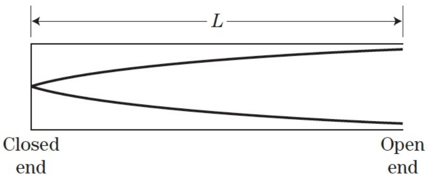
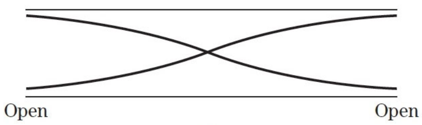
2025 YEAR 6 SUPERPOSITION TUTORIAL SOLUTIONS

1 For a closed pipe, at the fundamental frequency, the length of the tube is $\frac{1}{4}\lambda$



$$\begin{aligned} L &= \frac{1}{4}\lambda \\ &= \frac{1}{4}\left(\frac{v}{f}\right) \\ &= \frac{1}{4}\left(\frac{330}{256}\right) \\ &= 0.32 \text{ m (to 2sf)} \end{aligned}$$

For an open pipe, at the fundamental frequency, the length of the tube is $\frac{1}{2}\lambda$



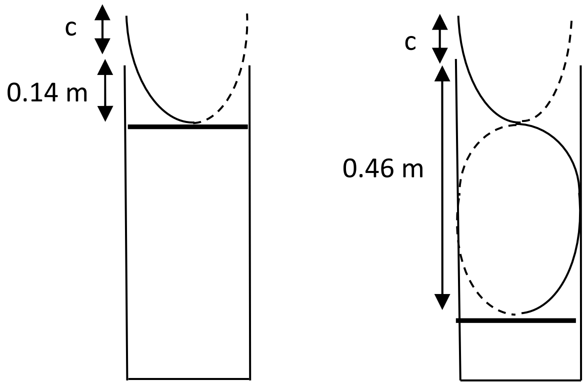
$$\begin{aligned} L &= \frac{1}{2}\lambda \\ &= \frac{1}{2}\left(\frac{v}{f}\right) \\ &= \frac{1}{2}\left(\frac{330}{256}\right) \\ &= 0.64 \text{ m (to 2sf)} \end{aligned}$$

2 A

3 D

4 (c + 0.46) – (c + 0.14) = λ / 2

➔ λ = 0.64 m



5 (a) (i) $\lambda / 2$

(ii) Phase difference = π rad

(b) $f\lambda$ of a stationary wave represents the speed of the two progressive waves that superpose to form the stationary wave. When two progressive waves of the same type of equal amplitude, equal frequency, equal wavelength, equal speed travelling in opposite directions meet and undergo superposition with each other, a stationary wave is formed.

6 C

7 D

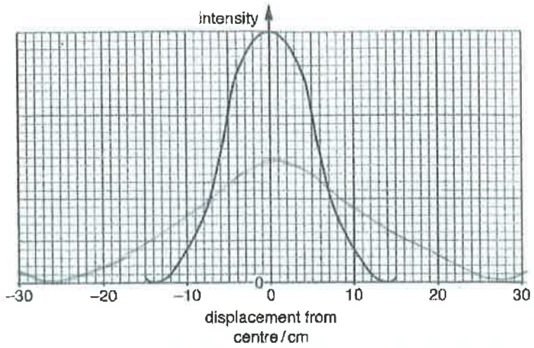
8 (a) Use $\sin\theta = \frac{\lambda}{b}$

since θ is small, $\sin\theta \approx \tan\theta$

$0.14/2.7 = \lambda/12 \times 10^{-6}$

$\lambda = 620$ nm

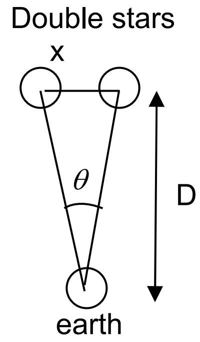
(b) More spreading with narrower slit with lower intensity.



(c) (i) From the formula $\sin\theta = \frac{\lambda}{b}$, when a longer wavelength is used with b unchanged, θ increases which means the central maxima is broader with the minima further on both sides further away from the central.

(ii) According to the formula $\sin\theta = \frac{\lambda}{b}$, for the same slit width, the different wavelengths in white light diffract at different angles, with the shorter wavelength (violet) nearer to the central and larger wavelength (red) further from the central, hence there is colour separation away from the central. At the central region, all the different colours have constructive interference, hence central is white.

9 The wavelength is estimated to be 500 nm.



$$\theta = \frac{\lambda}{b} = \frac{(500x10^{-9})}{3.0} = 1.57x10^{-7}$$

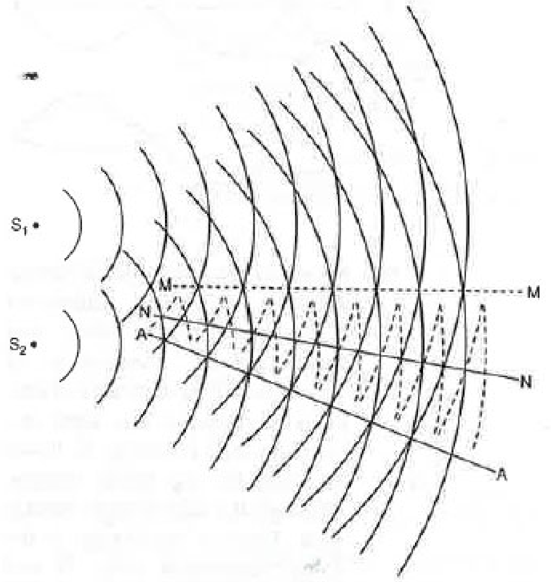
since θ is small, $\theta \approx \tan\,\theta = \dfrac{x}{D} = \dfrac{x}{20(9.5\times10^{15})}$

or x = 3.0 x 10^{10} m

(for this problem, any estimate of wavelength in the visible light region is correct).

10 B

11



12 B

13 Using $x = \frac{\lambda D}{a}$

(i) If *a* is decreased, fringe separation *x* increases from the formula. Intensity of fringes unaffected by changes in *a*.

(ii) If *D* is increased, fringe separation *x* increases from the formula. Intensity of fringes decreases as the distance from the source increases ($I \propto 1/r^2$).

(iii) Contrast between bright and dark fringes is reduced. This is because the bright fringe will be less bright while the dark fringes will becomes brighter due to incomplete destructive interference. Fringe separation is unchanged.

(iv) If one slit is covered with transparent medium, the fringe pattern is shifted. Fringe separation *x* is unaffected and the intensity of the fringes are reduced.

(v) Fringe separation remained the same. Bright fringes less bright now (as the intensity of light passing through the double slit is reduced). Dark fringe remains dark (no change). More fringes would be observed (due to more diffraction).

(vi) Fringe separation changes. Smaller D, smaller fringe separation (i.e. fringe separation closer on the screen nearer the slit) and fringes are brighter. Larger D, larger fringe separation and fringes are less bright.

14 (i) From $\sin\theta = \frac{\lambda}{b}$ and for small θ, $\sin\theta \approx \tan\theta$

thus, $5.90 \times 10^{-7} / 0.15 \times 10^{-3} = y / 2 (3.2)$ or $y = 0.025$ m

(ii) Rayleigh criterion states that for the two diffraction patterns to be <u>just distinguishable</u>, the central maximum of one must lie on the first minimum of the other. Thus, the minimum angle θ for two patterns to be just resolved is

$$\theta \approx \frac{\lambda}{b} \qquad = 5.90 \times 10^{-7} / 0.15 \times 10^{-3} = 3.9 \times 10^{-3} \text{ rad}$$

The angular separation θ_s of the two slits is $2.3 \times 10^{-3} / 3.2 = 7.2 \times 10^{-4}$ rad which is less than θ. Hence the diffraction patterns cannot be resolved and are not seen as separate.

(iii) From $x = \frac{\lambda D}{a} = 5.90 \times 10^{-7} (3.2) / 2.3 \times 10^{-3} = 8.2 \times 10^{-4}$ m

(iv) Number of fringes = $2.5 \times 10^{-2} / 8.2 \times 10^{-4}$

= 30 (round down to whole number)

15 D

16 C

17 $d \sin \theta = n\lambda$

First order:

$\sin \theta_V = 1 (450 \times 10^{-9}) / (10^{-2}/6000)$ ➔ $\theta_{V1} = 15.7°$

$\sin \theta_R = 1 (700 \times 10^{-9}) / (10^{-2}/6000)$ ➔ $\theta_{R1} = 24.8°$

angle of dispersion = $24.8^o - 15.7^o = 11.1^o$

Second order:

$\sin \theta_V = 2 (450 \times 10^{-9}) / (10^{-2}/6000)$ ➔ $\theta_{V2} = 32.7°$

$\sin \theta_R = 2 (700 \times 10^{-9}) / (10^{-2}/6000)$ ➔ $\theta_{R2} = 57.1°$

angle of dispersion = $57.1^o - 32.7^o = 24.4^o$

Third order:

$\sin \theta_V = 3 (450 \times 10^{-9}) / (10^{-2}/6000)$ ➔ $\theta_{V3} = 54.1° < \theta_{R2}$
Hence, there is overlap between 2nd order red and 3rd order violet.

$\sin \theta_R = 3 (700 \times 10^{-9}) / (10^{-2}/6000) = 1.26$ ➔ no third order red

18 (i) $\sin \theta < 1$ ➔ $(n \lambda / d) < 1$ ➔ $n < (d / \lambda) = [1/(4.00 \times 10^5)] / (589 \times 10^{-9}) = 4.24$

So, max n = 4.

(ii) 1. 4th order (greatest diffraction)

2. $\sin \theta = 4 \lambda / d$ ➔ $\theta_{589} = 70.459°$, and $\theta_{589.6} = 70.624°$
The 2 wavelengths can be distinguished since their angular separation is 0.165°.

19. (a) $d \sin \theta = n\lambda$
Where $\tan \theta = 0.55 / 1.50$ ➔ $\theta = 20.14°$
$\lambda = d \sin \theta / n = (1/ (5.5 \times 10^5)) (\sin 20.14°) /1 = 626$ nm

(b) Advantage: Higher order diffracted beams are more spaced out, and hence, the percentage error in θ is smaller.

Disadvantage: Higher order diffracted beams have a lower intensity due to diffraction, hence observation could be more difficult.