Topical Revision Paper 4

Term 2 Unit 4: Polynomials

Question 1:

It is given that $\mathrm{f}(x)=(x+3)(2x-1)\,\mathrm{g}(x)+mx+c$, where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are polynomials in terms of x, and m and c are constants.

The polynomial $\mathrm{f}(x)$ has a remainder of -11 when divided by $(x+3)$, and a remainder of -4 when divided by $(2x-1)$.

(i) Find the value of m and of c. [4]

(ii) Hence, find the remainder when $\mathrm{f}(x)$ is divided by $2x^2+5x-3$. [1]

Solution:

(i)

$f(-3)=-11=-3m+c$

$f(\frac{1}{2})=-4=\frac{1}{2}m+c$

$7=3.5m$

$m=2$

$c=-5$

(ii)

Remainder is $2x-5$

Question 2:

(i) Factorise x^3-5x^2+3x+9 completely. [4]

(ii) Hence, solve the equation $8x^6=20x^4-6x^2-9$. [3]

Solution:

(i)

Let $f(x)=x^3-5x^2+3x+9$

$f(-1)=-1-5-3+9=0$

$f(x)=(x+1)(x^2+bx+9)$

$-5=1+b$

$b=-6$

$\mathrm{f}(x)=(x+1)(x^2-6x+9)$

$\mathrm{f}(x)=(x+1)(x-3)^2$

(ii)

$\mathrm{f}(x)=(x+1)(x-3)^2$

$(x+1)(x-3)^2=0$

$x=-1$ or $x=3$

$8x^6=20x^4-6x^2-9$

Let

$\left(2x^2\right)^3=5\left(2x^2\right)^2-3\left(2x^2\right)-9$

$y=2x^2$

$y^3-5y^2+3y+9=0$

$y=1$ or $y=3$

$2x^2=-1$ (n.a.) or $2x^2=3$

$x=\pm\sqrt{\frac{3}{2}}$ or

$x=\pm\frac{\sqrt{6}}{2}$ or $x=\pm 1.22$

Question 3:

The polynomial P is given by $P(x)=2x^3+7x^2-6x+1$.

(i) Verify that $x=\frac{1}{2}$ is a root of the equation $P(x)=0$. [1]

(ii) Express $P(x)$ in the form $(2x-1)\left(Ax^2+Bx+C\right)$ where A, B and C are constants to be determined. [2]

(iii) Find the remaining roots of $P(x)=0$ using completing square method, expressing your answer in exact form. [3]

(iii) Hence solve the equation $2+7e^x-6e^{2x}+e^{3x}=0$. [3]

Solution:

(i)

Substitute $x=\frac{1}{2}$:

$P(x)=2\left(\frac{1}{2}\right)^3+7\left(\frac{1}{2}\right)^2-6\left(\frac{1}{2}\right)+1=\frac{1}{4}+\frac{7}{4}-3+1=0$.

(ii)

Let $2x^3+7x^2-6x+1=(2x-1)\left(Ax^2+Bx+C\right)$

By comparing coefficients of x^2 and constant terms,

$A=1$, $C=-1$.

By comparing coefficients of x term,

$7=2B-A \Rightarrow B=4$.

(iii)

The remaining roots come from solving $x^2+4x-1=0$

$\Rightarrow x^2+4x-1=0$

$\Rightarrow (x+2)^2-4=1$

$\Rightarrow (x+2)^2=5$

$\Rightarrow x=-2-\sqrt{5}$ or $x=-2+\sqrt{5}$.

(iv)

Replace x by $\dfrac{1}{e^x}$:

$$2\left(\frac{1}{e^x}\right)^3+7\left(\frac{1}{e^x}\right)^2-6\left(\frac{1}{e^x}\right)+1=0$$

$$\frac{1}{e^x}=\frac{1}{2},\,-2+\sqrt{5},\,-2-\sqrt{5}\ \text{(rej)}$$

$$e^x=2,\ \frac{1}{-2+\sqrt{5}}$$

$x=\ln 2, 1.44$ or $x=0.693, 1.44$

<u>Question 4:</u>

Using **<u>long division</u>**, find the quotient when $12x^3+8x^2+x-2$ is divided by $(2x+1)^2$.

[3]

<u>Solution:</u>

Using long division,

$$\begin{array}{r l}
 & \phantom{12x^3+8x^2+{}}3x-1 \\
4x^2+4x+1 & \overline{)\,12x^3+8x^2+x-2} \\
 & \underline{-\left(12x^3+12x^2+3x\right)} \\
 & \quad -4x^2-2x-2 \\
 & \quad \underline{-\left(-4x^2-4x-1\right)} \\
 & \qquad\qquad 2x-1
\end{array}$$

Quotient = $3x-1$

<u>Question 5:</u>

(i) Find the values of p and q for which x^2-x-6 is a factor of $2x^3+px^2-17x+q$.

[4]

(ii) Using the values of p and q found in part **(i)**, solve the equation $2x^3+px^2-17x+q=0$.

[3]

(iii) Hence, sketch the graph of $y = 2x^3 + px^2 - 17x + q$.	[3]

Solution:

(i)

$x^2 - x - 6 = (x-3)(x+2)$

$f(3) = 54 + 9p - 51 + q = 0 \Leftrightarrow 9p + q = -3$
$f(-2) = -16 + 4p + 34 + q = 0 \Leftrightarrow 4p + q = -18$

$p = 3$ and $q = -30$

(ii)

$2x^3 + 3x^2 - 17x - 30 = 0$
$(x-3)(x+2)(2x+5) = 0$
$x = 3,\ -2$ or -2.5

(iii)