Name: _____ ( ) Class: _____ Date: _____

MA302.5.X2 – Simultaneous Equations with Matrices

Matrices can also be used to solve several unknowns quickly in simultaneous equations.

X1. For a pair of simultaneous equations involving two unknowns, it can be rewritten into a matrix equation. For example, given equations $2x+y=5$ and $4x+3y=21$, by writing them into a matrix form, we have

$$\begin{pmatrix} 2x+y \\ 4x+3y \end{pmatrix} = \begin{pmatrix} 5 \\ 21 \end{pmatrix}.$$

However, the matrix on the left hand side can be broken apart into a product of two:

$$\begin{pmatrix} 2x+y \\ 4x+3y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Hence we can rewrite the matrix equation as

$$\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 21 \end{pmatrix}.$$

Eventually, the problem simplifies into solving for the column matrix $\begin{pmatrix} x \\ y \end{pmatrix}$.

S1. **Rewrite** the following simultaneous equations into matrix equations.

(a) $2x-3y=5$ and $4y+6x=12$

(b) $4x-\frac{1}{y}=5$ and $3xy+2=7y$

(c) $2-x^2+3y=0$ and $4x^2+6y-9=0$

S2. Let $\mathbf{A}\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{B}$, where $\mathbf{A}$ is a two 2×2 matrix. Make $\begin{pmatrix}x\\y\end{pmatrix}$ the subject and explain the significance of the determinant of $\mathbf{A}$, i.e. $\det(\mathbf{A})$.

We have thus arrive at the following:

Given a pair of simultaneous equations with unknowns x and y, the solution can be obtained by

Step 1: Converting into the matrix equation $\mathbf{A}\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{B}$.

Step 2: Solve for $\begin{pmatrix}x\\y\end{pmatrix}$.

Of course, the method fails when $\det(\mathbf{A})=0$. In this case, the two equations may have no or infinite number of solutions, depending on the matrix B.

S3. Use a matrix method to solve the following pairs of simultaneous equations.

(a) $6x^2 - 11y = -28$ and $5y - 3x^2 = 13$

(b) $4\sqrt{x} - 5\sqrt{y} = 13$ and $10\sqrt{y} - 8\sqrt{x} = 18$

(c) $2x^2+3y^2=11$ and $x^2-y^2=3$

S4. Find the inverse of $\begin{pmatrix} 7 & -3 \\ 3 & 2 \end{pmatrix}$ and solve the simultaneous equations $7x-3y=-29$ and $12x+8y=16$.