both solid and dotted lines are possible answers

- Assume that the area between the poles of the horseshoe magnet is much larger than the area of the coil and assume that the magnetic field between the poles is uniform and zero outside the poles.
- When the magnet is rotated about axis CD at constant angular velocity, the rate of decrease of magnetic flux linkage Φ , through the coil is constant as the magnet's poles leaves the coil. {NB:- If the distance between the axis CD and the poles of the magnet is large, we may assume the rate of change of Φ through the coil to be constant, otherwise, the lines in the diagram should be slight curves}.
- As the rate of decrease of magnetic flux linkage is constant, the induced e.m.f. is also constant {Faraday's Law} and positive {Lenz's Law}. There is no change in magnetic flux linkage through the coil as long as the coil's entire area is still between the magnet's poles. {At the start, near $t = 0$ s of the cycle} Once the entire coil leaves the magnet's poles, the magnetic flux linkage drops to zero. With no magnetic flux linkage, there is no change in Φ , hence no induced e.m.f.
- Towards the end of one cycle of rotation of the horseshoe magnetic, the poles of the horseshoe magnet approaches the location of the coil again. At this stage the magnetic flux linkage through the coil increases uniformly and a constant negative polarity e.m.f. is induced until the coil is fully between the magnet's poles where the magnetic flux linkage becomes constant again. There is no induced e.m.f. at this stage. {At the end : near $t = T$ of the cycle}