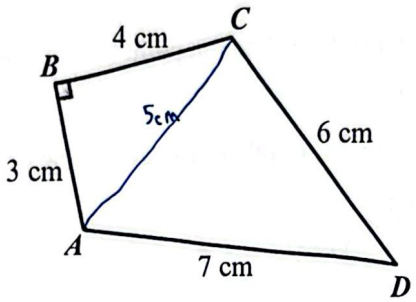
Answer 4 out of 6 questions

1 **(a)** Given that $x-y=2$, find the value of $x^4-y^4-2x^3y+2xy^3-16y$. [4]

(b) Simplify $\dfrac{3^{2x+1}+4(9)^x}{3^{2-3x}\times 9^{2x-1}}$ and express it in the form $m\left(n^x\right)$ where m and n are positive integers. [3]

(c) Quadrilateral $ABCD$ has side lengths 3 cm, 4 cm, 6 cm and 7 cm respectively. Given that $\angle ABC=90°$, find the exact area of the quadrilateral. [3]



2 **(a)** Let $\mathbf{M}$ be a matrix such that $\begin{pmatrix}3 & -4\\ 0 & 2\end{pmatrix}\mathbf{M}=\begin{pmatrix}4 & 0\\ 3 & 6\end{pmatrix}-\mathbf{M}$.

(i) State the order of $\mathbf{M}$. [1]

(ii) Determine $\mathbf{M}$ by solving the equation. [3]

(b) The circle $x^2+y^2=4$ and the parabola $y^2=ax+4$ intersect at exactly n points, where a is a non-zero real number and n is a positive integer.

(i) Find the value(s) of a if $n=3$. [4]

(ii) Write down all possible values of n. [2]

3 **(a)** Factorise $(x-2y)^3-8y^3-(x-4y)^3$ completely. [4]

(b) A polynomial $(x+2)^8$ leaves a remainder $R(x)$ when it is divided by $(x+1)(x+2)$. Find the remainder $R(x)$. [3]

(c) The variables x and y are related by the equation $x+y+axy=0$, where a is a constant. When $\dfrac{1}{y}$ is plotted against $\dfrac{1}{x}$, a straight line is obtained. Given that the straight line passes through point (2, 1) and makes an angle of 135° with the positive $\dfrac{1}{x}$-axis, find the value of a. [3]

4 (a) By using a suitable substitution, or otherwise, find the exact values of x satisfying the equation $\ln x + e(\log_x e) = 1 + e$. [5]

(b) (i) Simplify $\sqrt{13 - 4\sqrt{10}}$, leaving your answer in the form $a\sqrt{2} - \sqrt{b}$, where a and b are positive integers. [2]

(ii) Hence or otherwise, simplify $\left(\sqrt[4]{13 - 4\sqrt{10}} + \sqrt[4]{13 + 4\sqrt{10}}\right)^4$. [3]

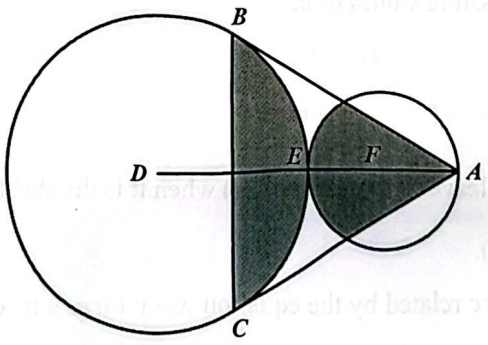
5 (a) A point $P(x, y)$ moves along the parabola $y = x^2 + 1$. The distance from P to the x-axis is twice the distance from P to the point (0, 5). Find the coordinates of all possible positions of P. [4]

(b) Using substitution $y = 2x - 1$, or otherwise, determine the range of values of x satisfying $\frac{4x^2 - 4x + 5}{5} \leq \frac{1}{2x - 1}$. [4]

(c) The graph $y = \ln x^2$ where $x > 0$ is obtained from the graph of $y = e^x$ after two successive transformations. Describe the transformations in sequence clearly. [2]

6 (a) Solve the equation $(\log_2 x)^3 = 3\log_8 x$. Leave your answers in exact form. [4]

(b) In the diagram below, a circle with centre D and radius 4 cm is externally tangent to a circle with centre F and radius 2 cm at point E. Line DF is extended and intersects the circle at A again. From point A, two tangent lines are drawn such that it meets the circle with centre D at B and C respectively.



(i) Explain why triangle ABC is an equilateral triangle. [2]

(ii) Find the exact area of the shaded region, overlapped by the triangle ABC and the two circles. Leave your answer in the form $\left(a\pi + b\sqrt{3}\right)$ cm^2 where a and b are constants. [4]

End of Paper

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Q	1	2	3	4				
M	7	1	1	5				

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1. (a) $x^4-2x^3y+2xy^3-y^4-16y=0$

$y=x-2$

$x^4-y^4-2x^3y+2xy^3-16y = x^4-(x-2)^4-2x^3(x-2)+2x(x-2)^3-16(x-2)$

$=[x^2+(x-2)^2][x^2-(x-2)^2]-(16+2x^3)(x-2)+2x(x-2)^3$

$=[x^2+x^2-4x+4][(x^2+x-2)(x^2-x+2)]-(2x^4-4x^3+16x-32)+2x(x-2)^3$

$=2(x^2-2x+2)(x+2)(x-1)(x-2)(x+1)-2x^4+4x^3-16x+32+2x(x^3-6x^2+12x-8)$

1

(b) Let 3^x be y

$\frac{3^{2x+1}+4(3^{2x})}{3^{2-3x}\times 9^{2x-1}} = \frac{3y^2+4y^2}{3^{2-3x}\times 3^{4x-2}}$

$=\frac{7y^2}{3^x}$

$=\frac{7y^2}{y}$

$=7y$

$=7(3^x)$

3

(c) $AC=\sqrt{3^2+4^2}$

$=5$

$s_1=\frac{3+4+5}{2}$

$=6$

$s_2=\frac{5+6+7}{2}$

$=9$

Area $=\sqrt{6(6-3)(6-4)(6-5)}+\sqrt{9(9-5)(9-6)(9-7)}$

$=\sqrt{36}+\sqrt{216}$

$=(6+\sqrt{216})\ cm^2$

$=6+6\sqrt{6}$

3

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Q								
M								

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2. (a)(i) order of $M = 2 \times 2$ ✓ 1

(ii) ? 0

(b) (i) ? 0

(ii) ? 0

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3. (a) $(x-2y)^3-8y^3-(x-4y)^3 = (x-2y)^3-(2y)^3-(x-4y)^3$

$= (x-4y)(x^2-4xy+4y^2+2xy-4y^2+4y^2)-(x-4y)^3$

$= (x-4y)(x^2-2xy+4y^2)-(x-4y)^3$

(b) $(x+2)^8 \div (x+1)(x+2) = \frac{(x+2)^7}{x+1}$

$x+1$ $R(x) = (-1+2)^7$

$= 1$

(c) $x+y+axy=0$

$y+axy=-x$

$y(1+ax)=-1$

$-\frac{1}{y} = 1+ax$

$\frac{1}{y} = -ax-1$

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Q								
M								

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4. (b)(i) $\sqrt{13-4\sqrt{10}} = \sqrt{(\sqrt{5})^2+(\sqrt{8})^2-2(\sqrt{5})(\sqrt{8})}$

$= \sqrt{(\sqrt{8}-\sqrt{5})^2}$ ✓

$= \sqrt{8}-\sqrt{5}$

$= 2\sqrt{2}-\sqrt{5}$ ✓

2

(ii) $\left(\sqrt[4]{13-4\sqrt{10}}+\sqrt[4]{13+4\sqrt{10}}\right)^4 = \left(\sqrt[4]{(\sqrt{8}-\sqrt{5})^2}+\sqrt[4]{(\sqrt{8}+\sqrt{5})^2}\right)^4$

$= \left(\sqrt{\sqrt{8}-\sqrt{5}}+\sqrt{\sqrt{8}+\sqrt{5}}\right)^4$ ✓

$= \left(\sqrt{8}-\sqrt{5}+2\sqrt{8-5}+\sqrt{8}+\sqrt{5}\right)^2$ ✓

$= (2\sqrt{8}+2\sqrt{3})^2$

$= (4\sqrt{2}+2\sqrt{3})^2$ ✓

$= 32+16\sqrt{6}+12$

$= 16\sqrt{6}+44$ ✓

3

(a) 7

0

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