

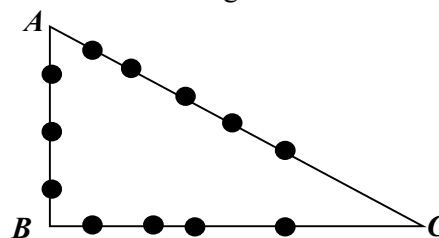


Additional Practice Questions for Chapter S1A:
Permutations and Combinations

- 1 From a group of 5 women and 7 men, one of whom is Mr Lee, find how many committees of size 4 can be formed in which
 - (i) there are 2 men and 2 women,
 - (ii) there is at least 1 man and at least 1 woman,
 - (iii) there is at least 1 man and Mr Lee is in the committee.
- 2 4 boys, 4 girls and a teacher are to be seated at a round table. How many ways can they be arranged if
 - (i) there is no restriction?
 - (ii) the teacher is to be seated between any 2 girls?
 - (iii) none of the boys are to be seated together?
- 3 How many 6-digit numbers
 - (i) are even?
 - (ii) begin and end with different digits?
- 4 A rectangular table has 7 secured seats, 4 being on one side facing the window and 3 being on the opposite side. In how many ways can 7 people be seated at the table
 - (i) if 3 people, X and Y and Z must sit on the side facing the window?
 - (ii) if 2 people, P and Q must sit on opposite sides?
- 5 A school is asked to send a delegation of six pupils selected from six badminton players, six tennis players and five squash players. No pupil plays more than one game. The delegation is to consist of at least one, and not more than three, players drawn from each sport. Giving full details of your working, find the number of ways in which the delegation can be selected.
- 6 Find the number of distinct arrangements of the letters of the word '*THERMOMETER*'
 - (i) if at least 2 '*E*'s are together,
 - (ii) which must start and end with '*T*' or '*R*'.

- 7 How many 4-digit numbers greater than 5000 can be formed from the digits 0, 1, 2, 3, 4, 5 if
- no digit may be repeated?
 - any digit may be repeated?
 - only the digit 4 may be repeated?
- 8 A photographer is positioning 5 men and 4 women for a photo shoot. Find the number of ways the photographer can position them in a row
- if no two people of the same sex are to stand next to each other,
 - if the photographer positions the men in the order from shortest on the left to tallest on the right. (Assume that all the men are of different heights and are not necessarily standing together in a group.)
- 9 A box contains 9 balls. Out of these 9 balls, there are 3 identical red balls, 2 identical yellow balls and 4 numbered green balls (each labelled with a different number from 1 to 4). 3 balls are to be picked out of the box, and the order in which they are picked out does not matter. Find the number of possible selections of 3 balls. [3]
- 10 A painter is given a job to paint the doors of 5 adjacent rooms. Given that he has 5 colours of paint; red, blue, green, yellow and purple, find the number of ways he could accomplish his task if no adjacent doors are to be painted the same colour **and**
- he must use two different colours only,
 - he can use some or all of the colours.
- 11 The following diagram shows 12 **distinct** points on the sides of a triangle ABC .

- How many line segments are there joining any two points on different sides?
- How many triangles can be formed by joining any one point on side AB , any one point on side BC and any one point on side AC ?



- 12** There are nine **different** waterslides at an amusement park. A person is allowed to go on each slide once only. However, he can choose to skip the slide or go on it.

- (i) If John goes on at least two slides at the amusement park, how many ways can he select the slides to go on?
- (ii) At another theme park, there are n different slides. John visits the theme park frequently and selects 2 slides to go on during every visit. At each visit, he makes a different selection, and first realizes that this was no longer possible on the 29th visit. Determine algebraically, the value of n .

- (iii) Give an example of a situation involving waterslides to which the expression

$$\frac{n!}{r!(n-r)!} \text{ is the solution.}$$

- 13** A group of 10 people consists of 9 men and 1 woman. Find the number of ways which the group can be seated at a round table with identical chairs if

- (i) there is no restriction, [1]
- (ii) 2 particular men, Caleb and James, do not want to sit beside the woman, but will like to sit together. [3]

The chairs at the table are replaced with 10 chairs of different colours.

- (iii) Find the number of ways which the group can be seated at a round table if Caleb and James must still sit together, but they need not be separated from the woman. [3]

- 14** 2 men and 5 women go to a restaurant. They choose an outdoor round table with 7 seats for their meal. Find the number of ways the group can be seated if

- (i) the two men are not seated next to each other. [2]
- (ii) one of the women, Mary, is to be seated between the two men. [2]

Before their orders arrive, they request to shift to a table in the 'non-smoking' section of the restaurant. They are then given a round table with 10 seats.

Find the number of ways they can be seated if

- (iii) the empty seats are adjacent to each other. [2]
- (iv) none of the empty seats are adjacent to each other and there must be more than 1 person between any two empty seats. [2]

- 15** A soccer team consists of 3 goalkeepers, 8 defenders, 7 midfielders and 4 strikers. The coach has to select exactly 1 goalkeeper and 10 other players to start a match.

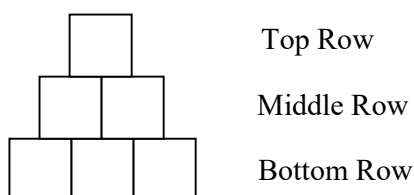
(i) In how many ways can he select the 11 players?

In each of the four sections (goalkeepers, defenders, midfielders and strikers), there is one STAR player. How many ways can the coach select his 11 players if

(ii) he must include all the STAR players?

(iii) he must include at least one STAR player?

- 16** Six identical boxes are arranged in 3 rows as shown in the following diagram.



Sandra is given 1 green, 2 blue and 3 red balls. The balls are identical except for their colour. She is to put one ball in each box. Find the number of ways she can do this when

- (i) there is no restriction, [2]
(ii) the balls in the bottom row are of different colours, [2]
(iii) there are at least 2 red balls in the bottom row. [3]

- 17** Three couples who each have a child are to be seated at a round table with ten secured seats. Find the number of ways the nine people can be seated if

- (i) they do not mind who they are sitting with, [1]
(ii) none of the children are to be seated on adjacent seats. [3]

Mr Bean, who knows the three families well, is invited to take a seat at the table. Find the number of ways to seat the ten people if each child is to be seated adjacent to both his or her parents. [3]

- 18** Seven cards each have a single digit written on them. The digits on the seven cards are 2, 2, 5, 7, 7, 7, 7 respectively.

Find the number of different 4-digit numbers that can be formed by placing four of the cards side by side. [4]

- 19** In how many ways can a committee of 3 men and 3 women be chosen from a group of 7 men and 6 women? [2]

The oldest of the 7 men is A and the oldest of the 6 women is B . It is decided that the committee can include at most one of A and B . In how many ways can the committee now be chosen? [3]

- 20** Consider the eight letters in the word GEOMETRY.

(a) Find the number of different arrangements of all eight letters in which the vowels are always separated. [3]

(b) Four letters are selected from the eight letters.

(i) How many possible selections can be made if all the letters are distinct and the order of selection is not relevant? [1]

(ii) Find the number of 4-letter code-words that can be formed if there is no restriction on the letters selected. [3]

- 21** Ten chefs, six males and four females, qualify for the final phase of a ‘Top Chef’ competition consisting of nine ‘Elimination Challenges’. One chef is eliminated after every challenge and the last chef remaining is the winner of the competition.

(i) In how many ways can the eliminations be done? [1]

(ii) For the first challenge, the ten chefs are divided into two groups of five. In how many ways can the two groups be formed such that each group has at least one female chef? [2]

(iii) After the first challenge, a particular male chef was eliminated. At a photography session before the next challenge, nine chairs are arranged in two rows: four in front and five at the back. In how many ways can the remaining chefs be arranged so that the male chefs and the female chefs must alternate? [3]

- 22 (a)** A student is arranging 4 blue flags, 4 red flags, 1 green, 1 yellow and 1 purple flag in a line.
All the flags are identical except for the colour. Find the number of different possible ways to arrange the flags if
- (i)** the green, yellow and purple flags must be placed together; [2]
 - (ii)** no blue flags are placed next to each other; [2]
 - (iii)** a red flag at the beginning and another red flag at the end of the line. [2]
- (b)** Ten people at a company dinner consist of 7 guests and 3 hosts. The group is to be seated at a round table.
- (i)** Find the number of arrangements such that there are at least two guests seated between any two hosts. [3]
 - (ii)** Two particular guests could not make it to the dinner. Find the number of ways to arrange the remaining guests and hosts with 10 identical chairs. [3]
- 23** The management board of a company consists of 6 men and 4 women. A chairperson, a secretary and a treasurer are chosen from the 10 members of the board. Find the number of ways the chairperson, the secretary and the treasurer can be chosen so that
- (i)** they are all women, [1]
 - (ii)** at least one is a woman and at least one is a man. [3]
- The 10 members of the board sit at random around a table. Find the probability that
- (iii)** the chairperson, the secretary and the treasurer sit in three adjacent places, [3]
 - (iv)** the chairperson, the secretary and the treasurer are all separated from each other by at least one other person. [3]

****THE END ****