**Paper H Sec 3 IP Solutions for students**

| Qn | Solutions |
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| **1.** | (a) Substitute $x = a$ into $f(x) = x^2 - x - 5$ to get<br>$a^2 - a - 5 = 1$<br>$a^2 - a - 6 = 0$<br>$(a-3)(a+2) = 0$<br>$\Rightarrow a = 3$ or $a = -2$ |
| | (b) $\dfrac{x^3 - 27}{(x-3)(x-4)(x+5)} - \dfrac{2x+29}{x^2+x-20}$<br>$= \dfrac{(x-3)(x^2+3x+9)}{(x-3)(x-4)(x+5)} - \dfrac{2x+29}{(x-4)(x+5)}$<br>$= \dfrac{x^2+3x+9-2x-29}{(x-4)(x+5)}$<br>$= \dfrac{x^2+x-20}{(x-4)(x+5)}$<br>$= 1$ |
| **2.** | (a) Substitute $3y = 1 - x$ into $x^2 + 3x - 3y = 4$.<br>$x^2 + 3x - (1-x) = 4$<br>$x^2 + 4x - 5 = 0$<br>$(x-1)(x+5) = 0$<br>$\Rightarrow x = 1$ or $x = -5$<br>$y = 0$ or $y = 2$<br>$\therefore (1,0)$ or $(-5,2)$<br>$AB = \sqrt{(1-(-5))^2 + (0-2)^2}$<br>$= 2\sqrt{10}$ or 6.32 (3 s.f.) or $\sqrt{40}$ units<br>Alternatively, students can make $x$ the subject and substitute into $x^2 + 3x - 3y = 4$ to find $y$ first. The equation will be $9y^2 - 18y = 0$. |
| | (b) $x^2 + (2x+c)^2 = 9$<br>$5x^2 + 4cx + c^2 - 9 = 0$<br>Does not intersect means no real solution of $x$<br>$\Rightarrow$ discriminant $< 0$<br>$(4c)^2 - 4(5)(c^2 - 9) < 0$<br>$\Rightarrow c^2 > 45$<br>$\Rightarrow c > 3\sqrt{5}$ or $c < -3\sqrt{5}$ |

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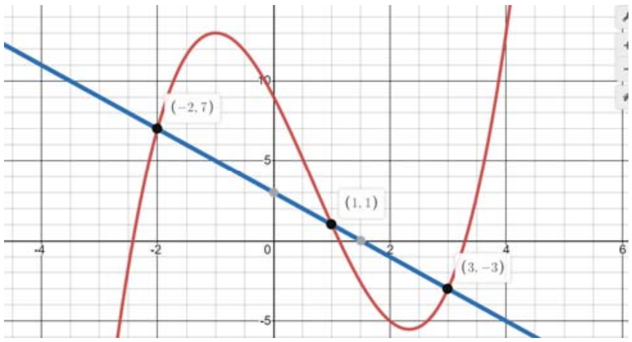
| **3.** | (i) Using the graph of $y = x^3 - 2x^2 - 7x + 9$<br>To solve $x^3 - 2x^2 - 5x + 6 = 0$<br>$\Rightarrow x^3 - 2x^2 - 5x + 6 - 2x + 3 = -2x + 3$<br>$\Rightarrow x^3 - 2x^2 - 7x + 9 = -2x + 3 = y$<br>Hence $y = -2x + 3$ is the graph to be drawn. |
| | (ii)<br><br><br>$x^3 - 2x^2 - 7x + 9 > -2x + 3$<br>From the graph, $-2 < x < 1$ or $x > 3$.<br>Alternatively, students can obtain<br>$(x + 2)(x - 1)(x - 3) > 0$, then solve it by regional test on a number line. |
| **4.** | (i) $\begin{pmatrix} 91 & 27 & 35 \\ 75 & 4 & 4 \end{pmatrix}$ |
| | (ii) $\begin{pmatrix} 4 \\ 9 \\ 4 \end{pmatrix}$ |
| | (iii) $\begin{pmatrix} 91 & 27 & 35 \\ 75 & 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 9 \\ 4 \end{pmatrix} = \begin{pmatrix} 747 \\ 352 \end{pmatrix}$<br>$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 747 \\ 352 \end{pmatrix} = (1099)$ (or by simple addition)<br>Total energy = 1099 calories |
| **5.** | (i) f is a 1-1 function and hence its inverse function exists. Alternatively, f passes the horizontal line test, hence its inverse function exists. |

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| | (ii) $f^{-1}(x) = \log_2 x$ or $\dfrac{\lg x}{\lg 2}$ or $\dfrac{\ln x}{\ln 2}$ |
| | (iii) $f(x)$ and $f^{-1}(x)$ are reflection of each other about the line $y = x$. |
| **6.** | (a) $2x^{-\frac{1}{3}} + y^3 = 28 \quad ...(1)$<br>$6x^{-\frac{1}{3}} - 2y^3 = -51 \quad ...(2)$<br><br>$2(1)+(2),$<br>$10x^{-\frac{1}{3}} = 5 \quad ...(2)$<br>$x^{-\frac{1}{3}} = \dfrac{1}{2} \Rightarrow x^{\frac{1}{3}} = 2 \Rightarrow x = 8$<br>Subst $x^{-\frac{1}{3}} = \dfrac{1}{2}$ into (1) gives $2\left(\dfrac{1}{2}\right) + y^3 = 28 \Rightarrow y^3 = 27 \Rightarrow y = 3$ |
| | (b) $2x = \sqrt{2x+5} + 1$<br>$2x - 1 = \sqrt{2x+5}$<br>$\Rightarrow (2x-1)^2 = 2x+5$<br>$4x^2 - 4x + 1 = 2x + 5$<br>$2x^2 - 3x - 2 = 0$<br>$\Rightarrow (x-2)(2x+1) = 0$<br>$\Rightarrow x = 2$ or $x = -\dfrac{1}{2}$ (na)<br>Overal P if does show rejection. |
| | (c)<br>Since $x - 3 \geq 0$, $x \geq 3$<br>$\Rightarrow \sqrt{3x+4} \geq \sqrt{3\times3+4} > 2$<br>That is $\sqrt{3x+4} + 2\sqrt{x-3} > 2$ for all real value of $x$.<br>Hence, no real solution. |
| **7.** | (i) $f(x)$<br>$= 2x^2 - 4x - 7$<br>$= 2(x^2 - 2x) - 7$<br>$= 2(x^2 - 2x + 1 - 1) - 7$<br>$= 2(x-1)^2 - 9$ |
| **7.** | (ii) |

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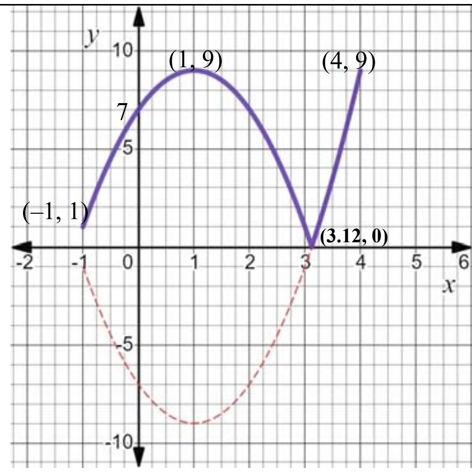
| |  |
| | (iii) $\lvert f(x)\rvert = 7$<br>$\lvert 2(x-1)^2 - 9\rvert = 7$<br>$2(x-1)^2 - 9 = \pm 7$<br>$2(x-1)^2 - 9 = 7$<br>$(x-1)^2 = 8$<br>$x = \pm 2\sqrt{2} + 1$<br>$x$ = 3.83 or -1.83 (out of domain)<br>or<br>$2(x-1)^2 - 9 = -7$<br>$(x-1)^2 = 1$<br>$x - 1 = \pm 1$<br>$x$ = 0 or 2<br>Hence, $x$ = 0 or 2 or 3.83 |
| **8.** | (i) Using cosine rule,<br>$AC^2 = 820^2 + 970^2 - 2(820)(970)\cos 56° \approx 723735$<br>$AC = 851$ m (nearest metre) |
| | (ii) $\tan 18° = \dfrac{BD}{970}$<br>$\Rightarrow BD = 970 \tan 18° = 315$ m (nearest metre) |
| | (iii) $\angle DTB$ is larger. |
| **9.** | (a) $y = \dfrac{1}{x}$<br>Stretch/scale the graph along the $x$-axis by a factor of 3 to obtain<br>$y = \dfrac{1}{\frac{x}{3}}$ i.e. $y = \dfrac{3}{x}$.<br>Translate the graph obtained downwards by 1 unit to<br>obtain $y = \dfrac{3}{x} - 1$. |

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| | **Alternative answer 2**<br>Stretch/scale the graph along the $y$-axis by a factor of 3 to obtain $\frac{y}{3}=\frac{1}{x}$ i.e. $y=\frac{3}{x}$.<br>Translate the graph obtained downwards by 1 unit to obtain $y=\frac{3}{x}-1$.<br><br>**Alternative answer 3**<br>Translate the graph obtained downwards by 1 unit to obtain $y=\frac{1}{x}-1$.<br>Stretch/scale the graph along the $x$-axis by a factor of 3 to obtain $y=\frac{1}{\frac{x}{3}}-1$ i.e. $y=\frac{3}{x}-1$.<br>**Alternative answer 4**<br>Translate the graph obtained downwards by $\frac{1}{3}$ unit to obtain $y=\frac{1}{x}-\frac{1}{3}$.<br>Stretch/scale the graph along the $y$-axis by a factor of 3 to obtain $\frac{y}{3}=\frac{1}{x}-\frac{1}{3}$ i.e. $y=\frac{3}{x}-1$. |
| | (b) $y=4\left(3^{2x}\right)-6$<br>$y=2\left(3^{2x}\right)-3$ (Compress along $y$ axis by factor 2) |
| **10.** | (i) Gradient of $BC$ = $-\frac{1}{2}$<br>Using $B$ (0, 3), equation of $BC$ is<br>$y-3=-\frac{1}{2}(x-0) \quad \Rightarrow y=-\frac{1}{2}x+3$<br>When $y$ = 0, $\Rightarrow 0=-\frac{1}{2}x+3 \Rightarrow \frac{1}{2}x=3 \Rightarrow x=6$<br>$\therefore \alpha=6$<br><br>Alternatively, |

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| | Gradient of $BC = \dfrac{3-0}{0-\alpha}$<br>$\Rightarrow \dfrac{3-0}{0-\alpha} \times 2 = -1 \therefore \alpha = 6$ (shown) |
| | (ii) Area of quadrilateral $ABCD$, units$^2$<br>$= \dfrac{1}{2}\begin{vmatrix} 0 & 6 & 18 & 6 & 0 \\ 3 & 0 & 6 & 15 & 3 \end{vmatrix}$<br>$= \dfrac{1}{2}(36 + 270 + 18 - (18 + 36))$<br>$= 135$ |
| | (iii) Midpoint of $CD = \left(\dfrac{6+18}{2}, \dfrac{0+6}{2}\right) = (12, 3)$<br>Gradient of $CD = \dfrac{6-0}{18-6} = \dfrac{1}{2}$<br>Gradient of perpendicular bisector of $CD = -2$<br><br>Equation of perpendicular bisector is<br>$y - 3 = -2(x - 12) \Rightarrow y = -2x + 27$ |
| | (iv) Yes point $A$ lies on the perpendicular bisector.<br>When $x = 6$, $y = -2(6) + 27 = 15$ which is consistent with the $y$-coordiante of $A$.<br>Alternatively, students can show $AC = AD = 15$. |
| **11.** | (a) $\log_5 \dfrac{75}{3} + 2(0) = \log_5 25 = 2$ |
| **11** | (b)<br>$\log_x (2x^2) = \log_x 2 + \log_x x^2$<br>$= \dfrac{1}{\log_2 x} + 2$<br>$= \dfrac{1}{u} + 2$ or $\dfrac{2u+1}{u}$ |
| | (c) $\log_{a^n}(b^n) = \dfrac{\log_a(b^n)}{\log_a(a^n)} = \dfrac{n\log_a b}{n\log_a a} = \log_a b$<br><br>$(\log_3 2)(\log_2 3 + \log_{2^2} 3^2 + \log_{2^3} 3^3 + \log_{2^4} 3^4 + ... + \log_{2^{2021}} 3^{2021})$<br>$= (\log_3 2)\left[\underbrace{\log_2 3 + \log_2 3 + \log_2 3 + \log_2 3 + ... + \log_2 3}_{\text{2021 terms}}\right]$ |

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| | $=(\log_3 2)(2021)(\log_2 3)$<br>$=\left(\dfrac{1}{\log_2 3}\right)(2021)(\log_2 3)$<br>$=2021$ |
| **12.** | (a) $y^2 3^{x-1}=81$<br>$\ln\left(y^2 3^{x-1}\right)=\ln 81$<br>$2\ln y+(x-1)\ln 3=4\ln 3$<br>$2\ln y=(-\ln 3)(x-1)+4\ln 3$<br>$\ln y=\left(-\dfrac{\ln 3}{2}\right)(x-1)+2\ln 3$ or $\ln y=\left(-\dfrac{\ln 3}{2}\right)(x-1)+\dfrac{\ln 81}{2}$ |
| | (b) Gradient $=\dfrac{6-(-2)}{5-3}=\dfrac{8}{2}=4$<br>*Method 1:*<br>Equation: $y\sqrt{x}-6=4(x^2-5)$<br>$\Rightarrow y\sqrt{x}=4x^2-14$<br>$y=4x^{\frac{3}{2}}-14x^{-\frac{1}{2}}$<br><br>*Method 2:*<br>Equation: $y\sqrt{x}=4x^2+c$<br>Substituting (5, 6), $6=4(5)+c\Rightarrow c=-14$<br>$\therefore y\sqrt{x}=4x^2-14$<br>$y=4x^{\frac{3}{2}}-14x^{-\frac{1}{2}}$<br>Hence, $a=4$ and $b=-14$<br>Note: students can manipulate $y=ax^{\frac{3}{2}}+bx^{-\frac{1}{2}}$ to get $y\sqrt{x}=ax^2+b$ first. |
| **13.** | (i) $P(-12, 4)$<br>Equation of $C_1$ *is* $(x+12)^2+(y-4)^2=16$. |
| | (ii)(a) $QR=(r+4)$ units |
| | (ii)(b) $QM=(r-4)$ units<br>Or $QM=\sqrt{(r+4)^2-12^2}=\sqrt{r^2+8r-128}$ units |
| | (iii) $(r+4)^2=(r-4)^2+12^2$<br>$r^2+8r+16=r^2-8r+16+144$<br>$16r=144$<br>$r=9$<br>Alternatively, $\sqrt{r^2+8r-128}+4=r$ can be formed. |

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| | (iv) $\tan \angle QPM = \dfrac{5}{12} \Rightarrow \angle SPM \approx 0.395$ (shown) |
| | (v) Area of minor sector, *APS*, units$^2$<br>$= \dfrac{1}{2} \times (4)^2 \left( \dfrac{\pi}{2} + 0.39479 \right) \approx 15.725$<br>Area of minor sector, *SQO*, units$^2$<br>$= \dfrac{1}{2} \times (9)^2 \left( \dfrac{\pi}{2} - 0.39479 \right) \approx 47.628$<br><br>Area of polygon, *APQRC*, units$^2$ $= \dfrac{1}{2} \times 24 \times 5 + 24 \times 4 = 156$<br>Area of shaded regions, units$^2$<br>$= 156 - 2(47.628) - 2(15.725) = 29.294 = 29.3(3sf)$ |