2. Forces & Moments

Content Page

Learning Objectives

2.1 Types of forces

(a) Describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate.

(b) H2: Show a qualitative understanding of forces including normal force, frictional force, buoyant force (upthrust), and viscous force, e.g. air resistance (knowledge of the concepts of coefficients of friction and viscosity is not required).

H1: Show a qualitative understanding of forces including normal force, frictional force and viscous force, e.g. air resistance (knowledge of the concepts of coefficients of friction and viscosity is not required).

(c) Recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems.

2.2 Moment and torque

(d) Define and apply the moment of a force and the torque of a couple.

(e) Show an understanding that a couple is a pair of forces which tends to produce rotation only.

(f) Show an understanding that the weight of a body[1] may be taken as acting at a single point known as its centre of gravity.

2.3 Translational and rotational equilibrium

(g) Apply the principle of moments to new situations or to solve related problems.

(h) Show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.

(i) Use free-body diagrams and vector triangles to represent forces on bodies that are in rotational and translational equilibrium.

[1] The term "body" (or "object") is used to refer to a collection of matter that can be treated as having no internal structure, or an internal structure that can be ignored in typical contexts.

2.1 TYPES OF FORCES

Self-study resources		
A SLS lesson on Hooke's law and the different types of forces.	for.edu.sg/types-of-forces	

2.1.1 What is a force?

In kinematics, we study the motion of objects, including accelerations which are changes in velocities. Physics is also a study of what can *cause* an object to accelerate. That cause is a **force,** which is, loosely speaking, a push or pull on the object. The force acts on the object to change its velocity.

We define the unit of force in terms of the acceleration a force would give to a 1 kg body. Suppose we put that body on a horizontal, frictionless surface and pull horizontally, as shown in Fig. 2.1, such that the body has an acceleration of 1 m s^{-2}. Then we can define our applied force as having a magnitude of 1 newton (abbreviated N). If we pulled with a force magnitude of 2 N, we would find that the acceleration is 2 m s^{-2}. Thus, the **acceleration is proportional to the force.**

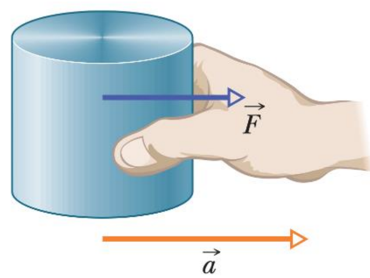


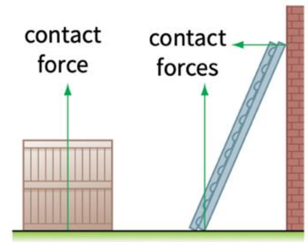
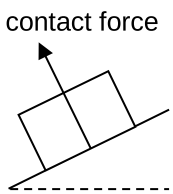
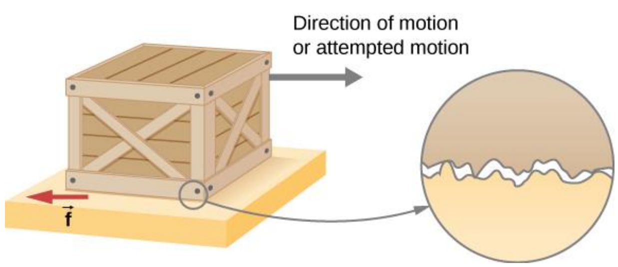
Fig. 2.1

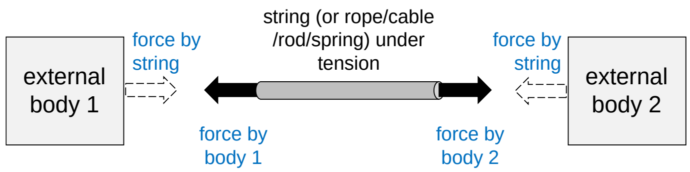
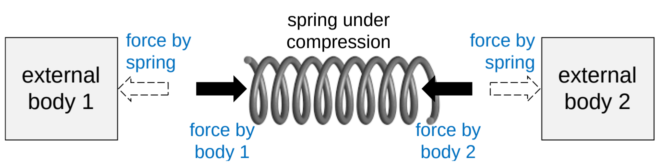
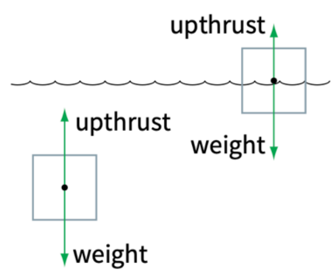
Force is a **vector** quantity and thus has not only magnitude but also direction. So, if two or more forces act on a body, we find the **resultant force** (or **net force**) by adding them as vectors, following the rules of "01 Quantities and Measurements". A single force that has the same magnitude and direction as the calculated resultant force would then have the same effect as all the individual forces added together. This is known as the **principle of superposition for forces**.

Forces are most often represented with a vector symbol such as $\vec{F}$ or $\boldsymbol{F}$, and a resultant force is represented with the vector symbol $\vec{F}_{\text{resultant}}$, $\vec{F}_{\text{net}}$, $\boldsymbol{F}_{\text{resultant}}$ or $\boldsymbol{F}_{\text{net}}$. For the rest of the notes, we drop the overhead arrows and just use the symbols to indicate the magnitude of the forces along that axis.

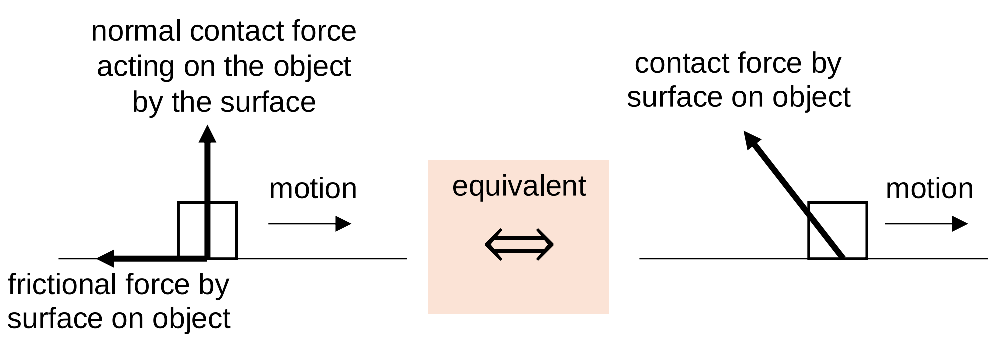
2.1.2 Contact forces

This type of force acts when two interacting objects have physical contact with each other.

Force	Important situations
Normal contact force. The force exerted by the surface of one object on the surface of another when they are in physical contact and prevents the objects from passing through each other. The force acts perpendicular (or "normal") to the surface.  	• standing on the ground • one object sitting on top of another • object on an inclined surface • leaning against a wall • one object bouncing off another
Friction. This is the force which arises when two surfaces rub over one another. Friction arises from the interlocking of the irregularities, as shown below.  The harder the surfaces are pushed together, the larger the frictional force. • If an object is sliding along the ground, friction acts in the opposite direction to its motion. • If an object is stationary, but tending to slide – perhaps because it is on a slope – the force of friction acts up the slope to stop it from sliding down.  object pulled along surface  object at rest on incline Friction always acts along a surface, never at an angle to it.	• pulling an object along the ground • vehicles cornering or skidding • sliding down a slope
Drag or viscous force. This force is similar to friction. When an object moves through air, there is friction between it and the air. Also, the object has to push aside the air as it moves along. Together, these effects make up drag. Similarly, when an object moves through a liquid, it experiences a drag force. Drag acts to oppose the motion of an object; it acts in the opposite direction to the object's velocity. It can be reduced by giving the object a streamlined shape. 	• vehicles moving • aircraft flying • parachuting • objects falling through air or water • ships sailing

Force	Important situations
Tension. This is the force in a rope or string when it is stretched. If you pull on the ends of a string, it tends to stretch. The tension in the string pulls back against you. It tries to shorten the string.  Tension can also act in springs. If you stretch a spring, the tension pulls back to try to shorten the spring. If you squash (compress) the spring, the tension acts to expand the spring.  	• pulling with a rope • squashing or stretching a spring
Upthrust. Any object placed in a fluid such as water or air experiences an upwards force. This is what makes it possible for something to float in water.  	• boats and icebergs floating • people swimming • divers surfacing • a hot air balloon rising

Combined effect of normal contact force and frictional force. The normal contact force and the frictional force on an object in contact with a rough surface can be represented by a single force, as shown in Fig. 2.2.



(a) **(b)**

Fig. 2.2

2.1.3 Fields of forces

Unlike the contact forces, gravitational force / electric force / magnetic force act on object without the need to be in contact. They are known as **fields of forces**.

A field of force is a **region of space** in which **a force is experienced**.

field	**acts on**	**definition**
gravitational	mass	A region of space in which gravitational force is experienced by a mass placed in the region.
electric	charge	A region of space where an electric force acts on a stationary charge placed at any point within the region.
magnetic	magnet / moving charge / current-carrying conductor	A region of space where a magnetic force acts on a magnet or moving charge or current-carrying conductor.

We will study these fields of forces in later topics.

2.1.4 Hooke's law

Compressive and tensile force

A pair of forces is needed to change the shape of a spring (Fig. 2.3). If the spring is being squashed and shortened, we say that the forces are **compressive** (Fig. 2.3b). If the spring is being stretched and lengthen, we say that the forces are **tensile** (Fig. 2.3c).

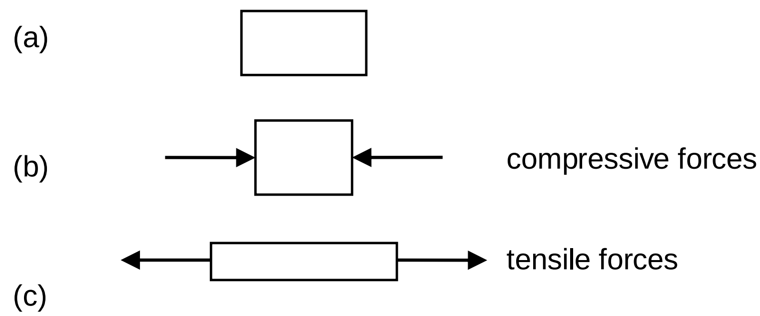


Fig. 2.3

Fig. 2.4 shows that when the wire is bent,

- line AA becomes longer and hence that part of the wire is in tension
- line BB becomes shorter and hence that part of the wire is in compression.

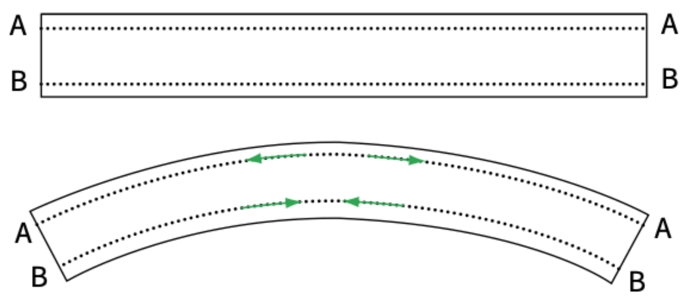


Fig. 2.4

The thicker the wire, the greater the compression and tension forces along its edges.

Hooke's law

We can investigate how the length of a helical spring is affected by the applied force or load. The spring hangs freely with the top end clamped firmly. A load is added and gradually increased. For each value of the load, the **extension** of the spring is measured (Fig. 2.5). We can plot a graph of force F against extension x to find the stiffness of the spring, as shown in Fig. 2.6.

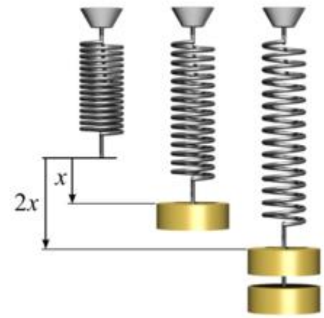


Fig. 2.5

For a typical spring, the section OA of the graph is a straight line passing through the origin. The extension x is directly proportional to the applied force (load) F. The behaviour of the spring in the linear region OA is

$$x \propto F$$

$$\therefore F = kx \qquad \text{(eq. 2.1)}$$

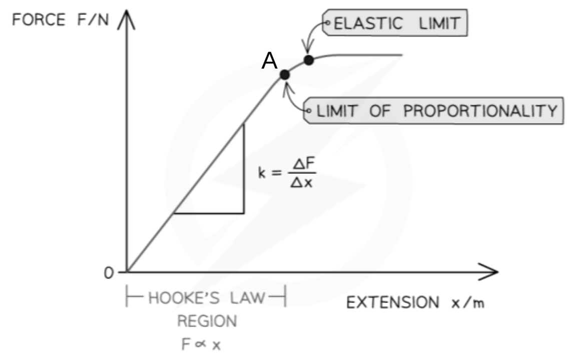


Fig. 2.6

where k is the force constant of the spring (sometimes called the stiffness or the spring constant of the spring).

We can also determine the force constant k from the gradient of section OA of Fig. 2.6.

Beyond point A in Fig. 2.6, the graph is no longer a straight line; its gradient changes and we can no longer use the equation $F = kx$. We say that the spring has been stretched beyond its limit of proportionality.

Rearranging eq. 2.1,

$$k = \frac{F}{x}$$

we see that the force constant k is the force per unit extension. So, the SI unit for the force constant is newtons per metre or N m^{-1}.

If a spring or anything else responds to a pair of tensile forces in the way shown in section OA of Fig. 2.6, we say that it obeys **Hooke's law**.

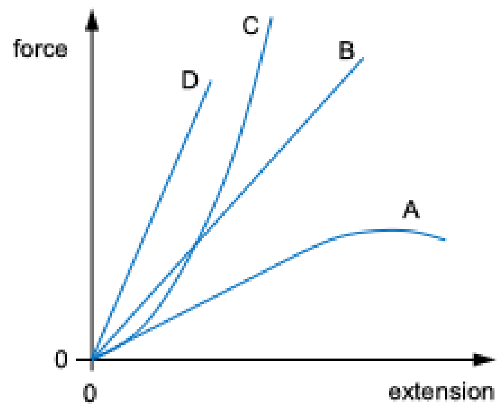
Hooke's law states that within the **limit of proportionality**, the **extension** produced in a material is **directly proportional to the load applied**.

You need to explain this

A spring obeying Hooke's law will deform if a small tensile force is applied to it and it will return to its original length when the force is not present. This behaviour is described as 'elastic'. However, if a large tensile force is applied, the spring may not return to its original length, i.e. permanently deformed. The force beyond which the spring becomes permanently deformed is known as the **elastic limit**.

Example 2.1

The figure shows the force–extension graphs for four springs, A, B, C and D.



(a) State which of the four springs does not obey Hooke's law.

(b) State which spring has the greatest value of force constant.

(c) State which is the least stiff.

Solution

(a) **Spring C** does not obey Hooke's law since the force-extension graph is a curve.

Spring A shows a segment where the extension is proportional to the force and this segment obeys Hooke's law. For the rest of the curve, it does not obey Hooke's law.

(b) **Spring D** has the largest gradient and therefore the greatest value of force constant.

(c) The stiffer the spring, the larger the force constant.

Spring A has the least value of force constant within its limit of proportionality and therefore is the least stiff.

Example 2.2 A unstretched spring has a length of 5.0 cm. It obeys Hooke's law and has a force constant of 400 N m^{-1}. Calculate the tension in the spring when its overall length is 7.0 cm.

Solution

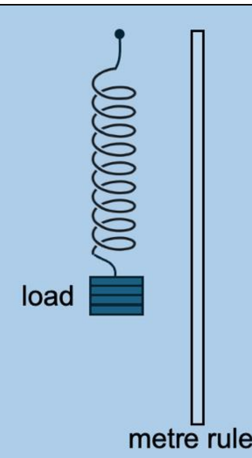
Extension of the spring = 7 – 5 = 2 cm

Magnitude of the tension in the spring
= magnitude of force applied to the spring
= kx = (400)(0.02)
= 8.0 N

Note: Ensure that the extension and the force constant are expressed to the same unit for length.

Class exercise

The spring constant *k* of a spring may be determined by finding the extension of the spring and the load applied, using the apparatus shown. A student obtained the following readings.



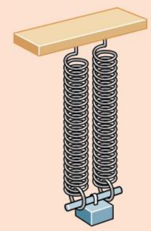
reading on the rule for the lower end of the unextended spring	= 13.60 ± 0.05 cm
reading on the rule for the lower end of the extended spring	= 17.95 ± 0.05 cm
load	= 4.00 ± 0.02 N

Determine *k* and express it with its actual uncertainty.

Combining springs

Springs can be combined in different ways, end-to-end (in series) and side-by-side (in parallel).

(a) Determine the equivalent force constant of two identical springs, each of force constant *k*, in series and two springs in parallel, as shown below.

<table>
<tr>
<td>
springs in series</td>
<td>Hint for spring in series:
1. A same force equal in magnitude to the load acts on each spring.
2. The equivalent spring will have the same amount of extension as the total extension of the two springs.</td>
<td>
springs in parallel</td>
<td>Hint for spring in parallel:
1. The two springs must have the same amount of extension.
2. The equivalent spring will have the same amount of extension as the two springs.</td>
</tr>
</table>

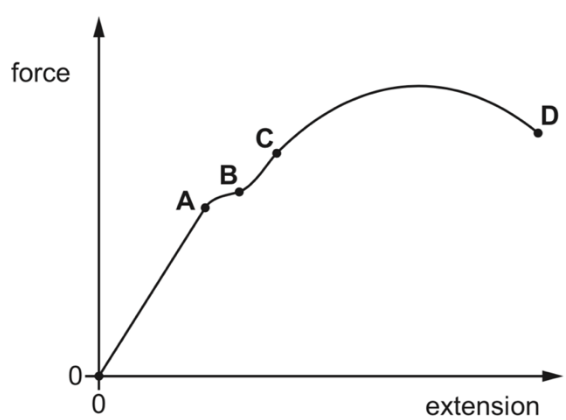
(b) Determine the equivalent force constant of three or more springs with difference force constants when they are combined in series and parallel.

Exercise 1: Hooke's law

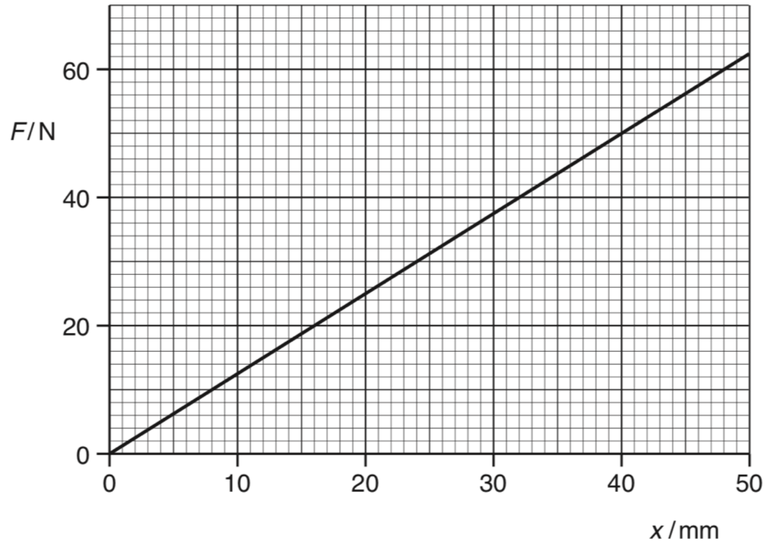
You should take about 15 minutes to complete this exercise.

1.1 The force-extension graph of a metal wire is as shown.

At which point on the graph does the metal wire stop obeying Hooke's law?



1.2 A spring is compressed by applying a force. The variation with compression x of the force F is shown below.



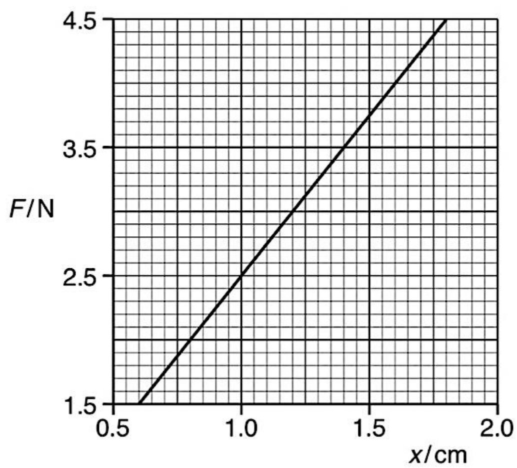
Calculate the force constant of the spring.

1.3 A spring is supported so that it is horizontal with one end attached to a fixed structure. The spring is extended by different forces and the extension x of the spring is measured for each force F.

The variation with x of F between 1.5 N and 4.5 N is shown.

(a) Show that the spring obeys Hooke's law.

(b) Determine the force constant of the spring.



1.4 An arrangement is used to determine the length l of a spring when different masses M are attached to the spring. The variation with mass M of l is shown below.

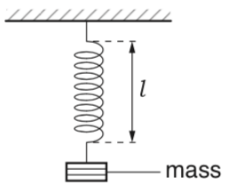
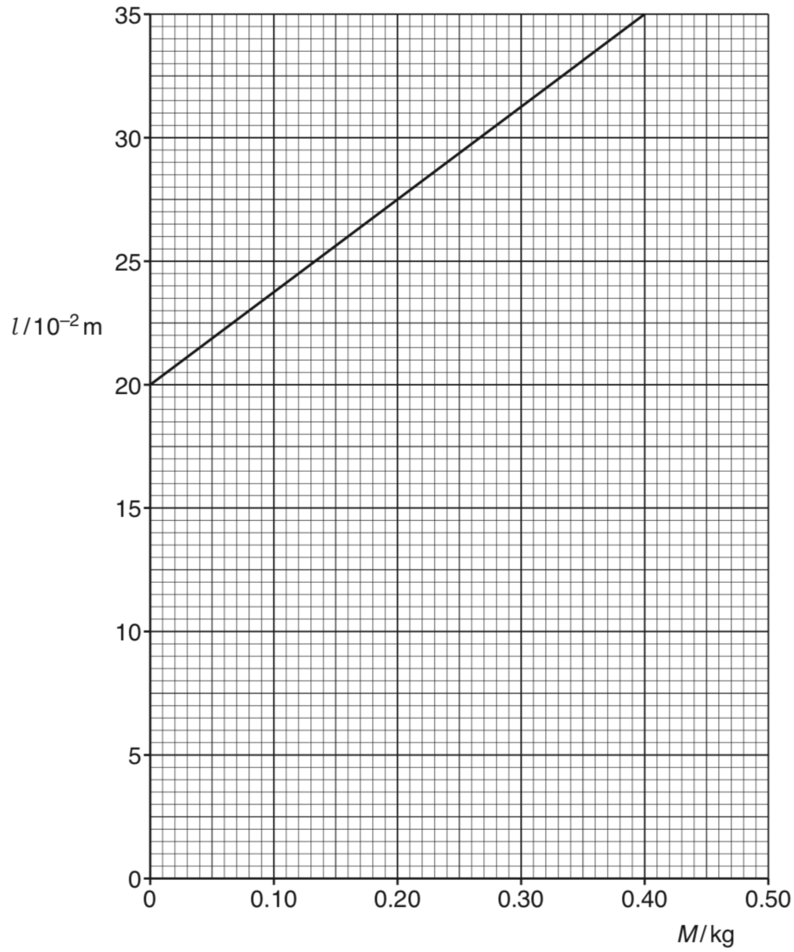


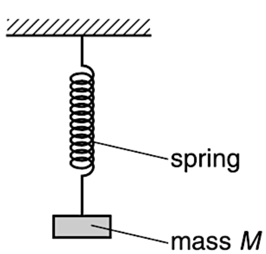
Diagram 1

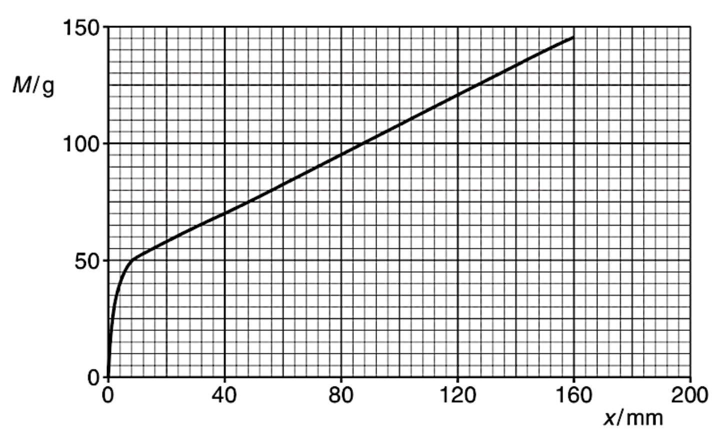


(a) Use the graph to deduce that the spring obeys Hooke's law.

(b) Show that the force constant of the spring is 26 N m^{-1}.

1.5 A spring hangs vertically and different masses are attached to the lower end of the spring. The extension x of the spring is measured for each mass M. The variation with x of M is shown below.





State and explain whether the spring obeys Hooke's law.

2.2 DRAWING FORCE DIAGRAMS

Self-study resources		
A revision SLS lesson on the drawing force diagram that you should have mastered when you studied Physics at secondary school level.	for.edu.sg/fbd	

A force diagram (or free body diagram) represents the forces acting on a body. Research had shown that this is a critical skill that you must master to improve the success rate of solving problems.

Key steps in drawing force diagrams

1. **Isolate the body of interest**. If there are more than one body, draw a separate force diagram for each body (represented by a box or a dot).

2. Identify **ALL forces acting on that body**. DO NOT show any forces that the object exerts on other bodies.

3. Draw the vector representing each force:

   a. Each force should be drawn **as straight arrows acting at the point of application and in the direction of application**.

   b. Length of arrow should reflect the relative magnitude of the force.

IMPORTANT NOTES:
- Do not draw any resultant forces acting on the object.
- If information of the positions where the forces act is provided, it usually means that you will need to consider the torque acting on the body. Therefore, the position where these forces act should be represented properly.

Let's walk through the process for creating the force diagram for a log pulled by a tractor shown in Fig. 2.7.

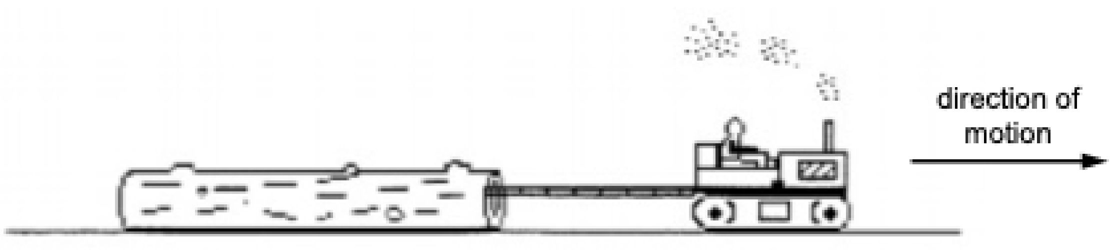
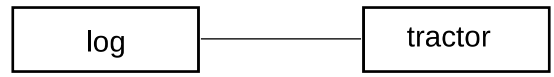
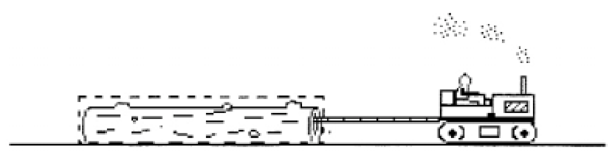
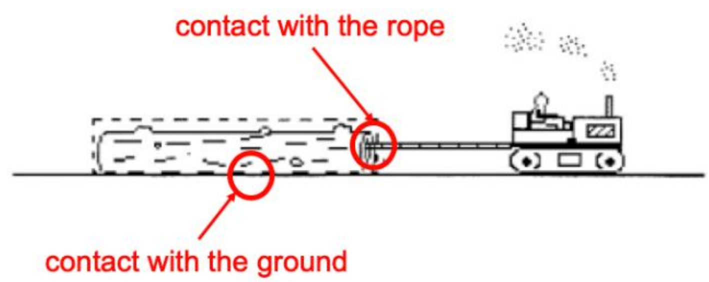
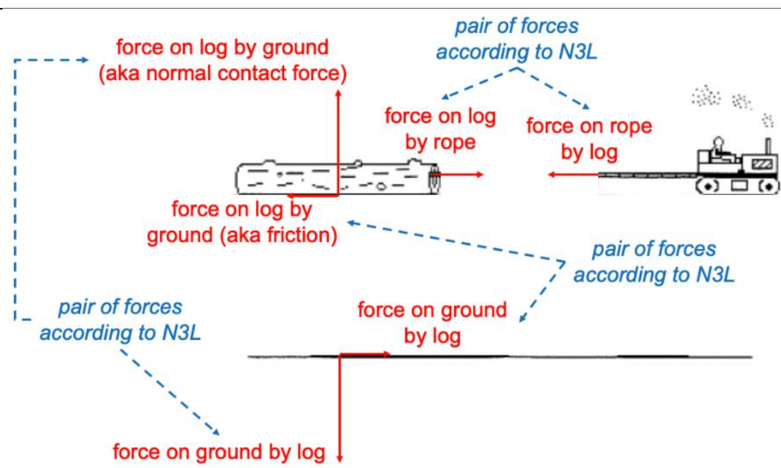
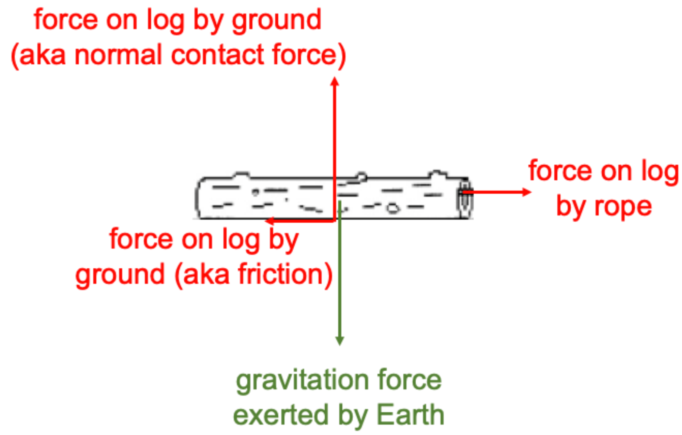
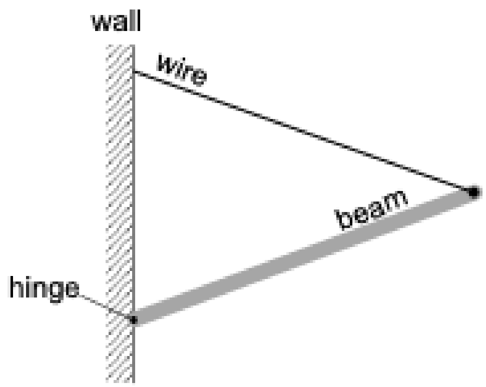
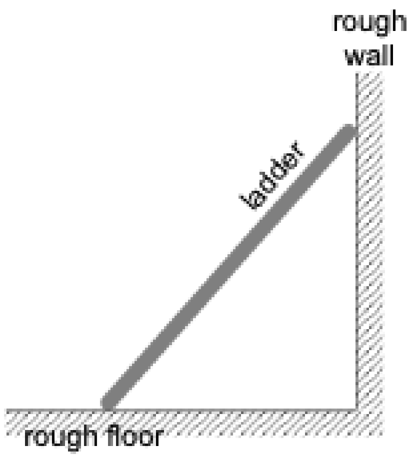
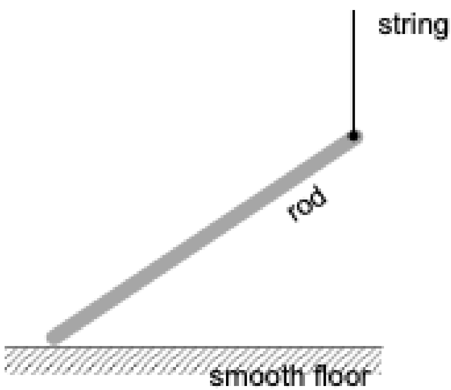
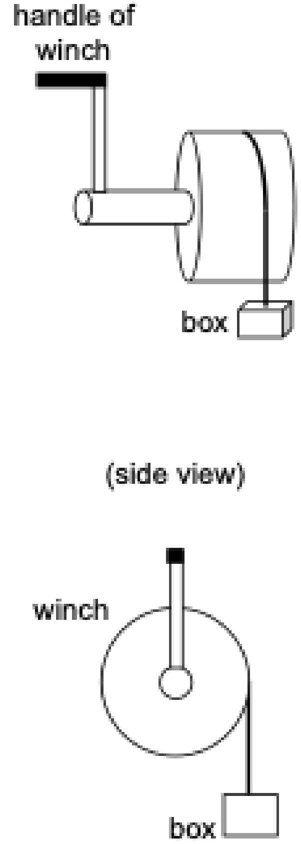


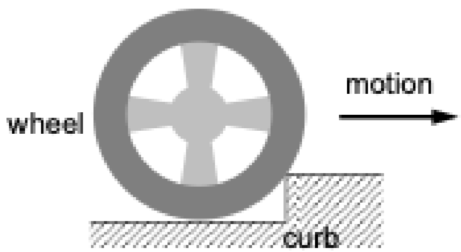
Fig. 2.7

	Description	Illustration
1	Sketch the problem if picture not provided. Note: The sketch does not need to be to scale or pretty.	Note: your simplified diagram can be: 
2	Isolate the object of interest.	The object of interest is the log, let's study the log (enclosed by the dotted line). 
3	Identify the contact points the object has with its surrounding.	
4	Separate the object from the surroundings. Draw the contact forces in pairs on the different objects (consequence of Newton third law). Note: There must be at least one pair of forces per contact point.	

	Description	Illustration
5	Include all non-contact forces (e.g. gravitational force).	

Class exercise Draw the force diagrams for the following objects (underlined item).	
(a) A uniform <u>beam</u> hinged to a wall and supported by a wire. 	**(b)** A <u>ladder</u> resting on a rough wall and rough floor. 
(c) A uniform <u>rod</u> supported by a vertical string on one end and the other end resting on a smooth floor. 	**(d)** A light <u>winch</u> holding up a heavy box. Draw the forces in the side view diagram. 

(e) A <u>wheel</u> rolling forward with a forward driving force. It is just about to mount up a curb.



Exercise 2: Drawing force diagrams

You should take about 15 minutes to complete this exercise.

Sketch the force diagram of the object highlighted in the following scenarios.

Note the conditions:

smooth ⇒ frictionless
constant speed / rest ⇒ no resultant force
acceleration ⇒ resultant force ≠ 0

2.1 A box pushed on a *rough*, flat table so that it has a *rightward acceleration*.

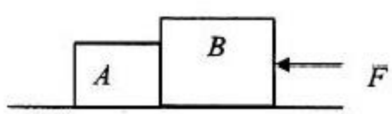
2.2 A box is sliding down a *smooth* slope.

2.3 A skydiver is descending with a *constant speed*.

2.4 An airplane is flying horizontally at *constant speed*.

2.5 A box is at rest relative to a platform and the platform is moving up with *upward acceleration*.

2.6 Two blocks, block A and block B, lie side-by-side on a *smooth* table. A force of magnitude F is exerted on block B and block B pushes on block A. Both blocks start to *accelerate*.



2.3 MOMENT AND TORQUE

(e) define and apply the moment of a force and the torque of a couple *(f) show an understanding that a couple is a pair of forces which tends to produce rotation only* *(g) apply the principle of moments to new situations or to solve related problems*

Self-study resources		
A SLS lesson on the turning effects of forces.	for.edu.sg/turning-effect	

2.3.1 Moment of a force

When a force acts on an object, the force may cause the object to move in a straight line. It could also cause the object to turn or spin (rotate).

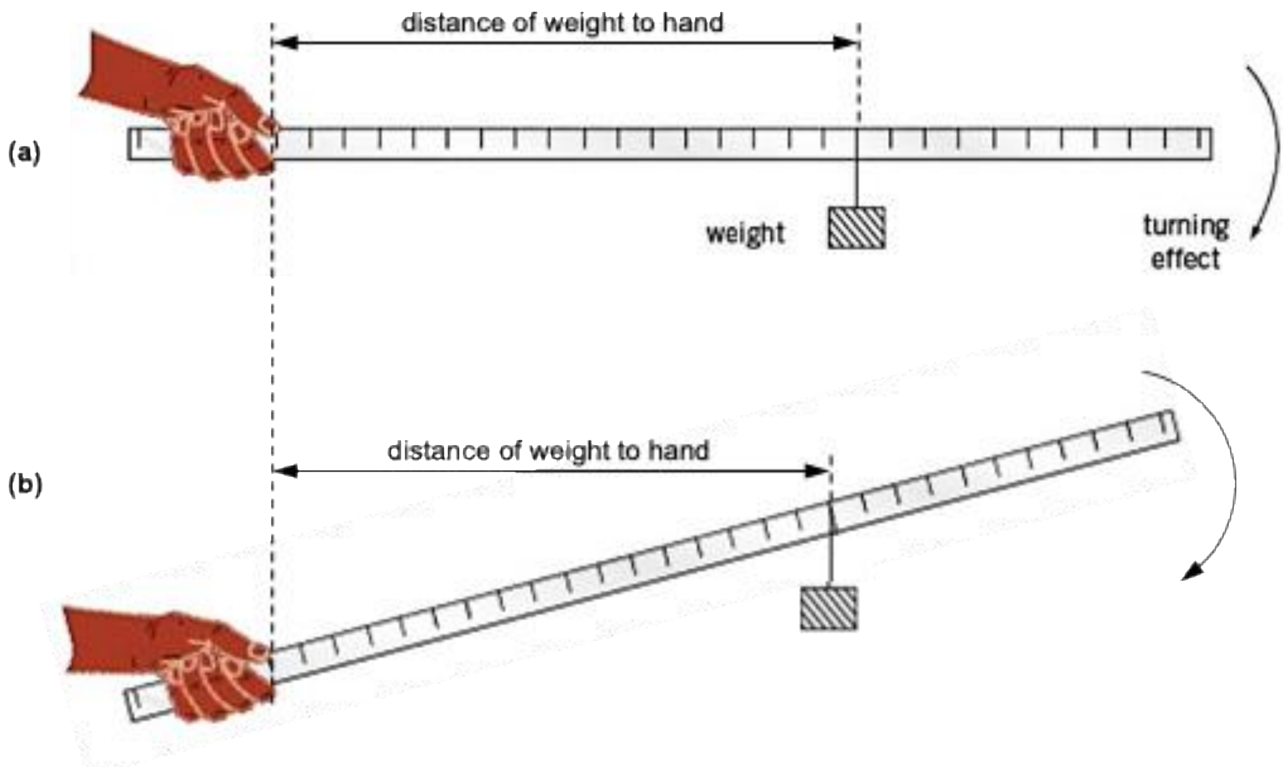


Fig. 2.8

Consider a metre rule held in the hand at one end so that the rule is horizontal (Fig. 2.8a). If a weight is hung from the ruler, we can feel a turning effect on the ruler. The turning effect acts at the hand where the metre rule is pivoted.

> The turning effect of a force is called the **moment of the force**.

The turning effect changes when

- the weight changes or it is moved closer or further from the hand along the ruler,
- the ruler is held at an angle to the horizontal (Fig. 2.8b) and decreases as the rule approaches the vertical position.

The effect described above can be summarised as a formal definition:

> The moment of a force about a pivot is equal to the **product** of the **force** and the **perpendicular distance** from the **line of action of the force** to **the pivot**.

You need to explain this

Fig. 2.9 shows a force of magnitude F acting on an object at a point distance l from the pivot. The line of action of the force is at a perpendicular distance d from the pivot. Then

$$\text{moment of force} = F \times d \qquad \text{(eq. 2.2)}$$

$$= F \times l \sin\theta$$

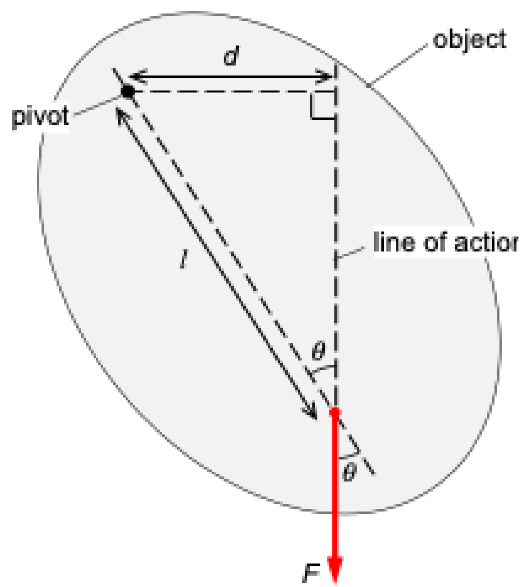


Fig. 2.9

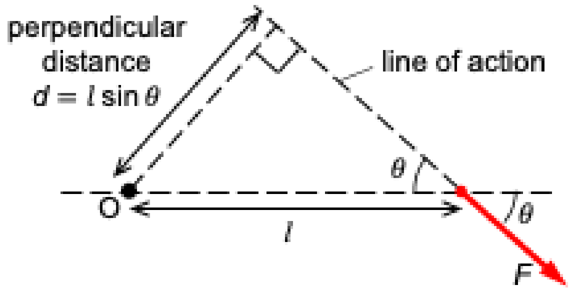
The SI unit for the moment of a force is newton-metre or N m.

From the figure, it is not difficult to see that the force will cause the object to rotate clockwise. There is a “direction” – clockwise or anticlockwise – associated with the moment of a force. Therefore, it is a vector[2].

There are two methods to determine the moments of forces. You should know how to use both methods depending on the situation.

[2] In vector notation, the moment of a force is calculated using $\vec{\tau} = \vec{r} \times \vec{F}$.

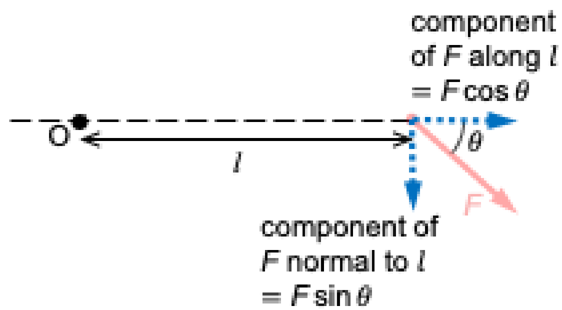
Method 1 – Find the perpendicular distance



Note: This method is used if the perpendicular distance is given or it can be deduced easily with trigonometry.

moment of force F about O
$= F \times d$
$= F \times l \sin\theta$

Method 2 – Resolution of force



The line of action of the component of F along l passes through O, so it does not contribute to the moment. Therefore,

moment of force F about O = ($F \sin\theta$) $\times$ l

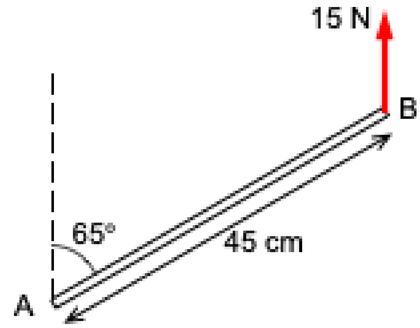
Example 2.3

A light rod AB of length 45 cm makes an angle of 65° to the vertical. A vertical force of 15 N acts on the rod at B.

Determine

(a) the moment of the force about A,

(b) the moment of the force about B.



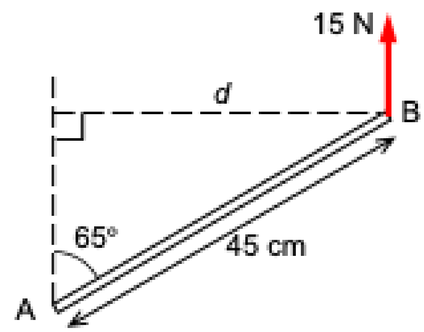
Solution

(a) [Method 1 – find the perpendicular distance]

The perpendicular distance from A to the line of action of the force d = 0.45 sin 65°

Note: Remember that distance is in metres.

Moment of the force about A
= $F \times d$ = (15)(0.45 sin 65°)
= 6.1 N m
(in anti-clockwise direction)

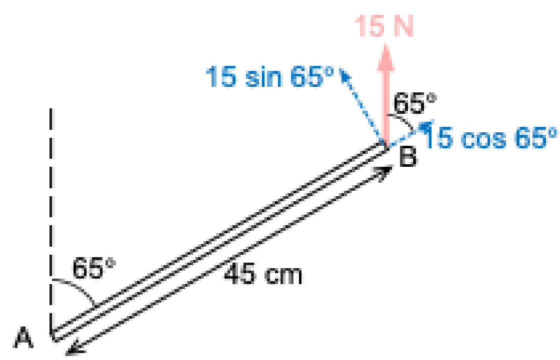


[Method 2 – resolution of force]

Resolve the force along and perpendicular to the rod.

The line of action of the component along the rod passes through A, so it does not contribute to the moment.

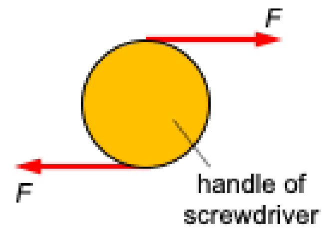
Moment of the force about A
= $F \times d$ = (15 sin 65°)(0.45)
= 6.1 N m
(in anti-clockwise direction)



(b) Since the force passes through B, the force does not produce a moment about B.

2.3.2 Torque of a Couple

Suppose you are using a screwdriver. You will apply a turning effect to the handle in order to tighten or loosen a screw. However, you do not just apply one force to the handle because this would cause the screwdriver to move sideways. Rather, you apply two forces of equal magnitude but in opposite direction on the opposite sides of the handle, as shown in Fig. 2.10.



A **couple** consists of two forces **equal in magnitude** but **opposite in direction** whose **lines of action do not coincide**.

The turning effect is not called a moment because it is produced by two forces, not one. This turning effect is referred to as a **torque**.

The **torque of a couple** is the turning effect of the couple and it is the product of one of the forces and the perpendicular distance between the lines of action of the forces.

You need to explain this

Consider two parallel forces, each of magnitude F acting on an object as shown in Fig. 2.11. From the figure, it is not difficult to see that the force will cause the object to rotate clockwise.

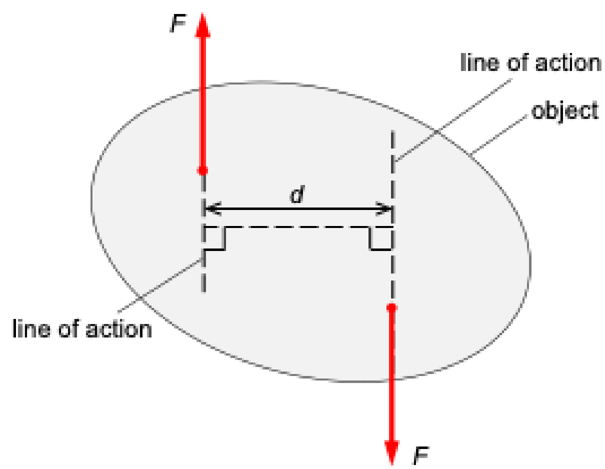


Fig. 2.11

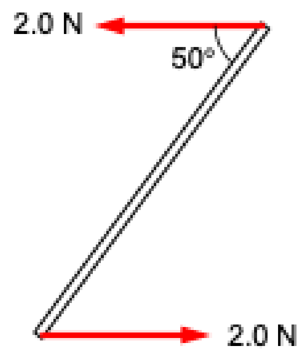
Mathematically, the torque produced by the pair of forces is

$$\text{torque of couple} = F \times d \qquad \text{(eq. 2.3)}$$

The SI unit for torque is newton-metre or N m.

Example 2.4

A couple acts at the ends of on a metre rule as shown.
Determine the torque on the metre rule.



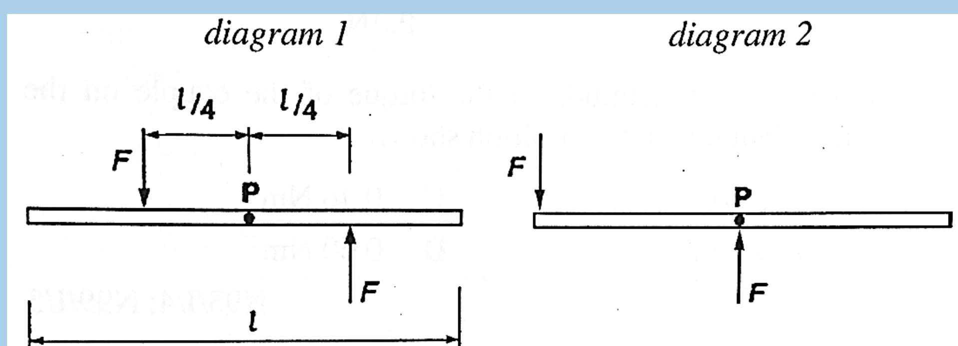
Solution

The perpendicular distance between the lines of action of the forces $d = 1 \times \sin 50^\circ$

Torque of the couple = $F \times d = (2.0)(1 \times \sin 50^\circ) = 1.5$ N m (in anti-clockwise direction)

Class exercise

Diagram 1 shows two parallel forces F acting on a bar of length l pivoted at **P**. The forces give rise to a couple of torque M. Diagram 2 shows the lines of action of the forces moved $\frac{l}{4}$ to the left.

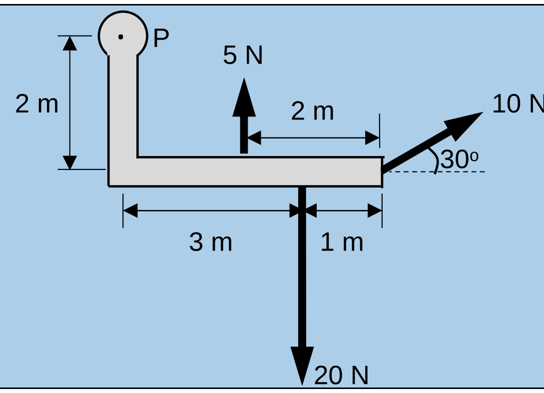


Determine the new torque of the couple in diagram 2 in terms of M.

Class exercise

A L-shaped lever is pivoted at point P. The forces acting on the lever are as shown.

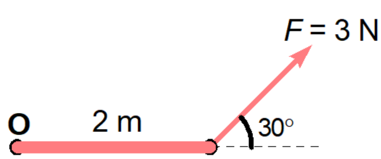
Determine the resultant moment about the point P.

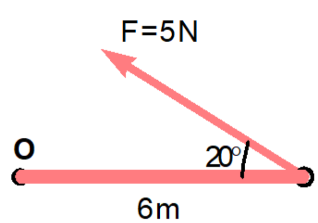


Exercise 3: Turning effects of forces

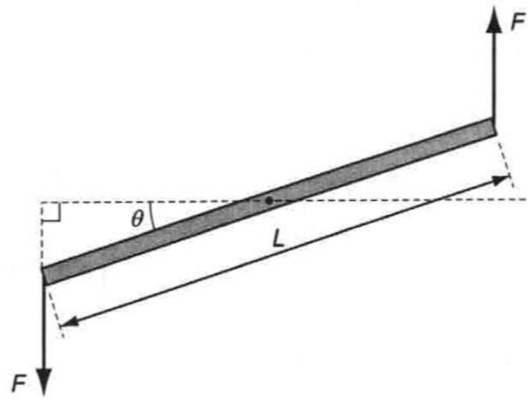
You should take about 10 minutes to complete this exercise.

3.1 Calculate the moment of each force about point O in the diagrams below.





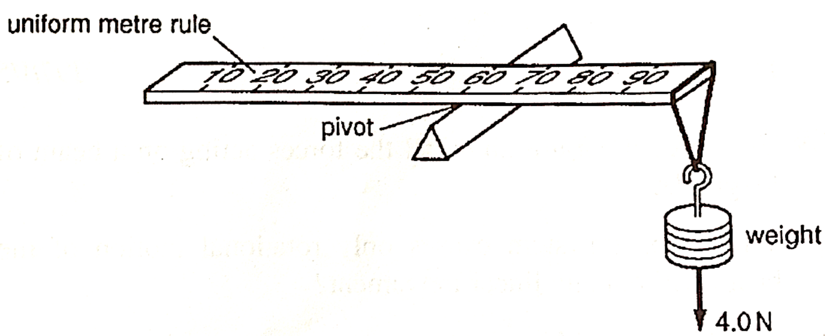
3.2 The diagram shows two equal and opposite forces applied to the ends of a pivoted bar of length *L*.



What is the magnitude of the torque exerted by these forces on the bar?

A FL **B** $2FL$ **C** $FL\cos\theta$ **D** $2FL\cos\theta$

3.3 A uniform metre rule of weight 2.0 N is pivoted at the 60 cm mark. A 4.0 N weight is suspended from one end, causing the rule to rotate about the pivot.



At the instant the rule is horizontal as shown, calculate the resultant turning moment about the pivot.

2.4 TRANSLATIONAL AND ROTATIONAL EQUILIBRIUM

Self-study resources		
A SLS lesson on the equilibrium of extended bodies.	for.edu.sg/equilibrium	

2.4.1 Principle of moments

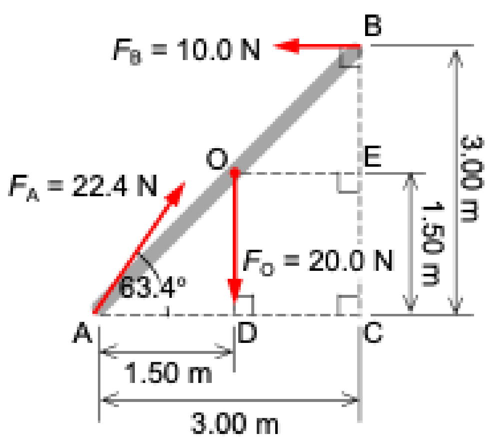
The principle of moment states that, for a body to be in rotational equilibrium, the sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about the same point.

You need to explain this

An object in rotational equilibrium

What does it mean that moments can be taken about any point in the definition of the principle of moment?

Consider a light rod AB in rotational equilibrium under the forces shown. The force F_O = 20.0 N acts at the midpoint O of the rod vertically downwards, the force F_A = 22.4 N acts at an angle of 63.4° to the horizontal at the end A and the force F_B = 10.0 N acts horizontally at the end B.



Let look at the total clockwise and anticlockwise moments of the forces about two points on the rod (A and B) and a point outside the rod (C).

taking moment about point	clockwise moment	anticlockwise moment
A	$F_O \times$ AD = (20.0)(1.50) = **30 N m**	$F_B \times$ BC = (10.0)(3.00) = **30 N m**
B	(F_A sin 63.4°) × AC = (22.4 sin 63.4°)(3.00) = **60 N m**	(F_A cos 63.4°) × BC + $F_O \times$ DC = (22.4 cos 63.4°)(3.00) + (20.0)(1.50) = **60 N m**
C	(F_A sin 63.4°) × AC = (22.4 sin 63.4°)(3.00) = **60 N m**	$F_B \times$ BC + $F_O \times$ DC = (10.0)(3.00) + (20.0)(1.50) = **60 N m**

From the calculations above, it can be seen that when the object is in rotational equilibrium, the **total clockwise and anticlockwise moments are equal regardless of the point the moments are taken from**!

2.4.2 Centre of gravity

Consider a thin uniform rod balanced at a particular point, which is at the midpoint of the rod, with a pivot and it does not turn. The pivot applies an upward force that is equal in magnitude to the weight of the rod to stop it from falling. Imagine that the rod is made of many equal segments, as shown in Fig. 2.12a. Each segment has a fraction of the total weight of the rod and, to be in balance, the sum of the clockwise and anticlockwise moments due to these fractional weights must be equal.

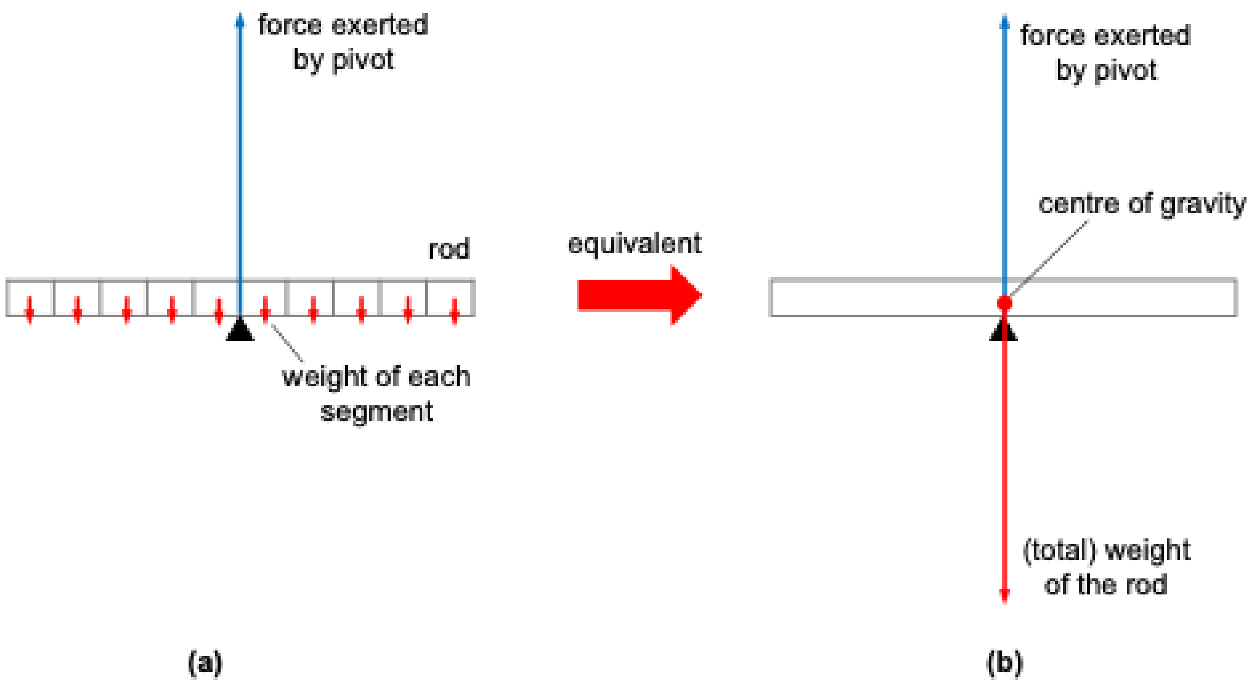


Fig. 2.12

Although all parts of the rod have weight, the whole weight of the rod appears to act at the balance point, as shown in Fig. 2.12b. This point is known as the **centre of gravity** of the rod.

The centre of gravity of a body is the point at which the whole weight of the body appears to act.

You need to explain this

In general, all objects has their own centres of gravity.

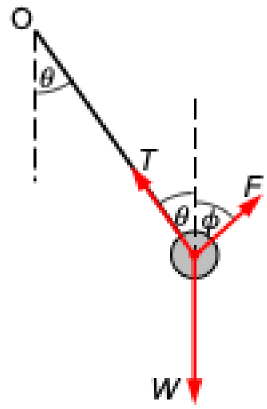
2.4.3 Equilibrium

The principle of moments gives the condition necessary for a body to be in rotational equilibrium. However, the body could still have a resultant force acting on it which would cause it to accelerate linearly. Therefore, to be in equilibrium, there must be zero resultant force acting on the body.

For a body or a system to be in equilibrium, there is no resultant force and no resultant torque acting on the body or the system.

You need to explain this

How do I make use of the condition "resultant force = zero"?

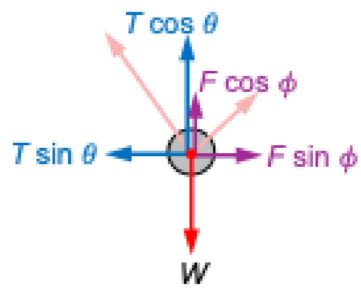


Consider a small bob of weight W suspended from a fixed point O by a thin light thread, as shown in Fig. 2.13. A force F is applied at an angle ϕ to the vertical so that the thread makes an angle θ to the vertical when box is at rest. A force T is exerted on the bob by the string.

Approach 1: Vector sum of all forces in any direction must be zero.

Procedure: Resolved all forces acting on the object in the chosen two perpendicular directions.

We choose the horizontal and vertical directions for the resolution of all the forces of Fig. 2.13 as shown.



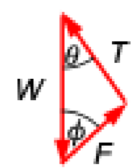
horizontal: $F \sin \phi - T \sin \theta = 0$

vertical: $T \cos \theta + F \cos \phi - W = 0$

Approach 2: The forces form a closed polygon.

Procedure: Arrange the forces such that the tail of the subsequent vector is connected to the head of the previous vector.

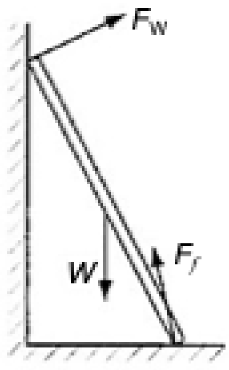
We arrange the force vectors in Fig. 2.13 and since there is not resultant force, the vectors form a closed triangle as shown.



Note:
1. This method is appropriate if there are only three forces acting on the body so that a closed triangle is produced.
2. If two of three forces are at right angle to one another, this method is usually faster than resolution of forces because we can apply Pythagoras theorem and simple trigonometry (otherwise, you will need to use cosine and sine rules).

How do I make use of the condition "resultant torque = zero"?

A ladder, resting on a rough floor and leaning against a rough wall, is on the point of slipping. It is of weight W and the contact forces exerted on the ladder by the wall and floor are F_W and F_f respectively, as shown in Fig. 2.14.



Since the ladder is at rest, we know that both the resultant force and resultant torque is zero. For the latter condition, we have two approaches.

Approach 1: Apply principle of moments.

Step 1: Inspect the question for the information on the unknown forces.

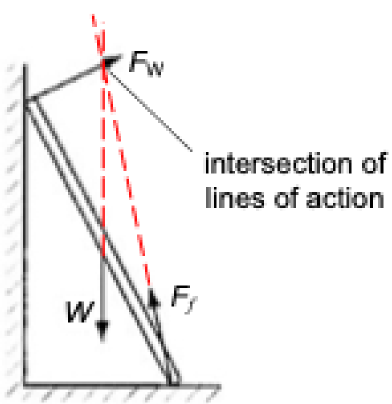
Step 2: Determine the priority, e.g. we may have more information to find F_W then F_f.

Step 3: Apply principle of moments by taking moment about the point where the line of action of the lower priority force passes through (so the moment due to this force is zero).

Approach 2: The lines of action of three non–parallel forces pass through the same point in space.

For three non-parallel forces acting on a body and it is in equilibrium, the lines of action of the three forces will have a common point of intersection. If we take moments about this point, the total moment of the body is zero and, according to the principle of moments, the sum of the clockwise and anticlockwise moments about any other points will be equal.

For Fig. 2.14, when we draw the lines of action of the forces, they will intersect at a single point as shown.

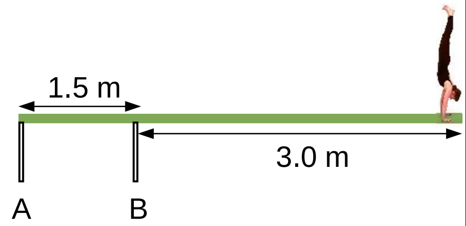


Note: This method can only work for three non-parallel forces situations.

Example 2.5

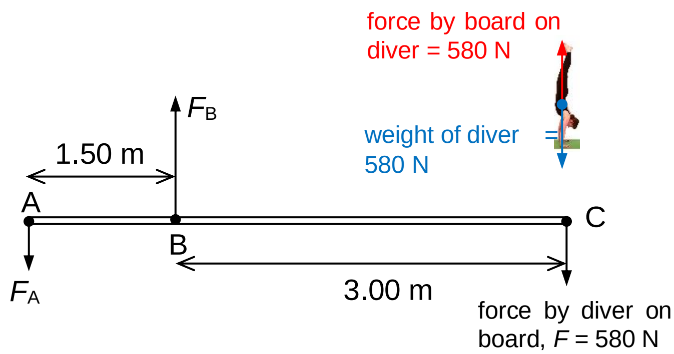
The figure shows a diver of weight 580 N standing at the end of a 4.5 m diving board of negligible weight. The board is attached by two pedestals 1.5 m apart.

Determine the force in each of the two pedestals.



Solution

The force diagram of the diving board is shown below.



Analysis: There are two unknown forces F_A and F_B. To calculate F_A first, we will take moments about point B so that F_B produces zero moment. (You may also calculate F_B first by taking moment about point A.)

Take moments about B and apply principle of moments,

sum of clockwise moments = sum of anticlockwise moments

$$F \times BC = F_A \times AB$$

$$580 \times 3.00 = F_A \times 1.50$$

$$\therefore F_A = 1160 \text{ N}$$

There are two approaches to calculate F_B.

Method 1 – Resultant force = 0

Since vector sum of all forces in any direction must be zero,

$$F_B - F_A - F = 0$$

$$F_B = F_A + F$$

$$F_B = 1160 + 580$$

$$\therefore F_B = 1740 \text{ N}$$

Method 2 – Resultant torque = 0

Take moment about A and apply principle of moments,

sum of clockwise moments = sum of anticlockwise moments

$$F \times AC = F_B \times AB$$

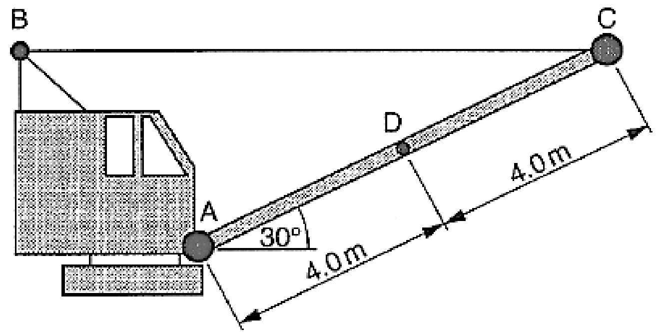
$$580 \times 4.50 = F_B \times 1.50$$

$$\therefore F_B = 1740 \text{ N}$$

Note: You may also take moments about C.

Example 2.6

A crane has a jib AC of mass 2500 kg pivoted to the cab at A. The jib is supported by horizontal cable BC. The centre of mass of the jib is at D. The dimensions are as shown on the diagram.



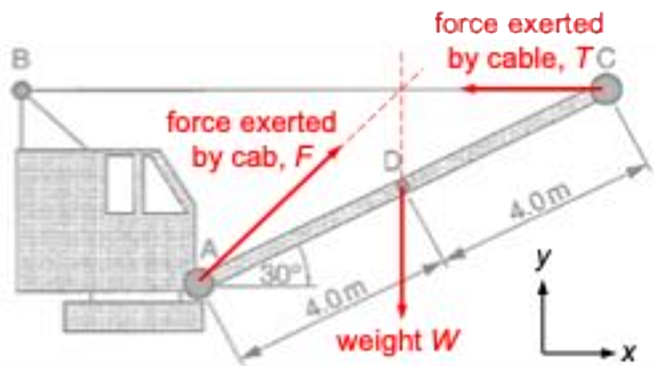
(a) Draw arrows to represent the forces acting on the jib. Label the forces clearly.

(b) Determine the tension in the cable BC and the force by the cab on jib at A.

Solution

(a) The jib has two points of contact at A (between the cab and the jib) and at C (between the cable and the jib). In addition the gravitational force (weight) acts at D. Therefore, we expect three forces acting on the jib as shown.

Note: The lines of action of the three forces must intersect a common point, as shown.



(b) Analysis: There are two unknown forces F and T. To calculate T first, we will take moments about point A (You may also calculate F first by taking moment about point C.)

Take moments about A and apply principle of moments,

sum of clockwise moments = sum of anticlockwise moments

$$W \times AD \cos 30^\circ = T \times AC \sin 30^\circ$$

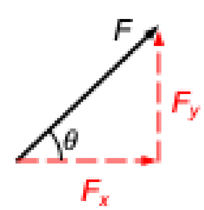
$$(2500 \times 9.81) \times 4.0 \cos 30^\circ = T \times 8.0 \sin 30^\circ$$

$$\therefore T = 21239 = 21000 \text{ N (2 s.f.)}$$

Since vector sum of all forces in any direction must be zero,

x-direction: $F_x - T = 0 \Rightarrow F_x = T$

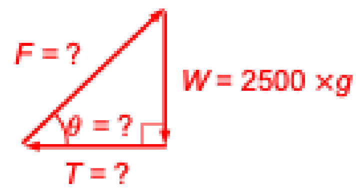
y-direction: $F_y - W = 0 \Rightarrow F_y = W$



$\therefore F = \sqrt{F_x^2 + F_y^2} = \sqrt{21239^2 + (2500 \times 9.81)^2} =$ 32000 N (2 s.f.)

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{2500 \times 9.81}{21239} = 49.1^\circ$$

Note: *W* and *T* are perpendicular, so the three forces will form a closed right-angled triangle which is a tempting approach. However, when you analyse this method further, you will see that it will not work because there are too many unknown quantities – *T*, *F* and θ, as shown.

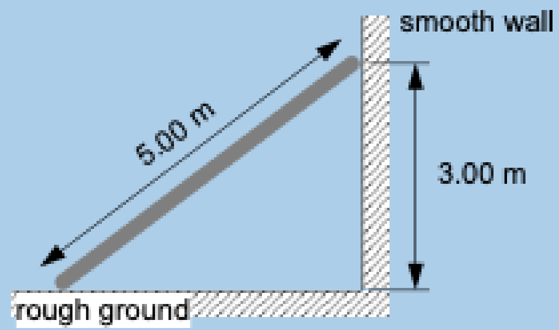


Class exercise

A uniform ladder is of length 5.00 m and mass 2.00 kg rests against a wall. Its upper end is a height of 3.00 m above the ground.

The wall is smooth, and the ground is rough.

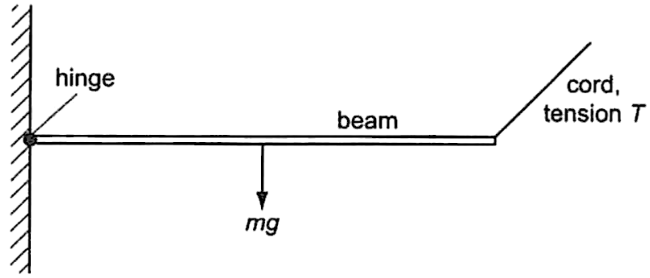
Determine the forces exerted by the wall and by the ground on the ladder.



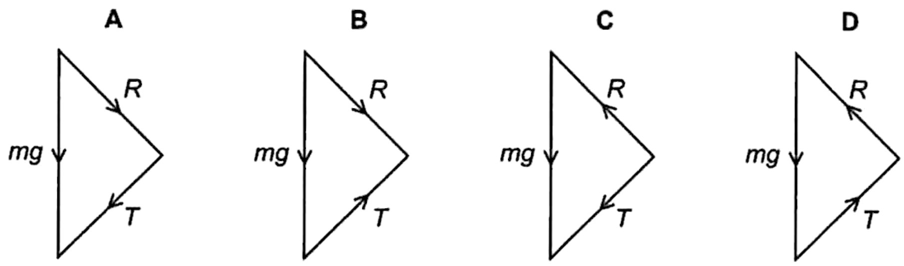
Exercise 4: Equilibrium

You should take about 30 minutes to complete this exercise.

4.1 A horizontal beam of weight mg is attached to a hinge at one end. At the other end it is supported in equilibrium by a cord which is under tension T. The hinge experiences a reaction force R.



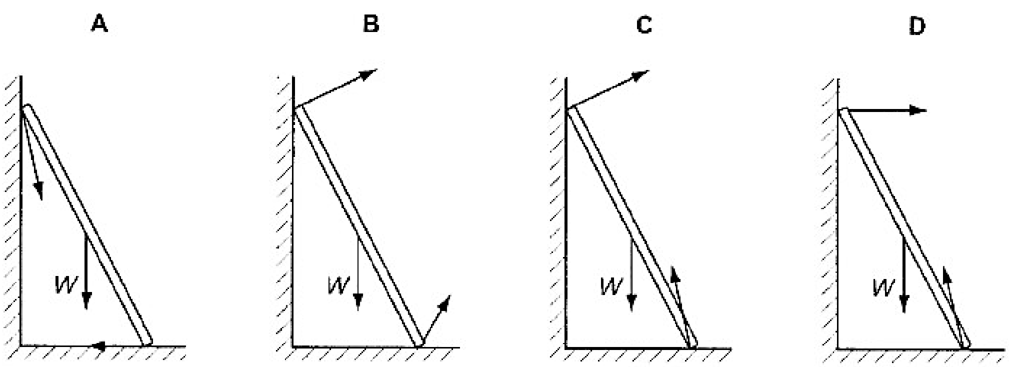
Which triangle of forces could correctly represent the three forces acting on the beam?



4.2 [2008 P1 Q7] A ladder of weight W rests against a vertical wall. The diagram shows two equal and opposite forces applied to the ends of a pivoted bar of length L.

Friction between the ladder and the ground and also between the ladder and the wall prevents the ladder from slipping.

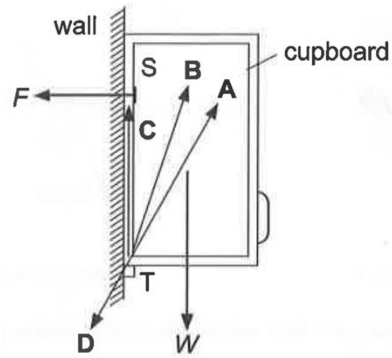
Which diagram shows the directions of the forces on the ladder?



4.3 [2012 P1 Q8]

A cupboard is secured to a wall by a screw at S. It rests on a support at T. Arrows show the directions of the weight *W* of the cupboard and the horizontal force *F* of the screw at S on the cupboard.

Which arrow best represents the direction of the force on the cupboard from the support T?



4.4 [2013 P1 Q8]

In order to pull a protruding tooth back into line, a metal band is attached to the tooth, as shown in diagram 1.

Diagram 2 shows the two forces of 2.5 N which the band exerts on the tooth. The angle between the two parts of the band is 150°.

This treatment makes the tooth move so slowly in the gum that it is effectively in equilibrium.

For equilibrium to exist, another force must act.

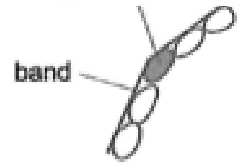


diagram 1

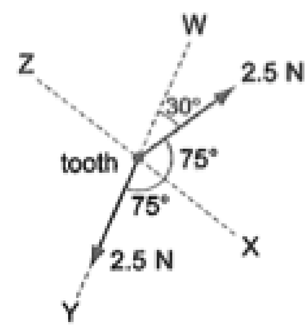


diagram 2

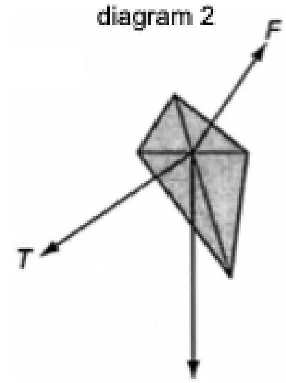
What is the magnitude and direction of this force?

	magnitude	direction
A	0.33 N	W
B	0.33 N	Y
C	1.3 N	X
D	1.3 N	Z

4.5 A stationary kite is held in equilibrium by three forces – its weight *W*, the tension *T* in the string and the force *F* exerted by the air.

The diagram shows the directions of the forces but they are **not** drawn to scale.

Which statement about the forces is correct?



A *F* must be the largest force.

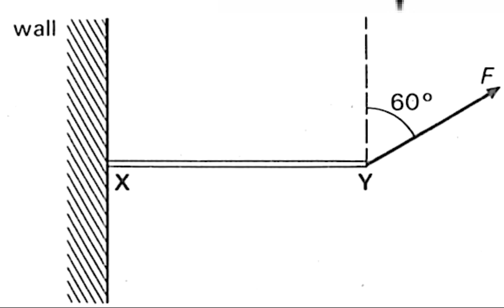
B *T* must be larger than *F*.

C *W* must be larger than *F*.

D *W* must be the smallest force.

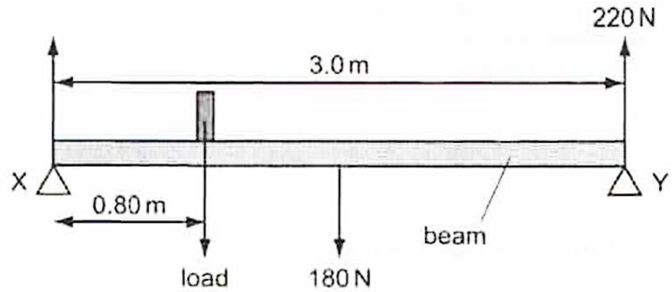
4.6 A uniform rod XY of weight 10.0 N is freely hinged to a wall at X. It is held horizontal by a force *F* acting from Y at an angle of 60° to the vertical as shown in the diagram.

What is the value of *F*?



A	20.0 N	**B**	10.0 N
C	8.66 N	**D**	4.33 N

4.7 [2010 P1 Q9] A uniform beam in a roof structure has a weight of 180 N. It is supported in two places X and Y, a distance 3.0 m apart. A load is placed on the beam a distance of 0.80 m from X. The support provided by Y is 220 N.

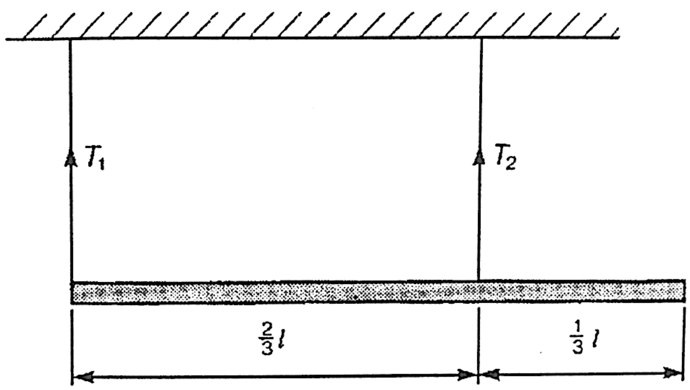


What is the value of the load?

A 270 N **B** 490 N

C 520 N **D** 830 N

4.8 A heavy uniform beam of length *l* is supported by two vertical cords as shown.



What is the ratio $\frac{T_1}{T_2}$ of the tensions in these cords?

A $\frac{1}{3}$ **B** $\frac{1}{2}$ **C** $\frac{2}{1}$ **D** $\frac{3}{1}$

4.9 A rod of length 1 metre has non–uniform composition, so that the centre of gravity is not at its geometric centre. The rod is laid on supports across two top–pan balances as shown in the diagram. The balances (previously set at zero) give readings of 360 g and 240 g.

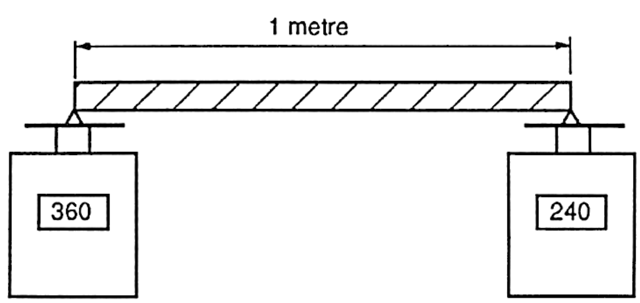
What is the centre of gravity of the rod relative to its geometric centre?

A $\frac{1}{10}$ to the left

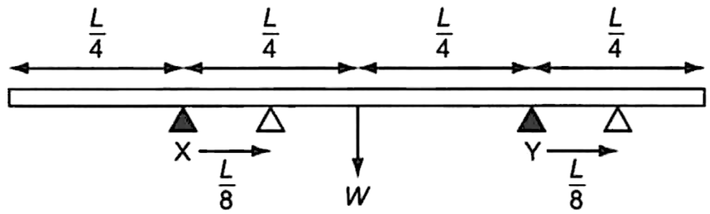
B $\frac{1}{10}$ metre to the right

C $\frac{1}{5}$ metre to the right

D $\frac{1}{5}$ metre to the left



4.10 A uniform plank, of weight W and length L is supported at points X and Y, each at distances $\frac{L}{4}$ from the ends of the plank.



What will be the increase of the force on the plank exerted by support X if both X and Y are moved a distance $\frac{L}{8}$ to the right from their original positions?

A $\frac{W}{16}$ **B** $\frac{W}{8}$ **C** $\frac{W}{4}$ **D** $\frac{5W}{16}$

2.5 Upthrust (H2 Physics only)

When an object is immersed in a fluid (a liquid or a gas), it appears to weigh less than in a vacuum because the immersion in the fluid provides a force of **upthrust** or **buoyant force** on the object.

- Force of upthrust U is an **upward force** on a body partially or fully immersed in a fluid (gas or liquid) **due to the pressure difference** in the fluid.
- It is proportional to the volume and density of fluid displaced by the body.

Principle of flotation

For an object floating in equilibrium and in the absence of other forces, upthrust is equal in magnitude and opposite in direction to the weight of the object.

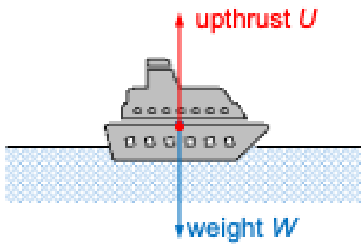


Fig. 2.18

This is also known as the *principle of flotation*.

Representing force of upthrust in force diagrams

The arrow representing the force of upthrust is **drawn from the centre of gravity of the immersed section** and the direction is **vertically upwards**, as shown in Fig. 2.19. The position of the upthrust is critical in the stability of ships as shown in Fig. 2.20.

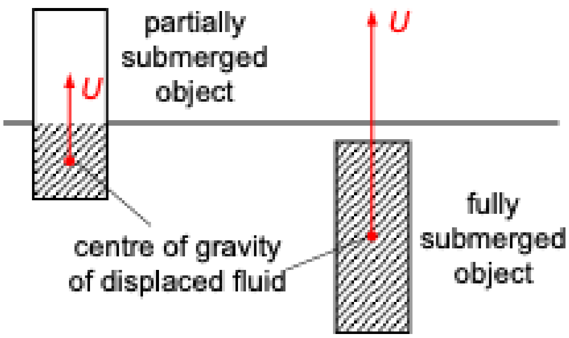


Fig. 2.19

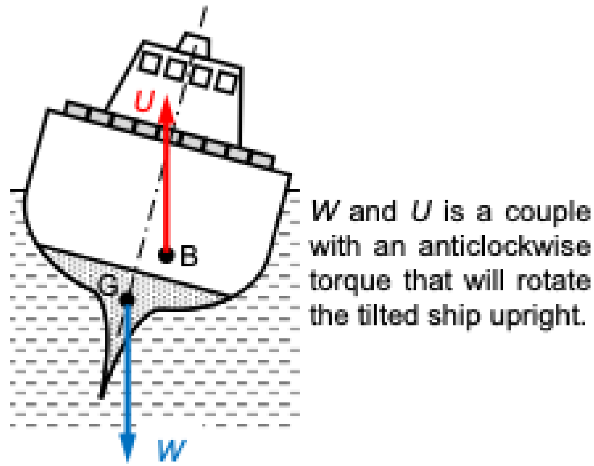


Fig. 2.20

Example 2.8

A solid, square wood raft measures 4.0 m by 4.0 m and is 0.30 m thick. The upthrust is measured to be 26000 N.

Explain, with relevant calculations, why the raft is floating in water.

[density of wood = 550 kg m^{-3}; density of water = 1000 kg m^{-3}]

Solution

Weight of raft = $\rho_{wood}\, V_{raft}\, g$ = 550 × (4 × 4 × 0.30) × 9.81 = 25898 N = 26000 N (to 2 sf)

Since upthrust = weight of raft, the raft is floating in water.

Solutions to Exercises

Exercise 1: Hooke's law

1.1 Answer: A

Point A is the limit of proportionality.

1.2 Force constant = gradient = $\frac{62-0}{0.050-0} = 1240 \text{ N m}^{-1}$

1.3 (a) Take two sets of co-ordinates taken to determine a constant value for *F/x*

e.g. when *x* = 0.6 cm, *F* = 1.5 N, so $k_1 = \frac{1.5}{0.006} = 250 \text{ N m}^{-1}$

when *x* = 1.8 cm, *F* = 4.5 N, so $k_2 = \frac{4.5}{0.018} = 250 \text{ N m}^{-1}$

since *F* /*x* constant hence obeys Hooke's law

(b) use one point on the line (e.g. use *x* = 1.8 cm, *F* = 4.5 N, so $k_2 = \frac{4.5}{0.018} = 250 \text{ N m}^{-1}$)

$k = 250 \text{ N m}^{-1}$

1.4 (a) When mass is zero, force on spring is zero and extension is zero. .

The variation of the extension with load or mass is a straight line graph passing through the origin.

So, spring obeys Hooke's law.

(b) Use of the gradient (not *F* = *kx*)

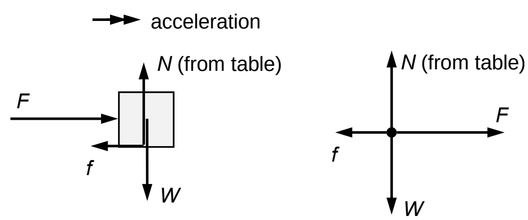
$k = (0.4 \times 9.81)/15 \times 10^{-2}$

$= 26 \text{ N m}^{-1}$ (2 s.f.)

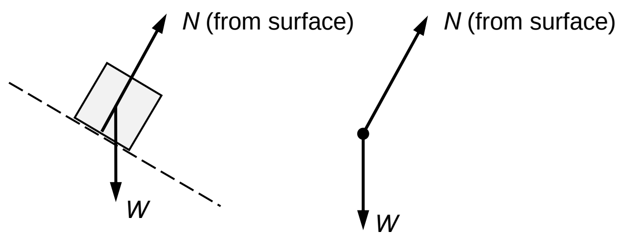
1.5 Straight line does not go through the origin, so the force is not proportional to extension. Spring does not obey Hooke's law.

Exercise 2: Drawing force diagrams

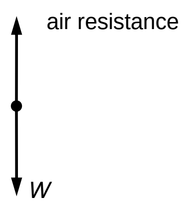
2.1 Use the direction of acceleration to figure the magnitude of the forces



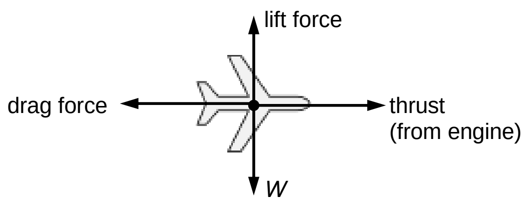
2.2



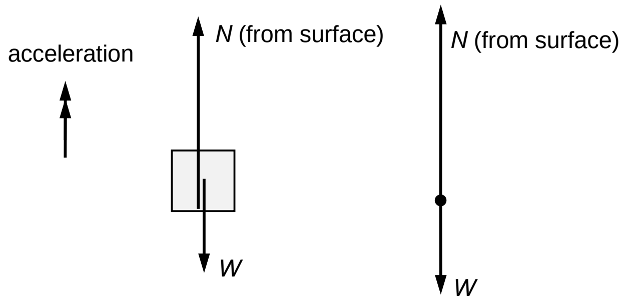
2.3 Falling at constant velocity means moving at terminal velocity, so resultant force is zero



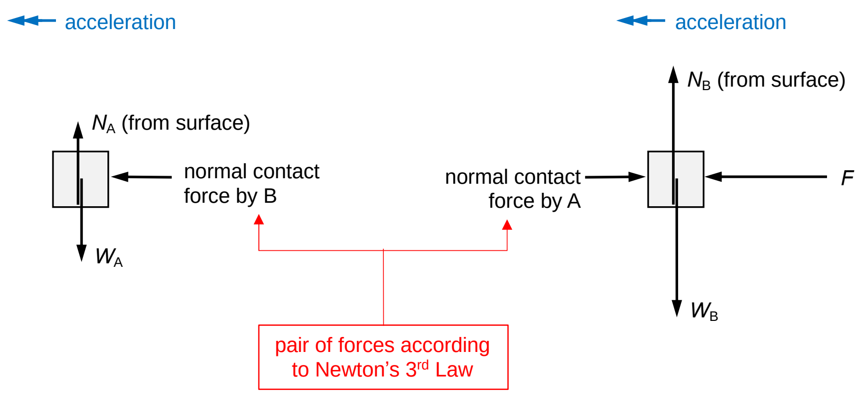
2.4



2.5 Box is at rest wrt to the surface means that the box and the surface have the same acceleration



2.6 The applied force will cause both the boxes to accelerate with the same acceleration to the left. Hence, when the FBD is drawn separately, they must show that there is a resultant force for each body so that both can have the same acceleration.



Exercise 3: Turning effects of forces

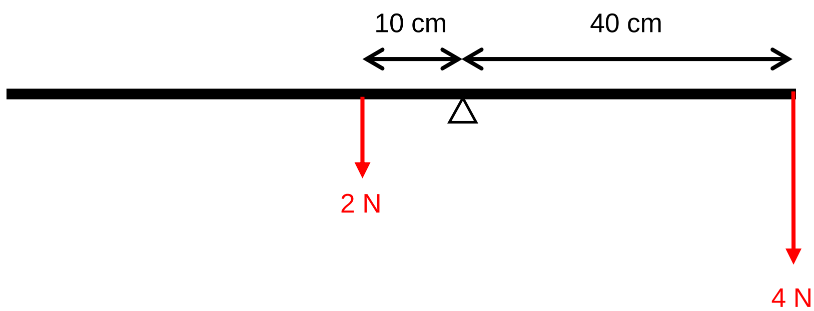
3.1 Moment = $F \sin 30^{o} \times 2.0 = 3.0$ N m

Moment = $F \sin 20^{o} \times 6.0 = 10$ N m

3.2 **Answer: C**

Perpendicular distance between couple $= L \cos \theta$

3.3 Weight of rule and force exerted by the suspended weight are acting on the beam as follow



Take moments about the pivot,

clockwise moment $= 4 \times 0.4 = 1.6 \text{ N m}$

anti–clockwise moment $= 2 \times 0.1 = 0.2 \text{ N m}$

resultant turning moment $= 1.6 - 0.2 = 1.4 \text{ N m (clockwise)}$

Exercise 4: Equilibrium

4.1 Answer: D

4.2 Answer: C

Option A has a resultant downward force,

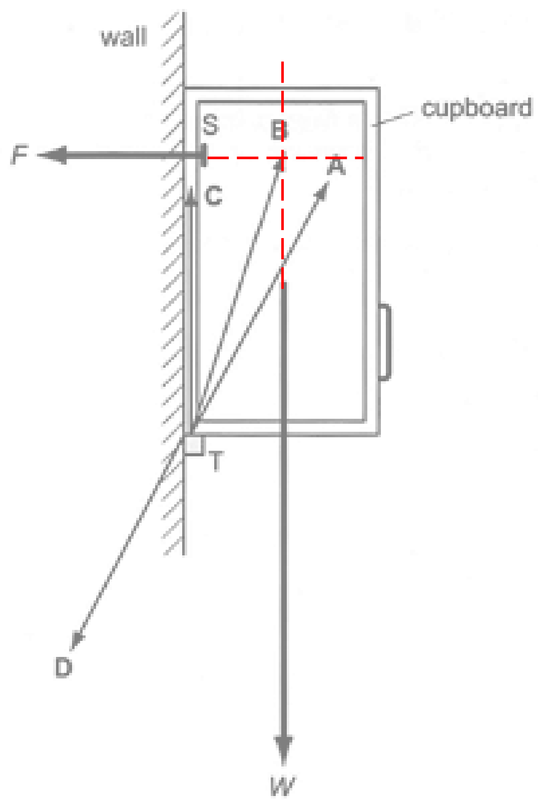
Option B has resultant rightward force.

Option D does not have friction between the wall and the ladder but the question indicated that there is friction

Therefore, by elimination, correct answer is C (note: lines of action of the forces also pass through a common point).

4.3 Answer: B

Lines of action of the forces also pass through a common point.



4.4 Answer: D

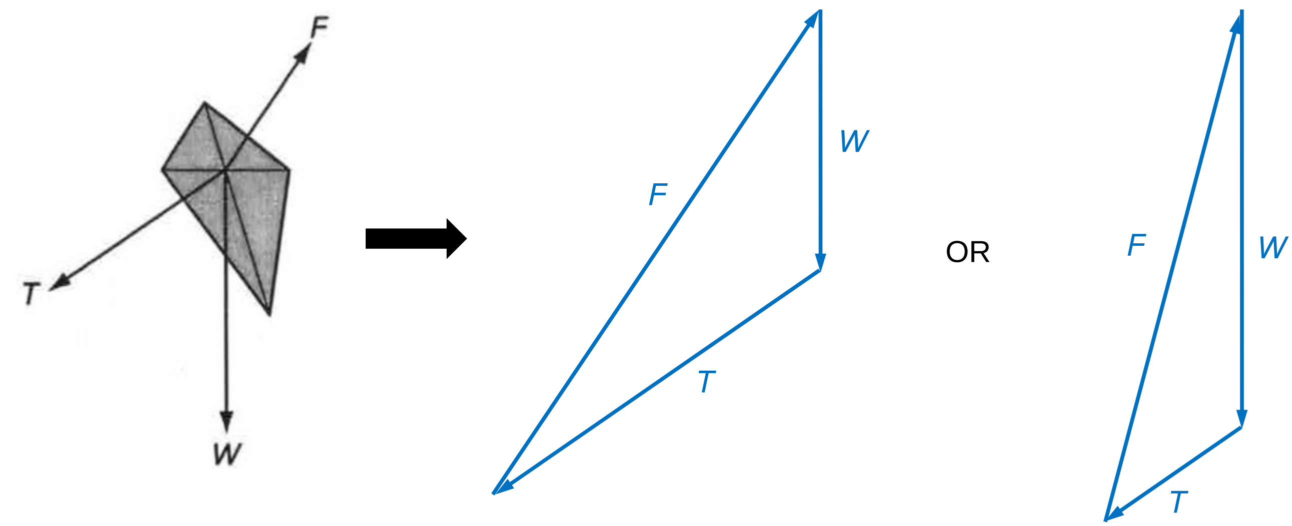
Resultant of the two 2.5 N forces is in direction towards X. So, the third force should be towards Z.

There is no need to calculate the magnitude of the force since you can arrive at the answer through elimination!

4.5 Answer: A

In the question, the diagram shows "the directions of the forces" but "not drawn to scale", so the diagram indicates the general directions of the forces but will vary depending on the magnitude of the forces

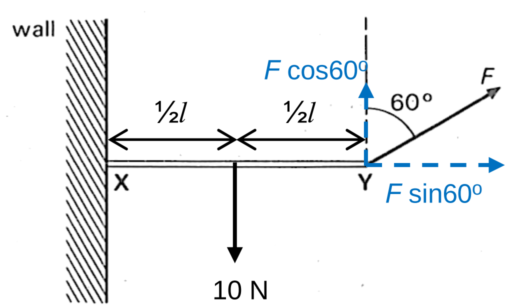
Kite is in equilibrium, so the three forces must form a closed polygon.



From the possible force triangles, it is quite clear that *F* is the largest. *W* can be smaller or larger than *T*.

4.6 Answer: B

Weight of rod acts at the mid–point of the length *l* of the rod. Resolve *F* as shown.



Rod is in equilibrium, apply principle of moments and take moments about X,

$$10 \times \frac{1}{2} l = F \cos 60^{o} \times l$$

$$F = 10 \text{ N}$$

Note: line of action of the component Fsin60^o passes through X so zero moments.

4.7 Answer: B

Force at X is unknown.

Beam is in rotational equilibrium, so take moments about X and apply principle of moments

sum of clockwise moments = sum of anticlockwise moments

load × 0.8 + 180 × 1.5 = 220 × 3

load = 490 N

4.8 Answer: A

– three forces – T_1, T_2 and weight – act on the beam

– since beam is uniform, centre of gravity is ½*l* from either ends

– beam is in rotational equilibrium, take moments about C.G and apply principle of moments

sum of clockwise moments = sum of anticlockwise moments

$$T_1 \times \frac{l}{2} = T_2 \times \left(\frac{l}{2} - \frac{l}{3}\right)$$

$$\frac{T_1}{T_2} = \frac{1}{6} \times 2 = \frac{1}{3}$$

Note: weight acts at the centre of gravity hence does not contribute any moment.

4.9 Answer: A

– three forces act on the rod – N_1 at the left balance, N_2 at the right balance and weight; $N_1 = 0.36g\,\text{(upwards)}$, $N_2 = 0.24g\,\text{(upwards)}$ and weight $= 0.6g\,\text{(downwards)}$

– rod is not uniform, let the centre of gravity (c.g.) be distance *x* from the left balance

– rod is in rotational equilibrium, so take moments about the left balance and apply principle of moments sum of clockwise moments = sum of anticlockwise moments

$$0.6g \times x = 0.24g \times 1$$

$$x = 0.4 \text{ m from the left}$$

– so the c.g. is 0.1 m left of the geometric centre

4.10 Answer: C

– three forces act on the plank – F_X upwards, F_Y upwards and W downwards

– since plank is uniform, centre of gravity is ½L from either ends

– plank is in rotational equilibrium, so take moments about Y and apply principle of moments

sum of clockwise moments = sum of anticlockwise moments

$$F_X \times \frac{L}{2} = W \times \frac{L}{4}$$

$$F_X = \frac{W}{2}$$

– when the supports are shifted, plank is still in rotational equilibrium, so take moments about Y and apply principle of moments

sum of clockwise moments = sum of anticlockwise moments

$$F_X' \times \frac{L}{2} = W \times \left(\frac{L}{4} + \frac{L}{8}\right)$$

$$F_X' = \frac{3}{4}W$$

– increase in the force $= \frac{3W}{4} - \frac{W}{2} = \frac{W}{4}$