

H2 Physics

Topic 17: Electromagnetic Induction

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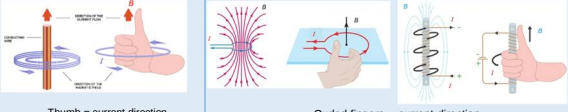
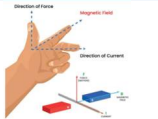
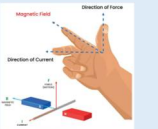
Key Differences Between Electromagnetism and Electromagnetic Induction

| Topic                     | Focus                                                                                                                            | Important Rules                                                                                                                                                                                                                                                                                                                                                                   |
|---------------------------|----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Electromagnetism          | How currents produce magnetic fields, and how these fields exert magnetic force on moving charges or current-carrying conductors | <ul style="list-style-type: none"><li>Right-hand grip rule (to find direction of <math>B</math> field around a current-carrying wire, produced by a current flowing through a circular loop of wire, and in solenoids)</li><li>Fleming's Left-hand Rule (to find the direction of magnetic force on a current-carrying conductor or moving charges in a magnetic field)</li></ul> |
| Electromagnetic Induction | How a changing magnetic field induces an e.m.f. (and a current if the circuit is closed)                                         | <ul style="list-style-type: none"><li>Fleming's Right-hand Rule (to find the direction of induced current)</li></ul>                                                                                                                                                                                                                                                              |

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Quick Reference for the Rules

| Rule                      | Fingers represent ... with relevant diagrams                                                                                                                                                                                                                                      |                           |
|---------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------|
| Right-hand grip rule      |  <ul style="list-style-type: none"><li>Thumb = current direction</li><li>Curled fingers = <math>B</math> field direction</li></ul>                                                              | Electromagnetism          |
| Fleming's Left-hand Rule  |  <ul style="list-style-type: none"><li>First finger = <math>B</math> Field (N to S)</li><li>seCond finger = Current direction</li><li>ThuMb = Motion (Force)</li></ul>                         |                           |
| Fleming's Right-hand Rule |  <ul style="list-style-type: none"><li>First finger = <math>B</math> Field (N to S)</li><li>ThuMb = Motion (movement of conductor)</li><li>seCond finger = Induced Current direction</li></ul> | Electromagnetic Induction |

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Contents

1. Introduction

2. Magnetic Flux and Magnetic Flux Linkage

3. Laws of Electromagnetic Induction
  - Faraday's law
  - Lenz's law

4. Applications

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## Guiding Question 1

From Electromagnetism, we learnt that:

electric current  produces magnetic field

What about?:

magnetic field  produces electric current ??

What is required for this?

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## Guiding Question 2

Can we achieve perpetual motion by invoking the magic of electromagnetism?

Electromagnetism is a fundamental force where electric currents generate magnetic fields, influencing current carrying conductors and moving charges, and enabling motors. A changing magnetic field induces a current, which in turn produces another magnetic field, potentially continuing the cycle.

Can we use **principles of electromagnetism & EMI** to create a machine forever without external energy?

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## Cause → Effect Relationships

### 1. Motion of Magnet

- Towards coil:
  - S-pole → Deflection left, N-pole → Deflection right
- Away from coil:
  - S-pole → Deflection right, N-pole → Deflection left

### 2. Speed of Movement

- Slow push/pull → Small deflection, Fast push/pull → Large deflection
- Shows: Induced current  $\propto$  Rate of change of magnetic flux

### 3. Number of Turns

- (10 turns → Small, 20 turns → Larger, 40 turns → Largest) deflection
- Shows: Induced current  $\propto$  Number of turns

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## Key Conclusions

### 1. Magnitude of induced current depends on:

- Rate of change of magnetic flux
- Number of coil turns

### 2. Direction of induced current depends on:

- Direction of magnet movement
- Pole orientation

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## Our Objective:

To be able to determine the **magnitude** and **direction** of the **induced e.m.f.**



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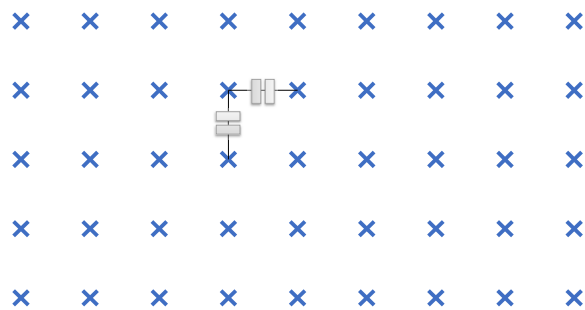
## Learning Outcomes

- |    |                                                                                                                          |
|----|--------------------------------------------------------------------------------------------------------------------------|
| a. | define magnetic flux as the product of an area and the component of the magnetic flux density perpendicular to that area |
| b. | recall and solve problems using $\Phi = BA$                                                                              |
| c. | define magnetic flux linkage                                                                                             |

MAGNETIC FLUX DENSITY,  $B$   
MAGNETIC FLUX,  $\Phi$   
MAGNETIC FLUX LINKAGE,  $N\Phi$

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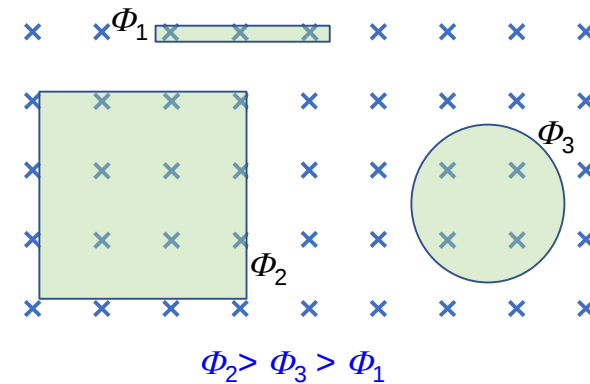
## Uniform Magnetic Flux Density, $B$



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## Magnetic Flux, $\Phi = B_{\perp} A$

**Magnetic Flux** is defined as the product of an area and the component of the magnetic flux density perpendicular to that area.



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### Magnetic Flux, $\Phi$

Top view

$B_{\perp}$  decreases from left to right, so  $\Phi$  decreases from left to right

Front view

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### Magnetic Flux, $\Phi = B A \cos \theta$

Top view

Front view

$\Phi = B_{\perp} A$   
 $\Phi = (B \cos \theta) A$   
 $\theta$  is angle between  $B$  and normal to  $A$

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### Magnetic Flux

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defined as the product of an **area** and the **component of the magnetic flux density** perpendicular to that area.

$\Phi = B_{\perp} A = B A \cos \theta$

SI units: weber (Wb)      scalar quantity

The weber is defined as the amount of magnetic flux with density 1 T passing through an area of 1 m<sup>2</sup>, and 1 T = 1 Wb m<sup>-2</sup>.

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### Magnetic Flux Linkage, $N\Phi = N B A \cos \theta$

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Magnetic Flux Linkage

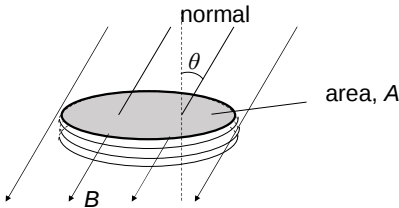
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defined as the product of the magnetic flux passing through the coil and the number of turns on the coil.

$$N\Phi = NBA \cos \theta$$

SI units: weber (Wb)

Scalar quantity



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Magnetic flux - examples



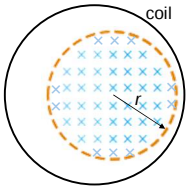
magnetic flux through the coil  
 $= B_{\perp} A$   
 $= B_{\perp} (\pi r^2)$

area is half the area of circle

magnetic flux through the coil  
 $= B_{\perp} (A/2)$   
 $= B_{\perp} (\pi r^2/2)$

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Magnetic flux - examples



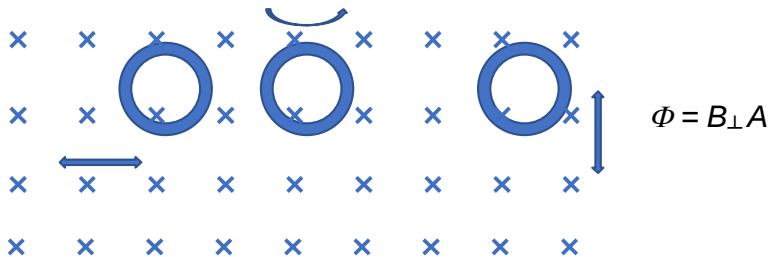
magnetic flux through the coil  
 $= B_{\perp} (\text{area of smaller circle})$   
 $= B_{\perp} (\pi r^2)$

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Example 1

The three loops of wire shown are all in a region of uniform magnetic field. Loop 1 swings back and forth as the bob on a pendulum, loop 2 rotates about a vertical axis and loop 3 oscillates vertically on the end of a spring. Which loop(s) have a magnetic flux that changes with time?

(Here, the magnetic field is directed into the plane of page. It is directed out of page in lecture notes.)



Only loop 2 has a changing magnetic flux as its orientation relative to the field, thus  $B_{\perp}$  changes as it rotates.

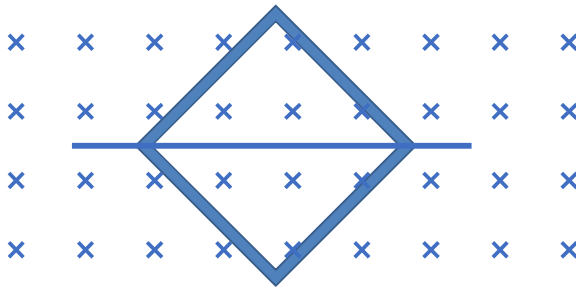
Since the magnetic field is uniform, moving loop 1 back and forth or moving loop 3 up and down does not change the magnetic flux through the loop, that is, the magnetic flux does not depend on the loop's position.

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Example 2

A coil of 10 turns and area  $0.35 \text{ m}^2$  is placed perpendicular to a magnetic field of density  $2.0 \text{ T}$  as shown in the diagram on the left. Points P and Q are pulled apart until the coil becomes a straight line as shown in the diagram on the right. What is the change in magnetic flux linkage through the coil?



Initial magnetic flux linkage,  $(N\Phi)_i = N B A \cos \theta = 10(2.0)(0.35) \cos 0^\circ = 7.0 \text{ Wb}$

Final magnetic flux linkage,  $(N\Phi)_f = 0$

Change in magnetic flux linkage  $(N\Phi)_f - (N\Phi)_i = 0 - 7.0 = -7.0 \text{ Wb}$

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Learning Outcomes

- |    |                                                                                                                                                                                                                                                                             |
|----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| d. | infer from appropriate experiments on electromagnetic induction:<br>i. that a changing magnetic flux can induce an e.m.f.<br>ii. that the direction of the induced e.m.f. opposes the change producing it<br>iii. the factors affecting the magnitude of the induced e.m.f. |
| e. | recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law                                                                                                                                                                                   |

DEMO

Faraday's Law and Lenz's Law

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Electromagnetic Induction

**Faraday's Law** of electromagnetic induction states that the induced e.m.f. is **directly proportional** to the rate of change of the magnetic flux linkage.

$$\mathcal{E} = \frac{-d(N\Phi)}{dt} = \frac{-d(NBA \cos \theta)}{dt}$$

- ❖ The **negative sign** is explained by Lenz's law. We can **drop the sign** when we are only interested in the magnitude of the e.m.f.
- ❖ It indicates that the direction of the induced e.m.f. always opposes the change in magnetic flux linkage.

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# Electromagnetic Induction

**Lenz's Law** of electromagnetic induction states the induced current or e.m.f. is in a direction to produce effects which oppose the change in magnetic flux that caused it.

Lenz's law is not written separately but combined with Faraday's law to form an equation.

$$\varepsilon = \frac{-d(N\Phi)}{dt} = \frac{-d(NBA \cos \theta)}{dt}$$

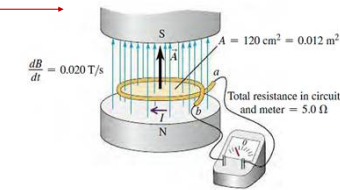
The negative sign represents that the direction of the induced current is always such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

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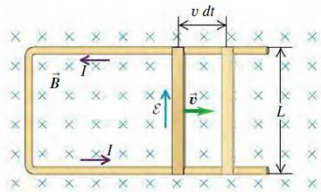
## Lenz's Law ⇒

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

The "cause" may be changing flux through a stationary circuit due to a **varying magnetic field**, **changing flux due to motion of the conductors that make up the circuit**, or any combination.



A stationary conducting loop in an increasing magnetic field.



A slidewire generator. The increase in magnetic flux (caused by an increase in area) induces the e.m.f. and current. (See Example 6 later.)

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Demonstration to show magnetic induction effect caused by changing flux through a stationary circuit

a stationary circuit wired in series with a galvanometer so that a current in the circuit will deflect the galvanometer immediately

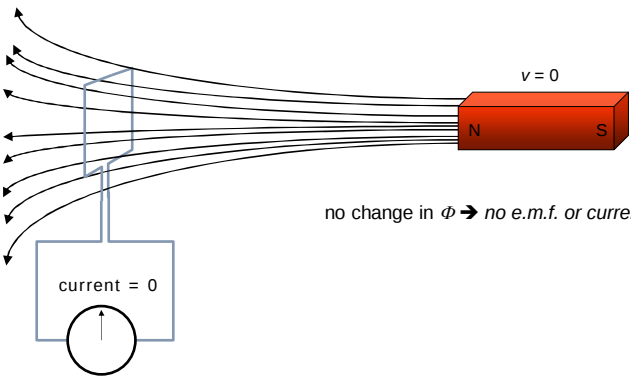


a single bar magnet is pushed into the circuit, North end first, with different speeds; the bar magnet is subsequently pulled out of the circuit with different speeds



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## Changing Magnetic Flux → E.M.F. & Current



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Changing Magnetic Flux → E.M.F. & Current

The diagram shows a red bar magnet with its North (N) pole facing a blue rectangular coil. An arrow labeled  $v$  indicates the magnet is moving to the left, towards the coil. Magnetic field lines are shown as curved arrows pointing from the N pole towards the coil. A circuit diagram below the coil shows a clockwise current flowing through a meter.

Faraday's law:  
small rate of increase of  $\Phi \rightarrow$  induced e.m.f.

Lenz's law:  
 $\Phi$  increases leftward, induced current produces a magnetic field (the effect) pointing to the right to oppose the increase.  
Using right hand grip rule, direction of current is anticlockwise (view from right to left)

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Changing Magnetic Flux → E.M.F. & Current

The diagram is similar to slide 9, but the magnet is moving faster, indicated by a longer arrow labeled  $v$ . The magnetic field lines are more densely packed. The circuit diagram shows a larger clockwise current.

Faraday's law:  
maximum rate of change of  $\Phi \rightarrow$  maximum induced e.m.f.

Lenz's law:  
maximum induced current to produce a strong magnetic field (the effect) pointing to the right to oppose the increase

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Changing Magnetic Flux → E.M.F. & Current

The diagram is similar to slide 9, but the magnet is moving slowly, indicated by a shorter arrow labeled  $v$ . The magnetic field lines are less dense. The circuit diagram shows a smaller clockwise current.

Faraday's law:  
small rate of increase of  $\Phi \rightarrow$  small induced e.m.f.

Lenz's law:  
small induced current to produce a magnetic field (the effect) pointing to the right to oppose the small increase in  $\Phi$

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Changing Magnetic Flux → E.M.F. & Current

The diagram shows the red bar magnet stationary next to the blue coil, with an arrow labeled  $v = 0$ . Magnetic field lines pass through the coil but are not changing. The circuit diagram shows a meter with zero deflection, labeled "current = 0".

no change in  $\Phi \rightarrow$  no induced e.m.f. or current

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Changing Magnetic Flux  $\rightarrow$  E.M.F. & Current

Faraday's law:  
small rate of decrease of  $\Phi \rightarrow$  induced e.m.f.

Lenz's law:  
 $\Phi$  decreases leftward, induced current produces a magnetic field (the effect) pointing to the left to oppose the decrease.  
Using right hand grip rule, direction of current is clockwise (view from right to left)

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Changing Magnetic Flux  $\rightarrow$  E.M.F. & Current

Faraday's law:  
maximum rate of change of  $\Phi \rightarrow$  maximum induced e.m.f.

Lenz's law:  
maximum induced current to produce a strong magnetic field (the effect) pointing to the left to oppose the decrease

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Changing Magnetic Flux  $\rightarrow$  E.M.F. & Current

Faraday's law:  
small rate of decrease of  $\Phi \rightarrow$  small induced e.m.f.

Lenz's law:  
small induced current to produce a magnetic field (the effect) pointing to the left to oppose the small decrease in  $\Phi$

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Changing Magnetic Flux  $\rightarrow$  E.M.F. & Current

no change in  $\Phi \rightarrow$  no induced e.m.f. or current

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To sum up ...

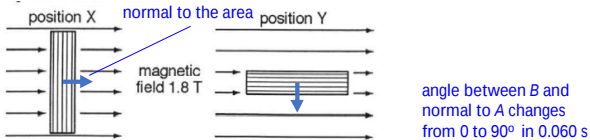
- If the flux in a stationary circuit changes, the induced current sets up a magnetic field of its own.
- Within the area bounded by the circuit, this field is **opposite** to the original field if the original field is **increasing** but is in the **same** direction as the original field if the latter is **decreasing**.
- That is, the induced current opposes the change in flux through the circuit (not the flux itself).

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Example 3

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A circular coil of diameter 2.0 cm has 3000 turns. It is rotated in a magnetic field of flux density 1.8 T from position X to position Y in 0.060 s.



What is the average e.m.f. induced in the coil during the rotation?

- A 28 V      B 36 V      C 113 V      D 1800 V

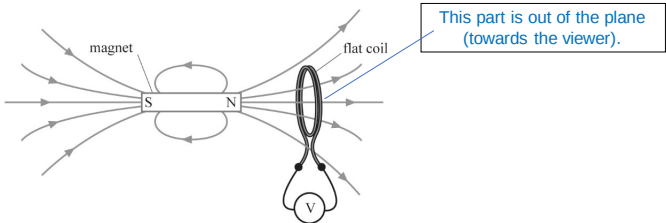
$$\begin{aligned}\epsilon &= \frac{d(N\Phi)}{dt} = \frac{d(NBA \cos \theta)}{dt} = NBA \left| \frac{\Delta \cos \theta}{\Delta t} \right| \\ &= (3000)(1.8)(\pi) (0.010^2) \left| \frac{\cos 90^\circ - \cos 0^\circ}{0.060} \right| = 28 \text{ V}\end{aligned}$$

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Example 4

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The diagram shows a magnet placed close to a flat circular coil.



- a) Explain why there is no induced e.m.f. even though there is magnetic flux linking the coil.

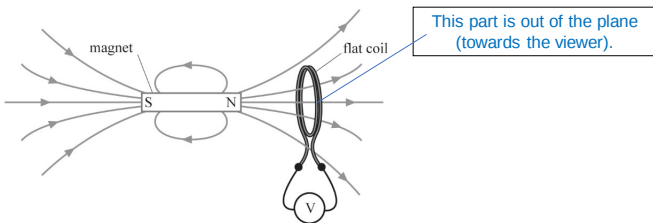
There is **no change** to the magnetic flux linking the coil, hence according to Faraday's law, there is no induced e.m.f.

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Example 4

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The diagram shows a magnet placed close to a flat circular coil.



- b) Explain why there is an induced e.m.f. when the magnet is pushed towards the coil.

The **magnetic flux density B** increases as the magnet moves towards the coil.

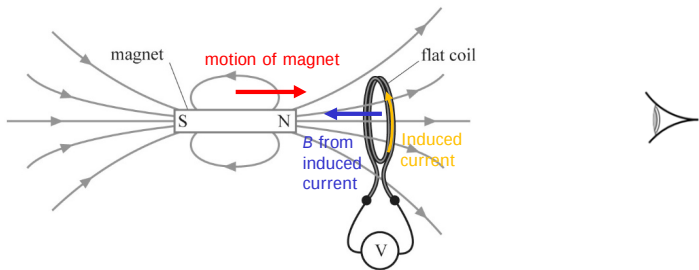
There is an **increase in the rightward magnetic flux** linking the coil, hence an e.m.f. is induced across the ends of the coil according to Faraday's Law.

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Example 4

Page 5

The diagram shows a magnet placed close to a flat circular coil.



c) Indicate direction of current in the coil by drawing arrows on the coil.

To oppose the increase in flux linkage through the coil, the induced current will produce a magnetic flux density that is **opposite direction** to that of the magnet.

From right hand grip rule, induced current flows as shown. The current flows clockwise when viewed along the axis of the coil from the right-hand side.

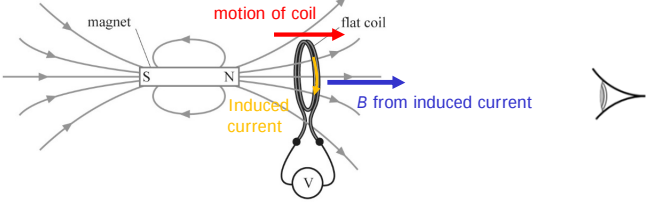
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Example 4

Page 5

The diagram shows a magnet placed close to a flat circular coil.



d) What **happened** if coil was pulled away from magnet?

The **magnetic flux density  $B$**  **decreases** as the coil moves away from the magnet. There is a **decrease in the rightward magnetic flux linking the coil**, hence an e.m.f. is induced across the ends of the coil according to Faraday's Law.

From Lenz's Law, to oppose the decrease in magnetic flux, the induced current will produce a magnetic flux density that is in **same direction** as that of the magnet (as shown). This means that the induced current flows anti-clockwise when viewed along the axis of the coil from the right-hand side.

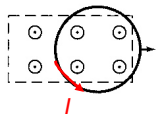
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Example 5

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Predict the direction of the induced current in the circular coil for each case:



Pulling the coil to the right out of a magnetic field pointing out of the plane of the page.

If the coil is pulled out of the field, outward magnetic flux linkage through the coil **decreases**.

The induced current will be in a direction to produce effects to oppose this decrease and hence will flow **counter-clockwise** to produce a magnetic field pointing out of the page.

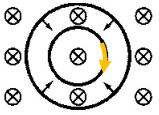
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Example 5

Page 6

Predict the direction of the induced current in the circular coil for each case:



Shrinking a coil in a magnetic field pointing into the plane of the page.

Since the coil area gets smaller, the inward magnetic flux linkage through the coil **decreases**.

Hence the induced current will be **clockwise** to produce its own magnetic field into the page to make up for this decrease in magnetic flux linkage.

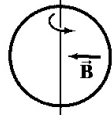
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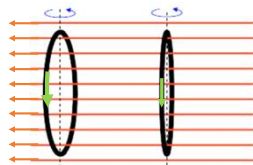
Example 5

Page 6

Predict the direction of the induced current in the circular coil for each case:



Rotating the coil by pulling the left side towards us and pushing the right side in with the magnetic field pointing right to left.



Initially, the flux through the coil is zero. When the loop is rotated, the flux linkage leftward through the coil begins to **increase**.

To produce effects to oppose this increase in leftward flux linkage, the current induced in the coil will flow **counter-clockwise** (when viewed along the axis of the coil from the right-hand side) to produce its own magnetic field pointing left to right.

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Example 5

Page 6

Predict the direction of the induced current in the circular coil for each case:



←  $I$  increasing

Increasing the current in a wire that is placed below a coil in the plane of the page.

The magnetic field produced within the coil by the current in the wire points into the page (plane of the loop). As the current increases, the inward magnetic flux linkage through the coil **increases**.

The induced current in the coil will flow **counter-clockwise**, to produce a magnetic field out of the plane of the page.

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Changing Magnetic Flux  
→ E.M.F. & Current

**Query:** The  $B$  field set up by the induced current points to the right to oppose the increase in flux linking the stationary circuit. Is it possible to cancel the original  $B$  field?

Even when fields oppose, they cannot cancel completely because:

- If fields cancelled → No change in flux
- If no flux change → No induced current
- If no induced current → No opposing field
- Therefore, original field would exist again, restarting the induction process

Always some net field present to maintain induction.

magnet approaches a stationary circuit at constant speed  $v$

Faraday's law:  
small rate of increase of  $\Phi \rightarrow$  induced e.m.f.

Lenz' law:  
 $\Phi$  increases leftward, induced current produces a magnetic field (the effect) pointing to the right to oppose the increase.  
Using right hand grip rule, direction of current is anticlockwise (view from right to left)

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Changing Magnetic Flux  
→ E.M.F. & Current

**Note:** When a magnet approaches a stationary circuit at **constant** speed, the induced current doesn't remain constant but varies with time.

Despite constant speed:

- Field strength follows inverse square law
- Rate of flux change varies with position
- Maximum current when rate of flux change greatest
- Current depends on  $d\Phi/dt$ , not just speed

This creates a bell-shaped variation of current with time.

magnet approaches a stationary circuit at constant speed  $v$

Faraday's law:  
small rate of increase of  $\Phi \rightarrow$  induced e.m.f.

Lenz' law:  
 $\Phi$  increases leftward, induced current produces a magnetic field (the effect) pointing to the right to oppose the increase.  
Using right hand grip rule, direction of current is anticlockwise (view from right to left)

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**Example 6 (moving rod) - A slidewire generator**

A rod of length 0.20 m moves at a steady speed of 3.0 m s<sup>-1</sup> at right angles to a magnetic field of flux density 0.10 T.

a) Use Faraday's law to determine the e.m.f. induced across the ends of the wire.

Also determine which end is the positive terminal.

rod moves a distance of  $\Delta s$  in time  $\Delta t$

$\Delta A = L \Delta s$

With a single conductor,  $N = 1$

Using Faraday's law,

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t} = -\frac{\Delta(NBA)}{\Delta t}$$

counterclockwise around the loop

$$= -\frac{\Delta(NBL\Delta s)}{\Delta t}$$
$$= -BLv$$
$$= -(0.10)(0.20)(3.0)$$
$$= -0.060 \text{ V}$$

The induced e.m.f. across the ends of the rod is 60 mV.

Electrons in the rod are made to move to the right, which corresponds to a conventional current flowing to the left.

Using Fleming's Left-Hand Rule, the magnetic force on the electrons acts downward along the rod, causing the lower end of the rod to accumulate negative charge and become negatively charged.

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**Example 6 (moving rod) - A slidewire generator**

A rod of length 0.20 m moves at a steady speed of 3.0 m s<sup>-1</sup> at right angles to a magnetic field of flux density 0.10 T.

a) Use Faraday's law to determine the e.m.f. induced across the ends of the wire.

Also determine which end is the positive terminal.

induced current

**Applying Fleming's Right-Hand Rule**

1. First finger → Magnetic field ( $B$ ) [into plane  $\otimes$ ]
2. Thumb → Motion ( $v$ ) [to the right →]
3. Second finger → Induced current ( $I$ ) [points up ↑]

**Result**

Current flows:

- Upwards in moving rod
- Around circuit through fixed conductors
- Completes circuit through resistor/load

The e.m.f. is directed **counterclockwise** around the loop. The induced current is also counterclockwise, as shown.

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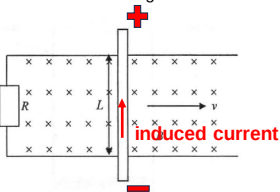
Example 6 (moving rod) - A slidewire generator

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A rod of length 0.20 m moves at a steady speed of 3.0 m s<sup>-1</sup> at right angles to a magnetic field of flux density 0.10 T.

b) Determine magnitude and direction of current if R = 2.0 Ω.

Indicate direction using arrows on the circuit diagram.



$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}} = \frac{60 \text{ mV}}{2.0 \, \Omega} = 30 \text{ mA}$$

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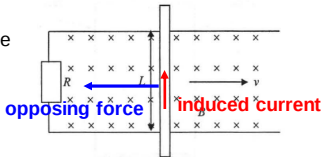
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Example 6 (moving rod) - A slidewire generator

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A rod of length 0.20 m moves at a steady speed of 3.0 m s<sup>-1</sup> at right angles to a magnetic field of flux density 0.10 T.

c) How to apply Lenz's law to example 6 (b)?



To oppose the motion of the rod to the right, Lenz's Law states that the induced current will flow in a direction that creates a magnetic force to the left, counteracting the motion.

Using Fleming's Left-Hand Rule, the induced current flows upward through the rod, as illustrated in the diagram.

6

6

Exploring Energy Conservation with Lenz's Law

Page 8

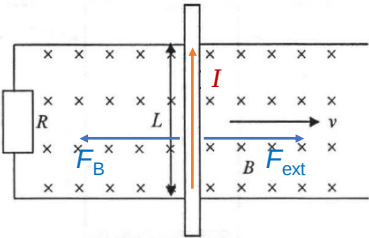
A magnetic force  $F_B$  is produced (to the left) due to the induced current, according to FLH rule:

$$F_B = BI_{\text{ind}}L$$

Hence an external force  $F_{\text{ext}}$  (same magnitude but opposite direction), need to be applied to maintain constant velocity  $v$ .

The rate of work done by external force  $F_{\text{ext}}$  is induced e.m.f.

$$F_{\text{ext}}v = BI_{\text{ind}}Lv = B\left(\frac{BLv}{R}\right)Lv = \frac{B^2L^2v^2}{R}$$



7

7

Exploring Energy Conservation with Lenz's Law

Page 8

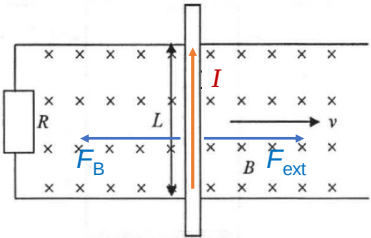
The electrical power dissipated in the resistor is

$$I_{\text{ind}}^2R = \left(\frac{BLv}{R}\right)^2R = \frac{B^2L^2v^2}{R}$$

So, work done against magnetic force by  $F_{\text{ext}}$  converts into electrical energy, and the electrical power = rate of doing work:

$$P = \frac{B^2L^2v^2}{R}$$

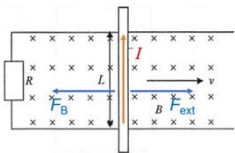
Lenz' law is a consequence of the principle of conservation of energy.



8

8

To sum up ...



The direction of the induced current in the circuit is such that the direction of the magnetic-field force is opposite in direction to its motion. Thus, the motion, which caused the induced current, is opposed. The induced current tries to preserve the status quo by opposing motion or a change of flux.

Lenz's Law is a direct consequence of the principle of energy conservation. If the induced current were to flow in the opposite direction, it could lead to a scenario where a conductor accelerates indefinitely without an external energy source, violating the conservation of energy.

9

# Learning Outcomes

f. explain simple applications of electromagnetic induction

EDDY CURRENTS  
AC GENERATOR  
TRANSFORMER

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## Example 7 (A Disc Rotating Perpendicularly in a Uniform Magnetic Field) - Faraday disk dynamo

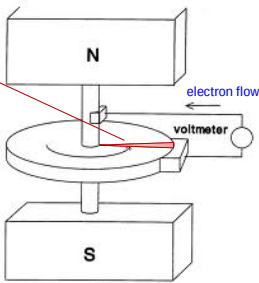
Page 9

When a disc of area  $A$  perpendicular to a uniform magnetic field  $B$  is rotated with constant frequency  $f$ , determine the e.m.f. induced between the **centre** and the **rim** of the disc.

We can regard the disc as a many-spoked wheel. The radius of the disc is sweeping through the magnetic field.

Area swept out by a radius per second,  $dA/dt$   
 $= (\pi r^2) f$   
Change in magnetic flux per second  $= B (dA/dt)$   
 $= B \pi r^2 f$

Thus,  $|\mathcal{E}| = B \pi r^2 f$



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11

## Example 7 (A Disc Rotating Perpendicularly in a Uniform Magnetic Field) - Faraday disk dynamo

Page 9

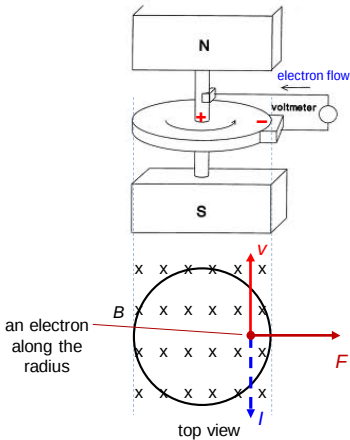
According to Fleming's LHR, the electrons in the rotating disc experience a magnetic force directed **radially outward**.

This force drives the electrons towards the rim of the disc, resulting in a net negative charge at the rim and a net positive charge at the center.

Consequently, the **center** of the disc has a **higher** electric potential than the rim.

If the rotation of the disc is reversed, the direction of the force changes, causing the rim to have a higher potential than the center.

This separation of charges generates an e.m.f. If a circuit is completed, such as by connecting a resistor between the rim and the center of the disc, the induced e.m.f. drives an electric current through the circuit.



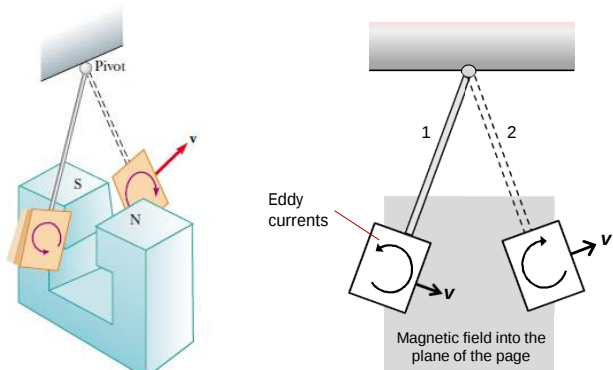
12

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Applications – Eddy currents

Page 10

Eddy currents braking system



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Applications – Eddy currents

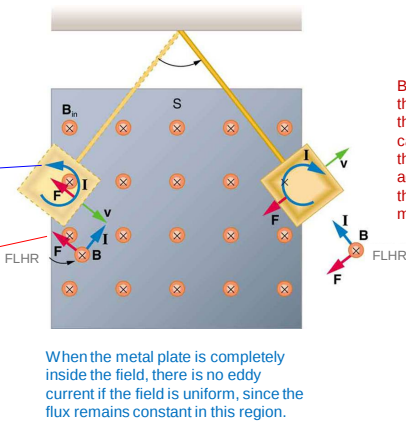
As it enters from the left, flux through the plate (into the page) increases, and so an eddy current is set up (Faraday's law).

By Lenz's Law: The induced eddy current creates its own magnetic field out of the page to oppose the change.

The direction of the eddy current is counterclockwise direction (Lenz's law), as shown.

Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (FLHR). [Return currents lie outside  $B$  so no  $F$  experienced.]

Heat energy is dissipated as eddy currents flows in a conductor (i.e., the conductor gets heated up).

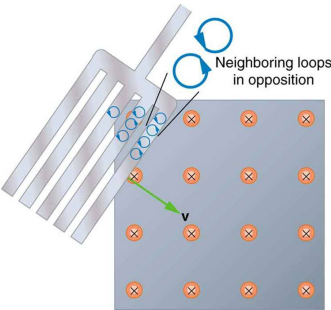


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Applications – Eddy currents

When a slotted metal plate enters the field, as shown, an e.m.f. is induced by the change in flux, but it is less effective because the slots limit the size of the current loops, by eliminating paths for the current flow.

Moreover, adjacent loops have currents in opposite directions, and their effects cancel.



When an insulating material is used, the eddy current is extremely small, and so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they can be slotted or constructed of thin layers of conducting material separated by insulating sheets.

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Magnetic Breaking

Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother.

However, eddy currents cannot bring the motion to a complete stop, since the force produced decreases with speed. Thus, speed can be reduced from say  $20 \text{ m s}^{-1}$  to  $5 \text{ m s}^{-1}$ , but another form of braking is needed to completely stop the vehicle.



Generally, powerful rare earth magnets such as neodymium magnets are used in roller coasters. The figure shows rows of magnets in such an application.

The vehicle has metal fins (normally containing copper) attached to the underside of the train.

As the train moves past the magnets, eddy currents are induced in the metal fins, creating opposing  $B$  field and generating a force to slow the vehicle down in much the same way as with the pendulum bob.

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Review our previous lecture:

- Faraday Disk Dynamo
- Eddy currents & braking force

Example 7 - Faraday disk dynamo

According to Fleming's LHR, the electrons in the rotating disc experience a magnetic force directed radially outward.

This force drives the electrons towards the rim of the disc, resulting in a net negative charge at the rim and a net positive charge at the center.

Consequently, the **center** of the disc has a **higher** electric potential than the rim.

This charge separation creates an electric field  $E$  from centre to rim.

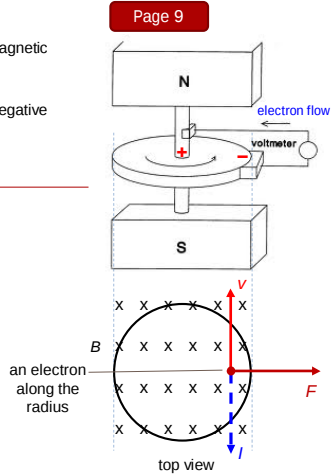
For any electron of charge  $q$  in the rotating disc, there are two forces: the magnetic force ( $q\mathbf{v} \times \mathbf{B}$ ) acting outward and the electric force ( $q\mathbf{E}$ ) acting inward.

$\langle v \rangle$  is the average speed of electrons along a rotating radius, as they move at different speeds  $v$ , depend on the distance from the rotating axis.

$$\langle v \rangle = \frac{1}{2} (0 + r\omega) = r\omega / 2 = \pi r f$$

At equilibrium, these forces balance ( $qE = q\langle v \rangle B$ ), giving  $E = \langle v \rangle B$ .

The p.d. across the centre and rim is  $\mathcal{E} = Er = \langle v \rangle B r = \underline{B \pi r^2 f}$



Example 7 - Faraday disk dynamo

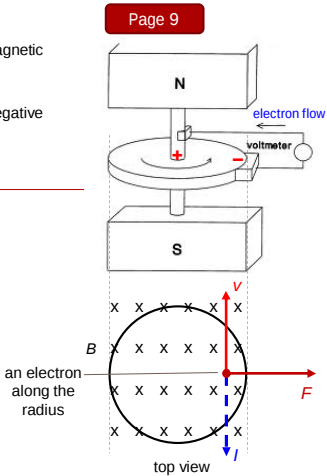
According to Fleming's LHR, the electrons in the rotating disc experience a magnetic force directed radially outward.

This force drives the electrons towards the rim of the disc, resulting in a net negative charge at the rim and a net positive charge at the center.

Consequently, the **center** of the disc has a **higher** electric potential than the rim.

The continuous disc rotation maintains this equilibrium state, sustaining the charge separation and thus the e.m.f.  $\mathcal{E}$ , much like how a chemical battery maintains its potential difference.

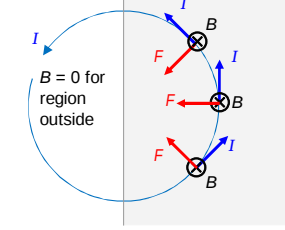
When the circuit is closed through sliding contacts, this e.m.f. drives current, converting mechanical energy to electrical energy.



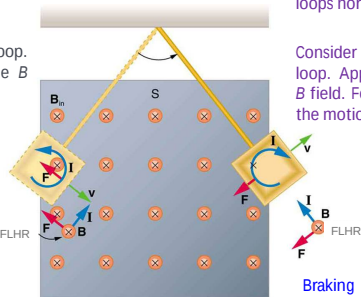
Eddy currents & braking force

At the leading edge of plate entering the field: Strong flux change occurs here. Strong eddy currents flow counter-clockwise in loops normal to both the motion and  $B$  field.

Consider current elements along one loop. Apply FLHR to each element within the  $B$  field.



Vector sum of forces on all the current elements points to the left.



Multiple eddy current loops all contribute to leftward braking force.

At the trailing edge leaving the field: Strong flux change occurs here too. Strong eddy currents flow clockwise in loops normal to both motion &  $B$  field.

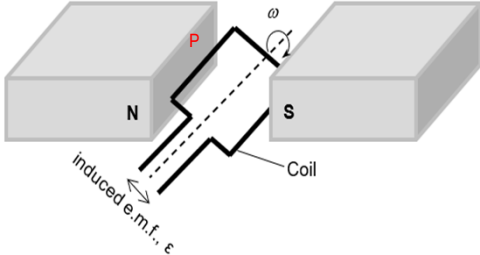
Consider current elements along one loop. Apply FLHR to each element within  $B$  field. Forces still sum to left, opposing the motion.

Braking force  $\propto$  velocity, leading to gradual deceleration of the plate. Oscillations decrease progressively until the plate eventually stops.

### Applications - A.C. Generator

Page 11

By turning the coil, the magnetic flux linkage  $\Phi$  through the coil is changing continuously, resulting in an induced e.m.f. in the coil.



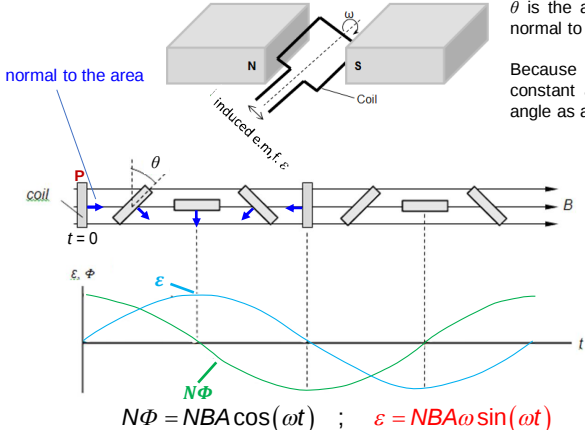
mechanical energy converted  electrical energy

5

### Applications - A.C. Generator

$\theta$  is the angle between  $B$  and the normal to the area  $A$ .

Because the angular speed  $\omega$  is constant and  $\theta = 0$  at  $t = 0$ , the angle as a function of time is  $\theta = \omega t$ .



$\epsilon = -\frac{d(N\Phi)}{dt}$

$N\Phi = NBA \cos(\omega t) \quad ; \quad \epsilon = NBA\omega \sin(\omega t)$  (See derivation)

6

### Applications - A.C. Generator

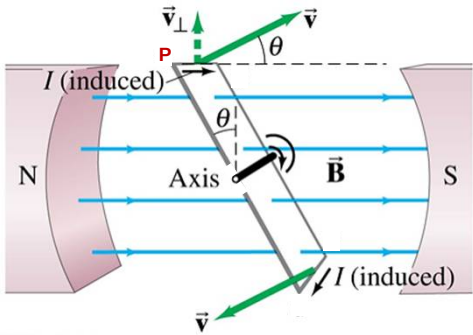
Page 11

$N\Phi = NBA \cos \theta \quad , \quad \theta = \omega t$

By Faraday's Law,

$$\epsilon = -\frac{d(N\Phi)}{dt}$$
$$= -\frac{d[NBA \cos(\omega t)]}{dt}$$
$$= -NBA \frac{d[\cos(\omega t)]}{dt}$$
$$= NBA\omega \sin(\omega t)$$

$N\Phi$  leads  $\epsilon$  by  $\frac{\pi}{2}$  rad



7

### Alternating Current Generator Simulation

[https://javalab.org/en/ac\\_generator\\_en/](https://javalab.org/en/ac_generator_en/)

**Variation with time of magnetic flux linkage and induced alternating e.m.f.**

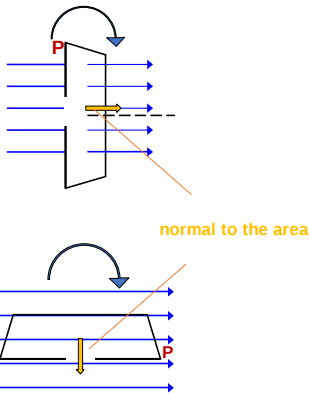
<https://www.physicslens.com/tag/ac-generator/>

8

Applications - A.C. Generator

Page 11

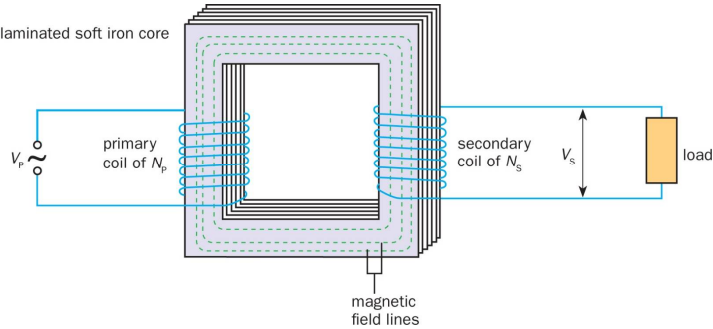
When the plane of the coil is perpendicular to the magnetic field ( $\theta = 0$ ), induced e.m.f. = 0.



When the plane of the coil is parallel to the field ( $\theta = 90^\circ$ ), the induced e.m.f. is at its peak value (peak e.m.f. =  $NBA\omega$ )

Applications - Transformer

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To be covered in the next topic **Alternating Current**.

Use the Faraday's law to explain the operation of a transformer:

Changing current  $\rightarrow$  changing flux in primary coil & core

Changing flux in secondary coil  
Faraday's law  $\rightarrow$  e.m.f. induced in secondary

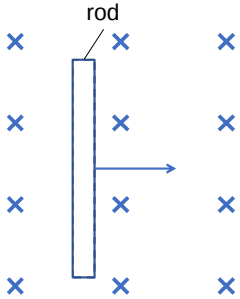
Core links the flux from primary to secondary coil

Summary

|   | Situation                                                                 | Formulae for induced e.m.f., $\varepsilon$ |
|---|---------------------------------------------------------------------------|--------------------------------------------|
| 1 | Moving rod in uniform $B$ field<br><i>Slidewire generator</i>             | $\varepsilon = BLv$                        |
| 2 | Rotating circular disc in uniform $B$ field<br><i>Faraday disk dynamo</i> | $\varepsilon = BAf = B\pi r^2 f$           |
| 3 | AC Generator                                                              | $\varepsilon = NBA\omega \sin(\omega t)$   |

Motional e.m.f.: Cutting of Magnetic Field Lines

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B into page

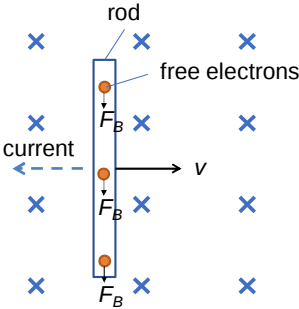
- A conductor moving at **constant velocity** in a **uniform magnetic field** cuts the magnetic field (flux).
- This motion induces an **e.m.f.** in the conductor.
- Induced e.m.f. is proportional to the rate of flux cutting.

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Motional e.m.f.: Cutting of Magnetic Field Lines

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B into page

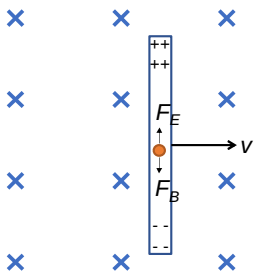
- The conductor moves to the **right** through a magnetic field (**B**) directed **into the page**.
- **Free electrons** in the conductor move to the **right**, causing conventional current to flow to the **left**.

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Motional e.m.f.: Cutting of Magnetic Field Lines

Page 14



Based on **Fleming's Left-Hand Rule**:

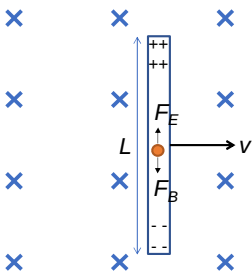
- Each electron experiences a **downward magnetic force**,  $F_B = Bqv$
- Electrons accumulate at the **lower end**, leaving a net **positive charge** at the **upper end**.

15

15

Motional e.m.f.: Cutting of Magnetic Field Lines

Page 14



As a result:

- A **downward electric field** (**E**) is generated inside the conductor.
- Charge accumulation continues until:
  - The **downward magnetic force** ( $F_B = qvB$ ) is balanced by the **upward electric force** ( $F_E = qE$ ).

$$\begin{aligned} F_E &= F_B \\ qE &= qvB \\ E &= vB \end{aligned}$$

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### Magnetic Field

**Electric interactions:**

1. A distribution of electric charge creates an electric field  $E$  in the surrounding space.
2. The electric field exerts a force  $F = qE$  on any other charge  $q$  that is present in the field.

Similarly,

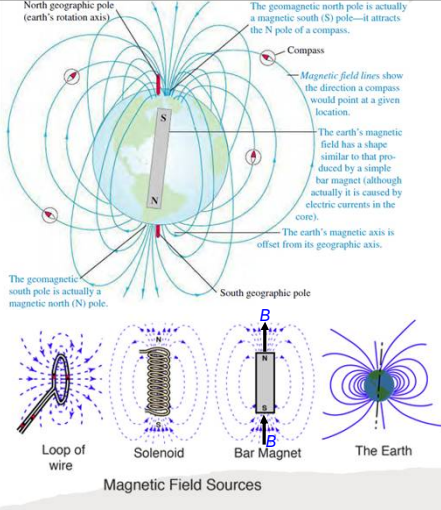
**Magnetic interactions:**

1. A moving charge or a current creates a magnetic field  $B$  in the surrounding space (in addition to its *electric* field).
2. The magnetic field exerts a force  $F$  on any other moving charge or current that is present in the field.

1

### Overview of Magnetic Fields

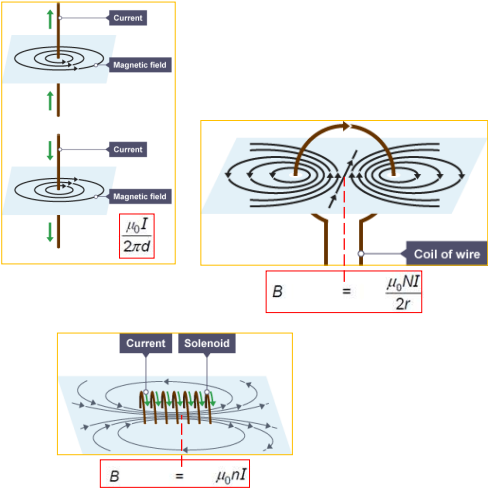
- Magnetic field  $B$  is a region of space where a magnetic pole, a current-carrying conductor or a moving charge particle will experience a force.
- $B$  is a vector field. At any position the direction of  $B$  is defined as the direction in which the north pole of a compass tends to point.
- The sources: current-carrying conductors and permanent magnets.
- The shape of the magnetic field pattern depends on the geometry of the current-carrying conductor.



2

### Magnetic Flux Patterns

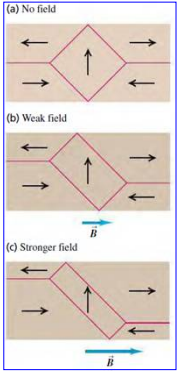
- Flux patterns for:
  - Long straight wire
  - Flat circular coil
  - Long solenoid
- Visual representations of field lines



3

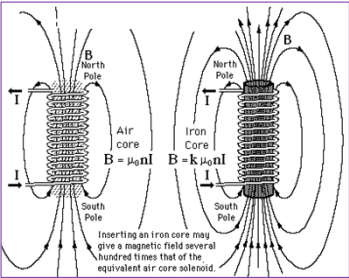
In ferromagnetic materials (iron, nickel, cobalt), strong interactions between atomic magnetic dipole moments cause them to line up parallel to each other in regions called **magnetic domains**, even when no external field is present.

The arrows show the directions of magnetization in the domains of a single crystal of nickel.



When there is no externally applied field, the domain magnetizations are randomly oriented.

Domains that are magnetized in the direction of an applied magnetic field **grow larger**.



The internal magnetic domains of the iron line up with the smaller driving magnetic field produced by the current in the solenoid.

The effect is the multiplication of the magnetic field by factors of tens to even thousands.

4

### Force on Current-Carrying Conductors in Magnetic Fields

$F = BIl \sin \theta$      $\theta$  is the smaller angle between  $I$  and  $B$

- Force exists at any angle except when conductor is parallel to field
- Maximum force occurs when conductor is perpendicular to field ( $\theta = 90^\circ$ )
- Force is always perpendicular to both:
  - The magnetic field direction
  - The direction of current flow

$\vec{F} = I \vec{l} \times \vec{B}$     (Cross product of its  $I \vec{l}$  and  $\vec{B}$ )

5

### Concept of magnetic flux density ( $B$ )

The *magnetic flux density*  $B$  of a magnetic field is defined as the force per unit length per unit current acting on a straight conductor placed normal to the magnetic field.

$B = \frac{F}{Il}$

Tesla: If a long straight conductor carrying a current of one ampere is placed normal to a uniform magnetic field of flux density one **tesla** (T), then the force per unit length on the conductor is one newton per metre.

6

### Measuring $B$ Using Current Balance

- By principle of a force acting on a current-carrying conductor in  $B$  field
- $I$  through wire PQRS
- $F$  acting downwards on QR
- Place rider of mass  $m$  at the other end to balance  $F$
- Moment due to  $F =$  moment due to weight of rider
- $(BIl)d = (mg)x \Rightarrow B = \frac{mgx}{Il d}$
- Only used in measuring steady  $B$

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### Forces between current-carrying conductors

The field due to  $I_1$  at a distance  $r$  is given to be

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

This field is uniform along wire 2 and perpendicular to it, so the force  $F_2$  it exerts on wire 2 is given by  $F = BIl \sin \theta$  with  $\sin \theta = 1$ :

$$F_2 = I_2 l B_1$$

Note that  $F_1 = -F_2$  based on Newton's 3<sup>rd</sup> law, and so we just write  $F$  for the magnitude of  $F_2$ .

Since the wires are very long, it is convenient to think in terms of  $F/l$ , the force per unit length.

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$F$  is **attractive** if the currents are in the **same** direction and **repulsive** if they are in **opposite** directions.

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### Force on moving charges

Four characteristics:

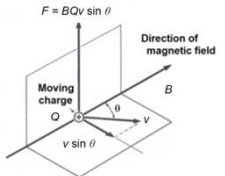
- its magnitude is proportional to the magnitude of the charge  $Q$
- the magnitude of the force is also proportional to  $B$
- depends on the particle's velocity  $v$ 
  - different from the electric-field force, which is the same whether the charge is moving or not
  - a charged particle at rest experiences no magnetic force
- does not have the same direction as the magnetic field  $B$  but instead, is always perpendicular to both  $B$  and the velocity  $v$ 
  - the magnitude  $F$  of the force is proportional to the component of  $v$  perpendicular to  $B$ ,  $v_{\perp} = v \sin \theta$
  - when that component is zero (that is, when  $v$  and  $B$  are parallel or antiparallel), the force is zero

$F = BQv \sin \theta$

$\theta$  is the smaller angle between  $v$  and  $B$

$F = Q \mathbf{v} \times \mathbf{B}$

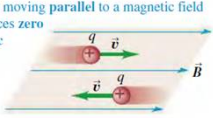
$(Q \text{ times the cross product of its } \mathbf{v} \text{ and } \mathbf{B})$



9

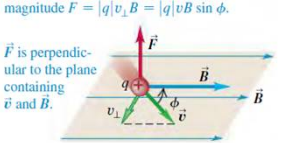
### Examples of the magnetic force $F$ acting on a moving charge $q$ moving at an angle $\phi$ :

A charge moving parallel to a magnetic field experiences zero magnetic force.

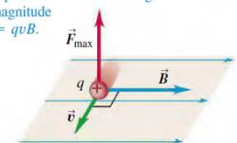


A charge moving at an angle  $\phi$  to a magnetic field experiences a magnetic force with magnitude  $F = |q|v_{\perp}B = |q|vB \sin \phi$ .

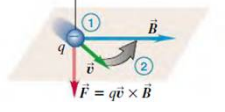
$\vec{F}$  is perpendicular to the plane containing  $\vec{v}$  and  $\vec{B}$ .



A charge moving perpendicular to a magnetic field experiences a maximal magnetic force with magnitude  $F_{\max} = qvB$ .



If the charge is negative, the direction of the force is opposite to that given by the Fleming's Left Hand Rule.



10

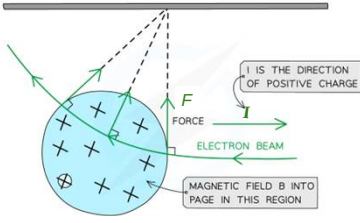
### Force on moving charges

The **direction** of magnetic force on the charged particle can be determined using **Fleming's Left Hand Rule**.

The image below shows an **electron** incident on a uniform magnetic field  $B$  directed into the page:

According to Fleming's Left Hand Rule:

- $B$  is directed into the page, therefore the first finger should point **into** the page
- The conventional current (or velocity of a **positive charge**) is directed to the **right** (because an electron is moving to the left), therefore the second finger should point to the **right**
- Therefore, the force on the electron as shown by the thumb is initially **upwards** as it enters the magnetic field



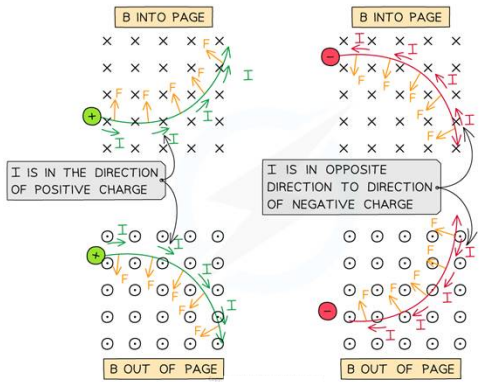
The force due to the magnetic field is **always perpendicular to the velocity** of the electron

- Note:** this is equivalent to **circular motion**
- Therefore, the magnetic force on a moving charge is a **centripetal force**

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### Force on moving charges

- The centripetal force is what keeps moving charges following a **circular trajectory**
- Examples of the direction of the magnetic force on positive and negative particles are:



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