Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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①

1 Factorise $27a^3 - 343$. [2]

$+21a$

$27a^3 - 343 = (3a-7)(9a^2 - \quad + 49)$

$21a$

$a^3 - b^3 = (a-b)(a^2 - ab + b^2)$

$+$

$27a^3 - 343 = (3a-7)(9a^2 + 21a + 49)$

②

2 Solve the inequality $x - 1 > 3(x^2 - 5)$. [3]

$x - 1 > 3(x^2 - 5)$

$x - 1 > 3x^2 - 15$

$-3x^2 + x + 14 > 0$

let $-3x^2 + x + 14 = 0$.

$$x = \frac{-1 \pm \sqrt{1^2 - (4)(-3)(14)}}{(2)(-3)}$$

$-2 < x < \frac{7}{3} / 2\frac{1}{3}$

$x < -2$ and $x > 2\frac{1}{3}$

not possible to be < -2

yet $> 2\frac{1}{3}$ at the same time.

3 **(i)** Express $y = x^2 - 6x + 13$ in the form $y = (x+h)^2 + k$, where h and k are constants. [2]

$(x-3)^2$
$= x^2 - 6x + 9$

5

$y = x^2 - 6x + 13$
$y = (x-3)^2 + 4$

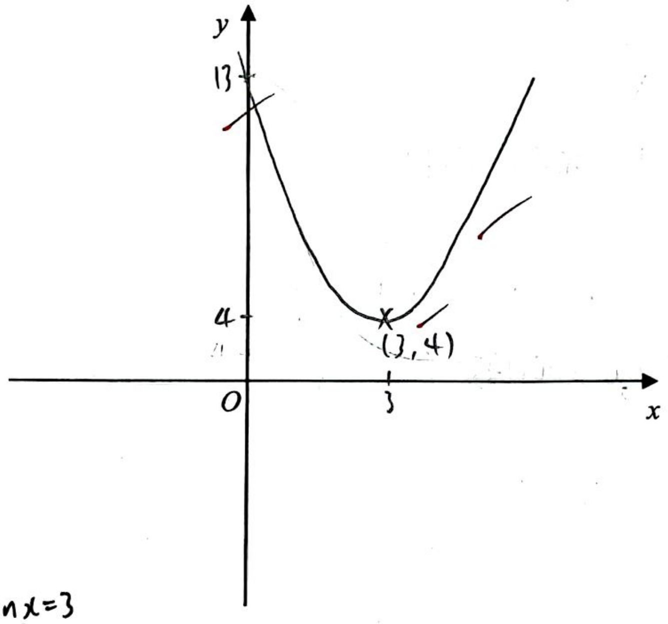
2

(ii) Sketch the graph of $y = x^2 - 6x + 13$, indicating clearly the y-intercept and the **coordinates** of the turning point. [3]

3

When $x = 0$
$y = 0^2 - (6)(0) + 13$
$y = 13$
y-int is $(0, 13)$

~~turning pt~~
[illegible]
~~$= 2$~~
~~coordinates of turning pt~~
[illegible]
~~when $x = 3$~~
~~$y = 3^2 - 6 \times 3 + 13$~~
~~$= 4$~~



When $y = 0$
~~$x^2 - 6x + 13 = 0$~~
[illegible]
~~$(x-3)^2 + 4 = 0$~~
~~$(x-3)^2 = 4$~~
~~$x - 3 = \pm\sqrt{4}$~~
~~$x - 3 = \pm 2$~~
~~$x = 5$ and $x = 1$~~
[illegible]
$(x-3)^2 + 4 = 0$
x is undefined.

when $y = 13$
$(x-3)^2 + 4 = 13$
$x = 0$ and $x = 6$
turning pt $= (6-0) \div 2 = 3$

when $x = 3$
$(3-3)^2 + 4 = y$
$y = 4$

(iii) State the range of values of $\frac{1}{y}$. [1]

0

~~$\frac{1}{y} = \frac{1}{x^2-6x+13}$~~ $y \geq 4$

~~$\frac{1}{y} = (x-3)^2 + 4$~~ ... $\frac{1}{y} \geq \frac{1}{4}$ $0 < \frac{1}{y} \leq \frac{1}{4}$

1

[Turn Over

4 **(a)** Solve the equation $2x^3 - 7x + 2 = 0$. Show your working clearly. [4]

8

4

Let $f(x) = 2x^3 - 7x + 2$

~~$f(0) = 2(0)^3 - 7(0) + 2$~~

~~$f(-\frac{1}{2}) = 2(-\frac{1}{2})^3 - 7(-\frac{1}{2})$~~

$f(-2) = 2(-2)^3 - 7(-2) + 2$

$= 0$

$x+2$ is a factor

$$
\begin{array}{r}
2x^2-4x+1 \\
x+2\,\overline{)\,2x^3-7x+2} \\
-(2x^3+4x^2) \\
\hline
-4x^2-7x+2 \\
-(-4x^2-8x) \\
\hline
x+2 \\
-(x+2) \\
\hline
0
\end{array}
$$

$2x^2 - 4x + 1 = 0$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - (4)(2)(1)}}{2(2)}$

$x = 1.71$ ~~or~~ and $x = 0.293$ (3sf)

not tally when expanded

$2x^3 - 7x + 2 = 0$

$(x+2)(x-1.71)(x-0.293) = 0$ } should omit this step

$x = -2$, $x = 1.71$ and $x = 0.293$

4 **(b)** Express $\dfrac{9x}{(2x-1)(x+1)^2}$ in partial fractions. [5]

4

$$\frac{9x}{(2x-1)(x+1)^2} = \frac{A}{2x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$A(x+1)^2 + B(x+1)(2x-1) + C(2x-1) = 9x$

Let $x = -1$

$C[2(-1)-1] = 9(-1)$

$-3C = -9$

$C = 3$

Let $x = \frac{1}{2}$

$A(\frac{1}{2}+1)^2 = 9(\frac{1}{2})$

$\frac{9}{4}A = \frac{9}{2}$

$A = 2$

Sub $A=2$, $C=3$

$2(x+1)^2 + B(x+1)(2x-1) + 3(2x-1) = 9x$

$2x^2 + 4x + 2 + B(2x^2 - x + 2x - 1) + 6x - 3 = 9x$

$2x^2 + 4x + 2 + 2Bx^2 - Bx + 2Bx - B + 6x - 3 = 9x$

$2x^2(1+B) + x(10+B) - 1 - B = 9x$

$10 + B = 9$

$B = 9 - 10$

$= -1$

$\therefore A = 2, B = -1, C = 3$

partial fractions form?

$$\frac{9x}{(2x-1)(x+1)^2} = \frac{2}{2x-1} - \frac{1}{x+1} + \frac{3}{(x+1)^2}$$

5 The function $\mathrm{f}(x) = x^4 + x^3 + mx^2 + nx - 6$, where m and n are constants, is exactly divisible by $(x+1)$ and $(x-2)$.

7

(i) Find the value of m and of n. [4]

4

$f(-1) = f(2)$

$f(-1) = (-1)^4 + (-1)^3 + m(-1)^2 + n(-1) - 6$

$= m - n - 6$

$f(2) = 2^4 + 2^3 + (2^2)m + 2n - 6$

$= 4m + 2n + 18$

$\begin{cases} m - n - 6 = 0 & -(1) \\ 4m + 2n + 18 = 0 & -(2) \end{cases}$

from (1)

$m = 6 + n \quad -(3)$

Sub (3) into (2)

$4(6+n) + 2n + 18 = 0$

$24 + 4n + 2n + 18 = 0$

$6n = -42$

$n = -7$

$m = 6 + (-7)$

$m = -1$

$\therefore m = -1, n = -7$

(ii) Using the values of m and n found in part **(i)**,

(a) find the remainder when $\mathrm{f}(x)$ is divided by $(x+3)$, [1]

1

$f(-3) = (-3)^4 + (-3)^3 - 1(-3)^2 - 7(-3) - 6$

$= 60$

(b) determine, showing all necessary working, the number of real roots of the equation $\mathrm{f}(x) = 0$. [3]

2

$(x+1)$ and $(x-2)$ are factors

$$\begin{array}{r}
x^3-x-6 \\
x+1\overline{)x^4+x^3-x^2-7x-6} \\
-(x^4+x^3) \\
\hline
-x^2-7x-6 \\
-(-x^2-x) \\
\hline
-6x-6 \\
-(-6x-6) \\
\hline
0
\end{array}$$

$$\begin{array}{r}
x^2+2x+3 \\
x-2\overline{)x^3-x-6} \\
-(x^3-2x^2) \\
\hline
2x^2-x-6 \\
-(2x^2-4x) \\
\hline
3x-6 \\
-(3x-6) \\
\hline
0
\end{array}$$

$x=\frac{-2\pm\sqrt{8}}{2}$

$x=\frac{-2\pm\sqrt{2^2-(4)(1)(3)}}{2(1)}$ } check again

$x=-\cancel{\tfrac{1}{2}}$ and $x=-\cancel{1\tfrac{1}{2}}$

$\therefore x=-1, x=2, x=-\frac{1}{2}, x=-1\frac{1}{2}$

$x^4+x^3-x^2-7x-6=(x+1)(x-2)(2x+1)(2x+3)$

$f(x)=0$ has ~~4~~ 2 real roots

When $f(x)=0$

$x^2+2x+3=0$

Discriminant $=2^2+4(3)=-8$

Since discriminant <0,

the equation $x^2+2x+3=0$ has no real roots

6 Show that the line $y = 2cx - \frac{15}{4}$ will intersect the curve $y = x^2 + 4x - 3c$ for all values of c. [5]

③

$\begin{cases} y=2cx-\frac{15}{4} & -(1) \\ y=x^2+4x-3c & -(2) \end{cases}$

sub (1) into (2)

$2cx-\frac{15}{4}=x^2+4x-3c$

~~[illegible]~~

~~$x^2-x(-4+2c)-3c+\frac{15}{4}=0$~~

~~Discriminant $=(-4+2c)^2-(4)(1)(-3c+\frac{15}{4})$~~

~~$=16+16c+4c^2+12c-15\frac{3}{4}$~~

~~$=4c^2+28c+\frac{1}{4}$~~

~~$c=\frac{-28\pm\sqrt{28^2-4(4)(\frac{1}{4})}}{2(4)}$~~

~~Discriminant $=(4+2c)^2-(4)(1)(-3c+\frac{15}{4})$~~

~~$=-16-16c-4c^2+12c-15\frac{3}{4}$~~

~~$=-4c^2-4c$~~

PP

$x^2+x(4-2c)-3c+\frac{15}{4}=0$

Discriminant $=(4-2c)^2-4(1)(-3c+\frac{15}{4})$ ✓

$=16-16c+4c^2+12c-15\frac{3}{4}$ 15

$=4c^2-4c+\frac{1}{4}$ ~~[illegible]~~

~~$c=\frac{-(-4)\pm\sqrt{(-4)^2-4(4)(\frac{1}{4})}}{2(4)}$~~

$=4(c-\frac{1}{2})^2-\frac{3}{4}$

$=(2c-1)^2$

~~[illegible]~~

As $(c-\frac{1}{2})^2$ will always be positive, and the constant $\frac{1}{4}$ is also positive, the discriminant ~~[illegible]~~ is positive. Thus ~~y~~ the line $y=2cx-\frac{15}{4}$ will intersect the curve $y=x^2+4x-3c$ for all values of c.

what about $-\frac{3}{4}$

$c^2-c+\frac{1}{4}$

$4c^2-4c+\underbrace{1-\frac{3}{4}}_{\frac{1}{4}}$

reasoning- Since $(2c-1)^2\geq0$ for all values of c, discriminant ≥0, the equation $x^2+(4-2c)x+\frac{15}{4}-3c=0$ has real roots.

conclusion- Hence the line will intersect the curve for all values of c.

7 Find the range of values of h for which the expression $(2h-4)x^2+8x+h$ is always positive for all real values of x. [4]

②

↳ doesn't touch x-axis → no real roots

$(2h-4)x^2+8x+h=0$

~~[illegible]~~

Discriminant ≥ 0 <0 and $2h-4>0$

$8^2-4(2h-4)(h)>0$

$64-8h^2+16h>0$

✓ $-8h^2+16h+64>0$ when $-8h^2+16h+64=0$

$h=\frac{-16\pm\sqrt{16^2-(4)(-8)(64)}}{2(-8)}$

$(h+2)(h-4)>0$

$h<-2$ or $h>4$

(rej bc always +ve) $\longrightarrow$ $(2h-4)x^2+8x+h=0$ has no real roots (need to expln if not no marks)

$\therefore h>4$

~~[illegible]~~ $-2<h<4$ M1 incomplete

8 Find the range of values of x for which $\dfrac{x^2+2}{2x^2-5x-3} < 0$. [3]

$\hookrightarrow \frac{+ve}{-ve} < 0$

	x	-3
x	x^2	$-3x$
$\frac{1}{2}$	$\frac{1}{2}x$	$-\frac{3}{2}$

$$\frac{x^2+2}{2x^2-5x-3} < 0$$

3

~~$\frac{A}{x-3} + \frac{B}{x+\frac{1}{2}} < 0$~~

~~$2[A(x+\frac{1}{2}) + B(x-3)] < 0$~~

since $x^2+2>0$ for all values of x

$2x^2-5x-3<0$

let $2x^2-5x-3=0$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-(4)(2)(-3)}}{2(2)}$$

✓

$-\frac{1}{2}<x<3$

✓

End of Paper

reflection

I think this paper was alright, just the time a bit tight. Happy with my results (I was