General Certificate of Education Ordinary Level 2016
Additional Mathematics 4047/01
Syllabus 4047 Paper 1
SUGGESTED Solutions & Marking Scheme Mark Allocation
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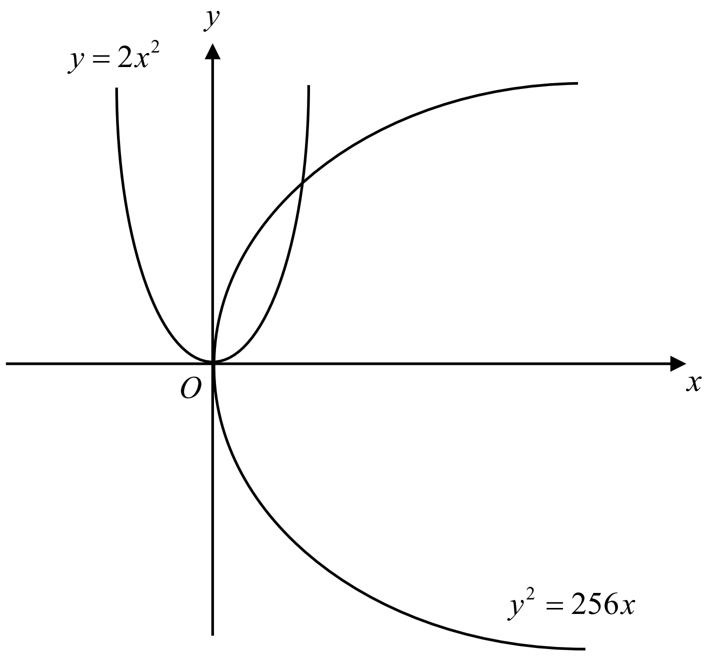
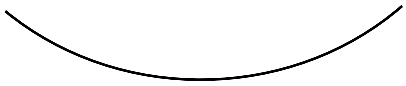
1	(i)	$y = 12 - 2x \ \ ---(1)$ $y = 2x^2 - 6x - 4 \ \ ---(2)$			
		Substitute (1) into (2) $2x^2 - 6x - 4 = 12 - 2x$ $2x^2 - 4x - 16 = 0$ $2\left(x^2 - 2x - 8\right) = 0$ $2(x+2)(x-4) = 0$	M1	Factorisation o.e.	
		$x = -2$ or $x = 4$			
		When $x = -2$, $y = 16$. When $x = 4$, $y = 4$.	M1	Attempt to find values of y	
		(–2, 16) and (4, 4)	A1		
	(ii)	$2x^2 - kx - 4 = 12 - 2x$ $2x^2 + (2-k)x - 16 = 0$			
		Discriminant $= (2-k)^2 - 4(2)(-16)$ $= (2-k)^2 + 128$	M1	Attempt to find discriminant	
		Since $(2-k)^2 \geq 0$, then $(2-k)^2 + 128 \geq 128 > 0$. Hence, discriminant > 0 for all values of *k*, the line intersects the curve at two distinct points. (Shown)	A1 AG	Must justify clearly that $D > 0$	
					[5]
2		$\left(\sqrt{6}+\sqrt{2}\right)^2 h = 16 + 4\sqrt{3}$ $h = \dfrac{16+4\sqrt{3}}{6+2\sqrt{12}+2}$	M1	Expand $\left(\sqrt{6}+\sqrt{2}\right)^2$ correctly	
		$h = \dfrac{16+4\sqrt{3}}{8+4\sqrt{3}} \times \dfrac{8-4\sqrt{3}}{8-4\sqrt{3}}$	M1	Multiply by conjugate s.o.i.	
		$= \dfrac{128+32\sqrt{3}-64\sqrt{3}-16(3)}{64-16(3)}$	M1	Expand	
		$= \dfrac{80-32\sqrt{3}}{16}$ $= 5 - 2\sqrt{3}$	A2	A1 for each term Deduct A1 for wrong form	
					[5]

3	(ai)	$-90° \le \sin^{-1} x \le 90°$ or $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$	B1	o.e.	
	(aii)	$0 \le \cos^{-1} x \le 180°$ or $0 \le \cos^{-1} x \le \pi$	B1	o.e.	
	(b)	$a = -1$	B1		
		$b = 6$	B1		
		$c = 2$	B1		
					[5]
4	(i)		B3	B1 for correct shape for each graph B1 for relative scale of graphs (i.e. $y^2 = 256x$ is much wider than $y = 2x^2$)	
	(ii)	$(2x^2)^2 = 256x$ $4x^4 - 256x = 0$ $4x(x^3 - 64) = 0$ $x = 0$ or $x = \sqrt[3]{64} = 8$	M1		
		$y = 8x$	A1		
					[5]

5		$\frac{2x^2+4x-31}{x^2+x-6}=2+\frac{2x-19}{x^2+x-6}$	M1	Use of long division o.e.	
		Let $\frac{2x-19}{x^2+x-6}=\frac{A}{x-2}+\frac{B}{x+3}$	M1	Partial fractions	
		$2x-19=A(x+3)+B(x-2)$			
		When $x=2$, $-15=5A\Rightarrow A=-3$	M1	FTA	
		When $x=-3$, $-25=-5B\Rightarrow B=5$	M1	FTA	
		$\frac{2x^2+4x-31}{x^2+x-6}=2-\frac{3}{x-2}+\frac{5}{x+3}$	A1		
					[5]
6	(i)	Since $x=4$ is the line of symmetry, then $\frac{p+6}{2}=4$. $p+6=8\Rightarrow p=2$ (Shown)	B1	Working shown clearly	
	(ii)	The equation of the curve is $y=\lvert a(x-6)(x-2)\rvert$.			
		When $x=4$, $y=6$. $6=\lvert -4a\rvert\Rightarrow a=\frac{3}{2}$ $(a>0)$	B1		
		$y=\frac{3}{2}(x-6)(x-2)$ $=\frac{3}{2}\left(x^2-6x-2x+12\right)$	M1	Expand	
		$=\frac{3}{2}x^2-12x+18$			
		$b=-12$	A1		
		$c=18$	A1		
	(iii)	$0<q<6$	B2	B1 for correct use of inequality B1 for correct limits used	
					[7]

<table>
<tr><td>7</td><td>(i)</td><td>$OP = 5t$ m
$OQ = (100-10t)$ m</td><td>B1</td><td>Both seen</td><td></td></tr>
<tr><td></td><td></td><td>By Pythagoras’ Theorem,
$s = \sqrt{(5t)^2 + (100-10t)^2}$</td><td>M1</td><td>s.o.i.</td><td></td></tr>
<tr><td></td><td></td><td>$= \sqrt{25t^2 + 10000 - 2000t + 100t^2}$
$= \sqrt{125t^2 - 2000t + 10000}$
$= \sqrt{125\left(t^2 - 16t + 80\right)}$ (Shown)</td><td>A1
AG</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(ii)</td><td>$\frac{\mathrm{d}s}{\mathrm{d}t} = \sqrt{125}\left(t^2 - 16t + 80\right)^{-\frac{1}{2}} \cdot \frac{1}{2} \cdot (2t-16)$</td><td>M1</td><td>Attempt to use chain rule</td><td></td></tr>
<tr><td></td><td></td><td>$= \frac{\sqrt{125}(t-8)}{\sqrt{t^2 - 16t + 80}} = \frac{5\sqrt{5}(t-8)}{\sqrt{t^2 - 16t + 80}}$</td><td>A1</td><td>o.e.</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(iii)</td><td>$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\sqrt{125}(t-8)}{\sqrt{t^2 - 16t + 80}} = 0$
$\Rightarrow t = 8$
When $t = 8$, $s = 44.72135955 = 44.7$ (3 s.f.)</td><td>B1</td><td></td><td></td></tr>
<tr><td></td><td></td><td>By first derivative test,
<table>
<tr><td>x</td><td>7.9</td><td>8</td><td>8.1</td></tr>
<tr><td>$\frac{\mathrm{d}y}{\mathrm{d}x}$</td><td>**–0.279**</td><td>**0**</td><td>**2.279**</td></tr>
<tr><td>Sketch of tangent</td><td>╲</td><td>—</td><td>╱</td></tr>
<tr><td>Sketch of curve</td><td colspan="3"></td></tr>
</table>
Thus, $s = 44.7$ is the least distance.</td><td>B1</td><td>Justify using 1st or 2nd derivative test
For 1st derivative test, values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be calculated</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>[7]</td></tr>
</table>

8	(i)	$2y=-3x+45\Rightarrow y=-\frac{3}{2}x+\frac{45}{2}\quad ---(1)$			
		Gradient of $AB=\frac{2}{3}$	M1	s.o.i.	
		$y=\frac{2}{3}x+c$ $6=\frac{2}{3}(-2)+c\Rightarrow c=\frac{22}{3}$ $y=\frac{2}{3}x+\frac{22}{3}\quad ---(2)$	M1	Find equation of *AB* s.o.i.	
		Substitute (1) into (2) $-\frac{3}{2}x+\frac{45}{2}=\frac{2}{3}x+\frac{22}{3}$	M1	Substitution or elimination s.o.i.	
		$\frac{13}{6}x=\frac{91}{6}$ $\Rightarrow x=7$ $\Rightarrow y=12$	M1	Find x or y s.o.i.	
		B (7, 12)	A1		
	(ii)	When $y=0$, $x=15$. C (15, 0)	B1	s.o.i.	
		$M=\left(\frac{15-2}{2},\frac{0+6}{2}\right)=\left(\frac{13}{2},3\right)$	M1		
		Let the coordinates of D be (p, q).			
		$\frac{p+7}{2}=\frac{13}{2}\Rightarrow p=6$ $\frac{q+12}{2}=3\Rightarrow q=-6$			
		D (6, –6)	A1		
					[8]

9	(i)	$\frac{dy}{dx} = -2x + \frac{32}{x^3}$	M1	o.e.	
		$\frac{dy}{dx} = 0 \Rightarrow -2x + \frac{32}{x^3} = 0$	M1	s.o.i.	
		$-2x^4 + 32 = 0$ $x^4 = 16$ $x = \pm 2$	M1	Solve for x	
		When $x = 2$, $y = -6$. When $x = -2$, $y = -6$.			
		(2, –6) and (–2, –6)	A2	A1 for each point	
	(ii)	$\frac{d^2y}{dx^2} = -2 - \frac{96}{x^4}$	M1	FT from (i)	
		When $x = 2$, $\frac{d^2y}{dx^2} = -2 - \frac{96}{2^4} = -8 < 0$. Hence, (2, –6) is a maximum point.	A1	Must calculate the value of $\frac{d^2y}{dx^2}$	
		When $x = -2$, $\frac{d^2y}{dx^2} = -2 - \frac{96}{(-2)^4} = -8 < 0$. Hence, (–2, –6) is a maximum point.	A1		
					[8]

10	(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{x^2}\right)=\frac{x^2\cdot\frac{1}{x}-\ln x\cdot 2x}{x^4}$	M2	M1 Attempt to use quotient rule M1 for correct differentiation	
		$=\frac{x-2x\ln x}{x^4}$ $=\frac{x}{x^4}-\frac{2x\ln x}{x^4}=\frac{1}{x^3}-\frac{2\ln x}{x^3}$ (Shown)	A1 AG		
	(ii)	$\int\frac{1}{x^3}-\frac{2\ln x}{x^3}\,\mathrm{d}x=\frac{\ln x}{x^2}+c_1$	M1	Knows to use result from (i)	
		$-2\int\frac{\ln x}{x^3}\,\mathrm{d}x=\frac{\ln x}{x^2}+c_1-\int x^{-3}\,\mathrm{d}x$ $=\frac{\ln x}{x^2}+c_1-\frac{x^{-2}}{-2}+c_2$	M1	Integration of x^{-3} s.o.i.	
		$\int\frac{\ln x}{x^3}\,\mathrm{d}x=-\frac{\ln x}{2x^2}-\frac{1}{4x^2}+d$, where $d=c_1+c_2$	A1		
	(iii)	$\mathrm{f}(x)=\int\frac{\ln x}{x^3}\,\mathrm{d}x=-\frac{\ln x}{2x^2}-\frac{1}{4x^2}+d$			
		$\mathrm{f}(1)=\frac{3}{4}\Rightarrow-\frac{1}{4}+d=\frac{3}{4}\Rightarrow d=1$	M1	Attempt to find arbitrary constant	
		$\mathrm{f}(x)=-\frac{\ln x}{2x^2}-\frac{1}{4x^2}+1$	A1	o.e.	
					[8]

11	(a)	$(\sec\theta-\tan\theta)^2$ $=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2$	M1	Change $\tan\theta$ to $\frac{\sin\theta}{\cos\theta}$ s.o.i.	
		$=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2$	M1	Join fraction	
		$=\frac{(1-\sin\theta)^2}{\cos^2\theta}$ $=\frac{(1-\sin\theta)^2}{1-\sin^2\theta}$	M1	Use of $\sin^2\theta+\cos^2\theta=1$ s.o.i.	
		$=\frac{(1-\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}$ $=\frac{1-\sin\theta}{1+\sin\theta}$	A1 AG		
	(bi)	$k=\frac{\pi}{30}$	B1		
		NOT within the 4049 syllabus.			
	(bii)	$\pm 40=80\sin kt$ $\sin kt=\pm\frac{1}{2}$ $\alpha=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$	M1	Find basic angle s.o.i.	
		$\frac{\pi}{30}t=\frac{\pi}{6},\ \pi-\frac{\pi}{6},\ \pi+\frac{\pi}{6},\ 2\pi+\frac{\pi}{6}$ $t=5,\ 25,\ 35,\ 55$	M1		
		40 minutes	A1		
					[8]

12	(i)	$\int_0^{\frac{\pi}{6}} \mathrm{f}(x)\,dx = \frac{3}{4} \Rightarrow \left[\sin x + k\cos 2x\right]_0^{\frac{\pi}{6}} = \frac{3}{4}$ $\sin\frac{\pi}{6} + k\cos 2\left(\frac{\pi}{6}\right) - \left(\sin 0 + k\cos 2(0)\right) = \frac{3}{4}$ $\frac{1}{2} + \frac{1}{2}k - k = \frac{3}{4} \Rightarrow k = -\frac{1}{2}$ (Shown)	B1 AG	Show substitutions clearly	
	(ii)	$\mathrm{f}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\sin x - \frac{\cos 2x}{2} - \frac{1}{2}\right)$	M1	Knows to differentiate	
		$\mathrm{f}(x) = \cos x + \sin 2x$	A2	A1 for each term	
	(iii)	$\mathrm{f}\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} + \sin 2\left(\frac{\pi}{6}\right) = \sqrt{3}$	M1		
		$\mathrm{f}'(x) = -\sin x + 2\cos 2x$	M1		
		$\mathrm{f}'\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} + 2\cos 2\left(\frac{\pi}{6}\right) = \frac{1}{2}$	M1	FT from $\mathrm{f}'(x)$	
		Gradient of normal $= -2$	M1	FT from $\mathrm{f}'\left(\frac{\pi}{6}\right)$	
		$y = -2x + c$ $\sqrt{3} = -2\left(\frac{\pi}{6}\right) + c \Rightarrow c = \sqrt{3} + \frac{\pi}{3}$ $y = -2x + \sqrt{3} + \frac{\pi}{3}$	A1		
					[9]

END OF MARKING SCHEME

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