Qn	Solution
1 [4]	Since θ is sufficiently small, $\sin\theta \approx \theta$. **<u>Method 1: Binomial Series (MF27)</u>** $\mathrm{f}(\theta) \approx \frac{1}{3-\theta}$ $= (3-\theta)^{-1}$ $= \left[3\left(1-\frac{1}{3}\theta\right)\right]^{-1}$ $= 3^{-1}\left[1+\left(-\frac{1}{3}\theta\right)\right]^{-1}$ $= \frac{1}{3}\left[1+\left(-\frac{1}{3}\theta\right)\right]^{-1}$ $= \frac{1}{3}\left[1+(-1)\left(-\frac{1}{3}\theta\right)+\frac{(-1)(-2)}{2!}\left(-\frac{1}{3}\theta\right)^2+\ldots\right]$ $= \frac{1}{3}\left(1+\frac{1}{3}\theta+\frac{1}{9}\theta^2+\ldots\right)$ $\approx \frac{1}{3}+\frac{1}{9}\theta+\frac{1}{27}\theta^2$ (shown) $\therefore p=\frac{1}{3}, q=\frac{1}{9}$ and $r=\frac{1}{27}$. **<u>Method 2: Repeated Differentiation</u>** $\mathrm{f}(\theta) \approx (3-\theta)^{-1}$ $\mathrm{f}'(\theta) \approx -(3-\theta)^{-2}(-1) = (3-\theta)^{-2}$ $\mathrm{f}''(\theta) \approx -2(3-\theta)^{-3}(-1) = 2(3-\theta)^{-3}$ Sub $\theta=0$: $\mathrm{f}(0)=3^{-1}=\frac{1}{3}$ $\mathrm{f}'(0)=(3-0)^{-2}=\frac{1}{9}$ $\mathrm{f}''(0)=2(3-0)^{-3}=\frac{2}{27}$ Using MF27, $\mathrm{f}(\theta) \approx \frac{1}{3}+\frac{1}{9}x+\frac{\frac{2}{27}}{2!}x^2+\ldots$ $\mathrm{f}(\theta) \approx \frac{1}{3}+\frac{1}{9}x+\frac{1}{27}x^2+\ldots$ (shown) $\therefore p=\frac{1}{3}, q=\frac{1}{9}$ and $r=\frac{1}{27}$.

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>2(i)
[3]</td><td>

From the question, the equation of the curve is $y=\dfrac{x^2+px+q}{x-2}$.

Since the vertical asymptote is $x=2 \Rightarrow r=2$.

C passes through the point $(0,-1.5)$.

$y=\dfrac{x^2+px+q}{x-2} \Rightarrow -1.5=\dfrac{0+0+q}{0-2} \Rightarrow q=3$

Hence, the equation of C is $y=\dfrac{x^2+px+3}{x-2}$ ----- (1)

C has asymptotes with equations $y=x-1$ and $x=2$.

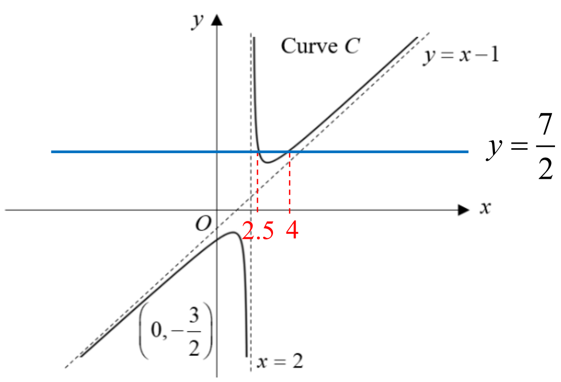
$\Rightarrow y=x-1+\dfrac{a}{x-2}$, where a is a constant.

$\therefore$ The equation of C is

$y=x-1+\dfrac{a}{x-2}=\dfrac{(x-1)(x-2)+a}{x-2} \Rightarrow y=\dfrac{x^2-3x+2+a}{x-2}$ -- (2)

Comparing (2) and (1), $p=-3$.

</td></tr>
<tr><td>2(ii)
[1]</td><td>



For $\mathrm{f}(x)=\dfrac{x^2-3x+3}{x-2}>\dfrac{7}{2}$, from graph,

$2<x<2.5$ or $x>4$.

</td></tr>
</table>

Qn	Solution
3(a) **[2]**	$\int \frac{x}{\sqrt{x^2-4}}\,\mathrm{d}x$ $=\frac{1}{2}\int 2x(x^2-4)^{-\frac{1}{2}}\,\mathrm{d}x$ $=\frac{1}{2}\left[\frac{(x^2-4)^{\frac{1}{2}}}{\frac{1}{2}}\right]+C$ $=\sqrt{x^2-4}+C$
3(b) **[3]**	Let $u=\ln(2x)$ $\quad \frac{\mathrm{d}v}{\mathrm{d}x}=x^2$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$ $\quad v=\frac{x^3}{3}$ $\int x^2\ln(2x)\,\mathrm{d}x$ $=\left(\frac{x^3}{3}\right)\ln(2x)-\int\frac{1}{x}\left(\frac{x^3}{3}\right)\mathrm{d}x$ $=\frac{1}{3}x^3\ln(2x)-\frac{1}{3}\int x^2 + \mathrm{d}x$ $=\frac{1}{3}x^3\ln(2x)-\frac{1}{9}x^3+C$
3(c) **[2]**	$\int\frac{\sin 2x}{1+\cos^2 x}\,\mathrm{d}x$ $=-\int\frac{-2\cos x\sin x}{1+\cos^2 x}\,\mathrm{d}x$ $=-\ln\left\|1+\cos^2 x\right\|+C$ $=-\ln\left(1+\cos^2 x\right)+C$

Qn	Solution
4(i) **[3]**	Let x, y and z be the usual selling price of Blend A, B and C coffee beans respectively. $x+y+z=176----(2)$ $0.85x+0.90y+0.95z=161.80----(2)$ $0.93x+0.95y+0.98z=169.56----(3)$ From GC: Hence x = \$32, y =\$44, z = \$100
4(ii) **[3]**	Price difference: $169.56-161.80=7.76$ To make it more attractive: Additional loyalty discount > price difference $\left(\frac{x}{100}\right)(32)+\left(\frac{x}{100}\right)(44)>7.76$ $x>10.2$ $x=11$ Note: $x\% = \frac{x}{100}$

Qn	Solution
5(i) [4]	Let $u = x^2 + 3 \Rightarrow \frac{du}{dx} = 2x$ $\int \frac{x^3}{\sqrt{x^2+3}}\,dx$ $= \int \frac{x^2}{\sqrt{x^2+3}}(x\ dx)$ $= \int \frac{u-3}{\sqrt{u}}\left(\frac{du}{2}\right)$ $= \frac{1}{2}\int u^{\frac{1}{2}} - 3u^{-\frac{1}{2}}\ du$ $= \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 3\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right) + C$ $= \frac{1}{3}u^{\frac{3}{2}} - 3u^{\frac{1}{2}} + C$ $= \frac{1}{3}\sqrt{\left(x^2+3\right)^3} - 3\sqrt{\left(x^2+3\right)} + C$ (shown) Or Let $u = x^2 + 3 \Rightarrow \frac{du}{dx} = 2x$ $\int \frac{x^3}{\sqrt{x^2+3}}\,dx$ $= \int \frac{(u-3)^{\frac{3}{2}}}{\sqrt{u}}\left(\frac{du}{2x}\right)$ Continue to replace x in terms of u if it can not be done easily with 1 step $= \int \frac{(u-3)^{\frac{3}{2}}}{\sqrt{u}}\left(\frac{du}{2(u-3)^{\frac{1}{2}}}\right)$ $= \int \frac{u-3}{\sqrt{u}}\left(\frac{du}{2}\right)$ $= \frac{1}{2}\int u^{\frac{1}{2}} - 3u^{-\frac{1}{2}}\ du$ $= \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 3\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right) + C$ $= \frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}} - 3\left(x^2+3\right)^{\frac{1}{2}} + C$ (shown)

5(ii) **[3]**

$$\int_{-1}^{3}\left|\frac{3x^3}{\sqrt{x^2+3}}\right|\,dx$$

$$=\int_{-1}^{0}\left|\frac{3x^3}{\sqrt{x^2+3}}\right|\,dx+\int_{0}^{3}\left|\frac{3x^3}{\sqrt{x^2+3}}\right|\,dx$$

$$=\int_{-1}^{0}\frac{-3x^3}{\sqrt{x^2+3}}\,dx \qquad +\int_{0}^{3}\frac{3x^3}{\sqrt{x^2+3}}\,dx$$

[original curve reflected in the x-axis] [Part of original curve]

$$=-\left[\sqrt{(x^2+3)^3}-9\sqrt{(x^2+3)}\right]_{-1}^{0}+\left[\sqrt{(x^2+3)^3}-9\sqrt{(x^2+3)}\right]_{0}^{3}$$

$$=-\left[\left(\sqrt{27}-9\sqrt{3}\right)-\left(\sqrt{64}-9\sqrt{4}\right)\right]+\left[\left(\sqrt{12^3}-9\sqrt{12}\right)-\left(\sqrt{3^3}-9\sqrt{3}\right)\right]$$

$$=-\left[\left(3\sqrt{3}-9\sqrt{3}\right)-\left(8-18\right)\right]+\left[\left(12\sqrt{12}-9\sqrt{12}\right)-\left(3\sqrt{3}-9\sqrt{3}\right)\right]$$

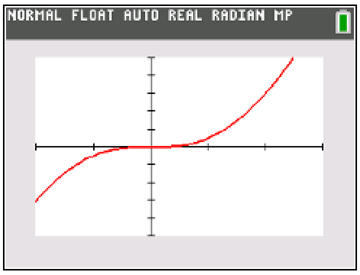
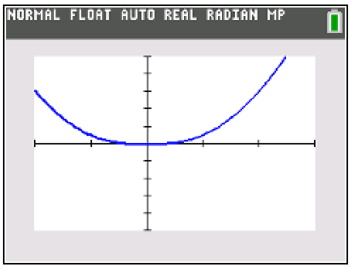
$$=-\left[10-6\sqrt{3}\right]+\left[\left(3\sqrt{12}\right)+6\sqrt{3}\right]$$

$$=-\left[10-6\sqrt{3}\right]+\left[\left(3\sqrt{4(3)}\right)+6\sqrt{3}\right]$$

$$=-\left[10-6\sqrt{3}\right]+\left[6\sqrt{3}+6\sqrt{3}\right]$$

$$=-10+18\sqrt{3}$$

Use GC:

Let $f(x)=\dfrac{3x^3}{\sqrt{x^2+3}}$	Let $f(x)=\left|\dfrac{3x^3}{\sqrt{x^2+3}}\right|$
	
$-1\le x<0$, we have $f(x)<0$ $0\le x\le 3$, we have $f(x)\ge 0$	$\int_{-1}^{0}\frac{-3x^3}{\sqrt{x^2+3}}\,dx$ [Reflection in x-axis] $+\int_{0}^{3}\frac{3x^3}{\sqrt{x^2+3}}\,dx$ [Part of original curve]

Qn	Solution
6(i) **[3]**	Area : $S = 2\times\left[\frac{1}{2}\times\pi\left(\frac{x}{2}\right)^2\right]+2xy$ $S=\frac{\pi x^2}{4}+2xy$ Perimeter: $25 = 2\times\left[\frac{1}{2}\times 2\pi\left(\frac{x}{2}\right)\right]+2x+2y$ $2y=25-\pi x-2x$ $S=\frac{\pi x^2}{4}+\left(25-\pi x-2x\right)x$ $S=\frac{\pi x^2}{4}+25x-\pi x^2-2x^2$ $S=25x-2x^2-\frac{3}{4}\pi x^2$ (shown)
6(ii) **[4]**	$S=25x-2x^2-\frac{3}{4}\pi x^2$ $\frac{\mathrm{d}S}{\mathrm{d}x}=25-4x-\frac{3}{2}\pi x$ At stationary point, $\frac{\mathrm{d}S}{\mathrm{d}x}=0$ $25-4x-\frac{3}{2}\pi x=0$ $50-8x-3\pi x=0$ $\left(8+3\pi\right)x=50$ $x=\frac{50}{8+3\pi}$ $\frac{\mathrm{d}S}{\mathrm{d}x}=25-4x-\frac{3}{2}\pi x$ $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=-4-\frac{3}{2}\pi = -8.71 \quad <0$ S is maximum area

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>7(i)
[2]</td><td>
GP: $a = 500,\ r = 0.6$
$u_0 = 500$
$u_1 = 0.6 \times 500 = 300$
$u_2 = 0.6 \times (0.6 \times 500) = 500 \times 0.6^2 = 180$ (verified)
</td></tr>
<tr><td>7 (ii)
[2]</td><td>
<table>
<tr><td>Week n</td><td>New case, u_n</td><td>Total Cumulative, S_n</td></tr>
<tr><td>0</td><td>500</td><td>500</td></tr>
<tr><td>1</td><td>500×0.6^1</td><td>$500 + 500 \times 0.6^1$</td></tr>
<tr><td>2</td><td>500×0.6^2</td><td>$500 + 500 \times 0.6^1 + 500 \times 0.6^2$</td></tr>
<tr><td>…</td><td>…</td><td>…</td></tr>
<tr><td>n</td><td>500×0.6^n</td><td>$500 + 500 \times 0.6^1 + 500 \times 0.6^2 + \ldots + 500 \times 0.6^n$</td></tr>
</table>
$S_n = 500 + 500 \times 0.6^1 + 500 \times 0.6^2 + \ldots + 500 \times 0.6^n$
$S_n = 500\left(1 + 0.6 + 0.6^2 + \ldots + 0.6^n\right)$
$S_n = 500\left[\dfrac{1\left(1 - 0.6^{n+1}\right)}{1 - 0.6}\right]$
$S_n = 1250\left(1 - 0.6^{n+1}\right)$ (shown)
</td></tr>
<tr><td>7
(iii)
[1]</td><td>
<u>Method 1: GC</u>
$1250\left(1 - 0.6^{n+1}\right) > 1240$
From GC
<table>
<tr><td>n</td><td>$1250\left(1 - 0.6^{n+1}\right)$</td></tr>
<tr><td>8</td><td>1237.4</td></tr>
<tr><td>9</td><td>$1242.4 > 1240$</td></tr>
</table>

<u>Method 2: Solving Inequality</u>
$S_n = 1250\left(1 - 0.6^{n+1}\right)$
$1250\left(1 - 0.6^{n+1}\right) > 1240$
$0.6^{n+1} < 1 - \dfrac{1240}{1250}$
$n > \dfrac{\ln\left(\dfrac{1}{125}\right)}{\ln(0.6)} - 1$
$\ln(0.6) < 0$, handle the inequality sign accordingly
$n > 8.45$
$n_c = 9$
</td></tr>
</table>

7 (iv) [2]	$S_n = 1250\left(1-0.6^{n+1}\right)$ $n \to \infty,\ 0.6^{n+1} \to 0,\ S_n \to 1250$ Since the maximum cumulative infection is 1250 < 1280, the patient capacity will be sufficient.

Qn	Solution
8 (i) [2]	**Method 1** $y = \mathrm{e}^{2\sin^{-1}x} \quad \Rightarrow \quad \ln y = 2\sin^{-1}x$ Differentiating both sides wrt x: $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x} = 2y \quad \text{(shown)}$ **Method 2** $y = \mathrm{e}^{2\sin^{-1}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2\sin^{-1}x} \times \frac{2}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{2\sin^{-1}x}$ $\sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x} = 2y \quad \text{(shown)}$
8 (ii) [4]	$\left(1-x^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 4y^2$ Differentiating both sides wrt x: $\left(1-x^2\right)\times 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) + (-2x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 8y\frac{\mathrm{d}y}{\mathrm{d}x}$ --- ***Alternatively,*** $\left(1-x^2\right)^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}x} = 2y$ Differentiating both sides wrt x: $\left(1-x^2\right)^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)\frac{\mathrm{d}y}{\mathrm{d}x} = 2\frac{\mathrm{d}y}{\mathrm{d}x}$ ---

	When $x=0$, $y=\mathrm{e}^{2\sin^{-1}0} \quad \Rightarrow \quad y=1$ $\sqrt{1-0^2}\,\dfrac{\mathrm{d}y}{\mathrm{d}x}=2(1) \quad \Rightarrow \quad \dfrac{\mathrm{d}y}{\mathrm{d}x}=2$ $\left(1-0^2\right)\times 2(2)\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+(0)(2)^2=8(1)(2) \quad \Rightarrow \quad \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=4$ -- ***Alternatively,*** $\left(1-0^2\right)^{\frac{1}{2}}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{1}{2}\left(1-0^2\right)^{-\frac{1}{2}}(0)(2)=2(2) \quad \Rightarrow \quad \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=4$ -- $\therefore$ The Maclaurin series for y: $\quad y=1+2x+4\left(\dfrac{x^2}{2!}\right)+\ldots$ $\Rightarrow \quad y=1+2x+2x^2+\ldots$
8 (iii) [2]	$\displaystyle\int_0^{0.1} x\mathrm{e}^{2\sin^{-1}x}\,\mathrm{d}x \approx \int_0^{0.1} x\left(1+2x+2x^2\right)\,\mathrm{d}x = 0.00571667$ $= 0.005717$ (4 s.f.)

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>9
(i)
[2]</td><td>

$\overrightarrow{AB}$ is parallel to $\begin{pmatrix} k \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{OB} - \overrightarrow{OA} = \lambda \begin{pmatrix} k \\ 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 17 \\ 10 \\ 19 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} k \\ 1 \\ 2 \end{pmatrix}$

$\lambda \begin{pmatrix} k \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 16 \\ 8 \\ 16 \end{pmatrix} = 8 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

By comparison:
when $\lambda = 8,$
$k = 2$ (shown)

</td></tr>
<tr><td>9
(ii)
[4]</td><td>

Observe the l_1 and l_2 are not parallel.
l_1 and l_2 either are skewed lines or intersect at a point.

We need to further check if the lines intersect.

$l_1 : \; \underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R}$

$l_2 : \; \underset{\sim}{r} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$

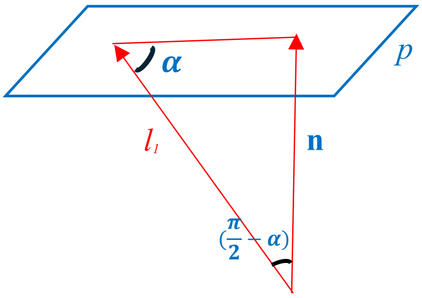
<u>Method 1: Use GC</u>

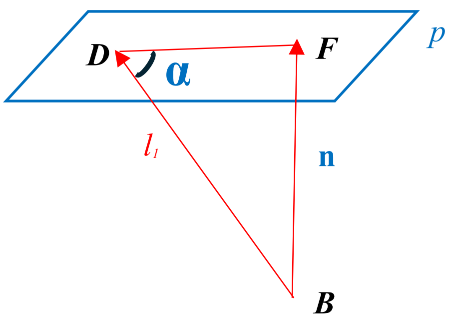
$\begin{pmatrix} 1+2\lambda \\ 2+\lambda \\ 3+2\lambda \end{pmatrix} = \begin{pmatrix} 3+2\mu \\ 4 \\ 6+\mu \end{pmatrix}$

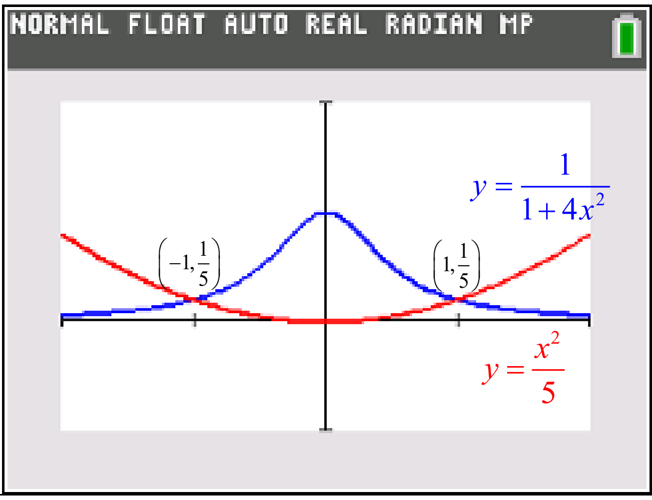
$2\lambda - 2\mu = 2$
$\lambda = 2$
$2\lambda - \mu = 3$
From GC, $\lambda = 2$ and $\mu = 1$.
The system of eqns are consistent and solution is unique.

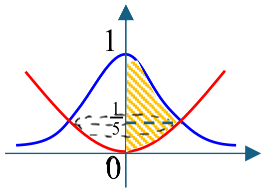
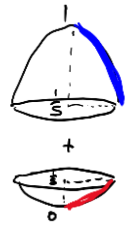
</td></tr>
</table>

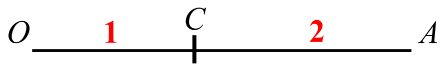
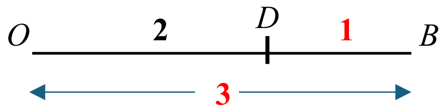
	Both lines intersect at position vector, The lines intersect. Sub $\mu = 1$ into l_2. $\begin{pmatrix}3\\4\\6\end{pmatrix}+(1)\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}5\\4\\7\end{pmatrix}$ **<u>Method 2: Solve manually</u>** If both lines intersect at point, there is an unique solution $\begin{pmatrix}1+2\lambda\\2+\lambda\\3+2\lambda\end{pmatrix}=\begin{pmatrix}3+2\mu\\4\\6+\mu\end{pmatrix}$ Only 1 unknown to start with! Compare j component $2+\lambda=4$ $\lambda=2$ Compare i component $1+2(2)=3+2\mu$ $\mu=1$ Note: To check if the system in consistent When $\lambda=2$ and $\mu=1$, $\text{LHS}=\begin{pmatrix}1+2[2]\\2+[2]\\3+2[2]\end{pmatrix}=\begin{pmatrix}5\\4\\7\end{pmatrix}=\begin{pmatrix}3+2[1]\\4\\6+[1]\end{pmatrix}=\text{RHS}$ Both lines intersect at position vector, $\begin{pmatrix}5\\4\\7\end{pmatrix}$
9(iii) [We 3]	Given α the acute angle between l_1 and p (refer to diagram below) $\cos\left(\frac{\pi}{2}-\alpha\right)=\left\lvert\frac{\begin{pmatrix}2\\1\\2\end{pmatrix}\bullet\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left(\sqrt{2^2+1+2^2}\right)\left(\sqrt{1^2+0+1^2}\right)}\right\rvert$ $\sin\alpha=\frac{4}{3\sqrt{2}}=\frac{2\sqrt{2}}{3}$

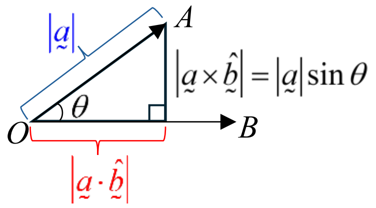
	$\sin^2\alpha + \cos^2\alpha = 1$ $\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} = \frac{1}{3}$ 
9 (iv) [3]	**<u>Method 1: Intersection of a Line and a Plane</u>** Point F, $\overrightarrow{OF}$ lies on ℓ_{BF} : $\underset{\sim}{r} = \begin{pmatrix}17\\10\\19\end{pmatrix} + \alpha\begin{pmatrix}1\\0\\1\end{pmatrix}$, for some $\alpha \in \mathbb{R}$ ----(1) Sub (1) into the eqn of plane p as Point F is also on plane p: $\begin{pmatrix}17+\alpha\\10\\19+\alpha\end{pmatrix}.\begin{pmatrix}1\\0\\1\end{pmatrix} = 16$ $17+\alpha+19+\alpha = 16$ $\alpha = -\frac{20}{2} = -10$ $\overrightarrow{OF} = \overrightarrow{OB} + \overrightarrow{BF}$ $\overrightarrow{OF} = \begin{pmatrix}17\\10\\19\end{pmatrix} + (-10)\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}7\\10\\9\end{pmatrix}$ Coordinates of point F is (7, 10, 9). **<u>Method 2: Find $\overrightarrow{BF}$ by projecting $\overrightarrow{BD}$ onto n, where D is a point on plane *p*.</u>** Obtain a specific point, D from plane p, $\overrightarrow{OD} = \begin{pmatrix}16\\0\\0\end{pmatrix}$.

	$\overrightarrow{BF}$ $=\left(\overrightarrow{BD}\cdot \underset{\sim}{n}\right)\underset{\sim}{n}$ $=\dfrac{\left\{\begin{pmatrix}16\\0\\0\end{pmatrix}-\begin{pmatrix}17\\10\\19\end{pmatrix}\right\}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left\|\sqrt{2}\right\|}\dfrac{\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left\|\sqrt{2}\right\|}$ $=\dfrac{-20}{2}\begin{pmatrix}1\\0\\1\end{pmatrix}=-10\begin{pmatrix}1\\0\\1\end{pmatrix}$ $\overrightarrow{OF}=\overrightarrow{OB}+\overrightarrow{BF}$ $\overrightarrow{OF}=\begin{pmatrix}17\\10\\19\end{pmatrix}+(-10)\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}7\\10\\9\end{pmatrix}$ 

Qn	Solution
10(i) **[2]**	$y=\dfrac{1}{1+4x^2}$ -------- (1) $y=\dfrac{x^2}{5}$ ----------- (2) Using GC: The intersection points are $\left(-1,\dfrac{1}{5}\right)$, $\left(1,\dfrac{1}{5}\right)$. 
10(ii) **[5]**	Exact area of region R $=\displaystyle\int_{-1}^{1}\left(\frac{1}{1+4x^2}\right)-\left(\frac{x^2}{5}\right)\,\mathrm{d}x$ $=\displaystyle\int_{-1}^{1}\frac{1}{4}\left(\frac{1}{\left(\frac{1}{2}\right)^2+x^2}\right)-\left(\frac{x^2}{5}\right)\,\mathrm{d}x$ $=\left[\dfrac{1}{4}\left[\dfrac{1}{\left(\frac{1}{2}\right)}\tan^{-1}\left(\dfrac{x}{\frac{1}{2}}\right)\right]-\dfrac{1}{15}x^3\right]_{-1}^{1}$ $=\left[\left[\dfrac{1}{2}\tan^{-1}(2x)\right]-\dfrac{1}{15}x^3\right]_{-1}^{1}$ $=\left[\left(\dfrac{1}{2}\tan^{-1}(2)-\dfrac{1}{15}(1)^3\right)-\left(\dfrac{1}{2}\tan^{-1}(-2)-\dfrac{1}{15}(-1)^3\right)\right]$ $=\dfrac{1}{2}\tan^{-1}(2)-\dfrac{1}{15}+\dfrac{1}{2}\tan^{-1}(2)-\dfrac{1}{15}$ $=\tan^{-1}(2)-\dfrac{2}{15}$ units^2

10(iii) **[5]**	$y=\dfrac{x^2}{5} \Rightarrow x_1^{\,2}=5y$ $y=\dfrac{1}{1+4x^2} \Rightarrow x_2^{\,2}=\dfrac{1}{4}\left(\dfrac{1}{y}-1\right)$ Exact volume generated $=\pi\displaystyle\int_0^{\frac{1}{5}} x_1^{\,2}\ \mathrm{d}y+\pi\int_{\frac{1}{5}}^{1} x_2^{\,2}\ \mathrm{d}y$ $=\pi\displaystyle\int_0^{\frac{1}{5}} 5y\ \mathrm{d}y+\pi\int_{\frac{1}{5}}^{1} \frac{1}{4}\left(\frac{1}{y}-1\right)\ \mathrm{d}y$ $=\pi\left[\dfrac{5}{2}y^2\right]_0^{\frac{1}{5}}+\dfrac{\pi}{4}\left[\ln y-y\right]_{\frac{1}{5}}^{1}$ $=\pi\left(\dfrac{1}{10}-0\right)+\dfrac{\pi}{4}\left[\left(\ln(1)-1\right)-\left(\ln\left(\dfrac{1}{5}\right)-\dfrac{1}{5}\right)\right]$ $=\dfrac{\pi}{10}+\dfrac{\pi}{4}\left[-\dfrac{4}{5}-\ln\left(\dfrac{1}{5}\right)\right]$ $=\dfrac{\pi}{4}\ln 5-\dfrac{\pi}{10}\ \text{unit}^3$  

Qn	Solution
11(i) **[2]**	$\overrightarrow{OC}=\frac{1}{3}\overrightarrow{OA}$  $\overrightarrow{OC}=\frac{1}{3}\underset{\sim}{a}$ $\overrightarrow{OD}=\frac{2}{3}\overrightarrow{OB}$  $\overrightarrow{OD}=\frac{2}{3}\underset{\sim}{b}$
11(ii) **[5]**	$\overrightarrow{OE}=\underset{\sim}{r}=\overrightarrow{OA}+\lambda\overrightarrow{AD}$ ---(1) $\overrightarrow{OE}=\underset{\sim}{r}=\overrightarrow{OB}+\mu\overrightarrow{BC}$ ---(2) Solve (1) = (2): $\Rightarrow \overrightarrow{OA}+\lambda\overrightarrow{AD}=\overrightarrow{OB}+\mu\overrightarrow{BC}$ $\overrightarrow{OA}+\lambda\left(\overrightarrow{OD}-\overrightarrow{OA}\right)=\overrightarrow{OB}+\mu\left(\overrightarrow{OC}-\overrightarrow{OB}\right)$ $\underset{\sim}{a}+\lambda\left(\frac{2}{3}\underset{\sim}{b}-\underset{\sim}{a}\right)=\underset{\sim}{b}+\mu\left(\frac{1}{3}\underset{\sim}{a}-\underset{\sim}{b}\right)$ $(1-\lambda)\underset{\sim}{a}+\frac{2}{3}\lambda\underset{\sim}{b}=\frac{1}{3}\mu\underset{\sim}{a}+(1-\mu)\underset{\sim}{b}$ Comparing coefficient of $\underset{\sim}{a}$: $1-\lambda=\frac{1}{3}\mu$ $\Rightarrow \lambda+\frac{1}{3}\mu=1$ ---(3) Comparing coefficient of $\underset{\sim}{b}$: $\frac{2}{3}\lambda=1-\mu$ $\Rightarrow \frac{2}{3}\lambda+\mu=1$ ---(4) Solving (3) & (4), by GC: $\lambda=\frac{6}{7}$, $\mu=\frac{3}{7}$ Sub $\lambda=\frac{6}{7}$ into (1): $\overrightarrow{OE}=\underset{\sim}{a}+\frac{6}{7}\left(\frac{2}{3}\underset{\sim}{b}-\underset{\sim}{a}\right)$ $=\underset{\sim}{a}+\frac{4}{7}\underset{\sim}{b}-\frac{6}{7}\underset{\sim}{a}$ $=\frac{1}{7}\underset{\sim}{a}+\frac{4}{7}\underset{\sim}{b}$ (shown)

11 (iii) [3]	Area of $\Delta OAE = \frac{1}{2}\left\lvert \overrightarrow{OA} \times \overrightarrow{OE} \right\rvert$ $= \frac{1}{2}\left\lvert \underset{\sim}{a} \times \left(\frac{1}{7}\underset{\sim}{a} + \frac{4}{7}\underset{\sim}{b}\right)\right\rvert$ $= \frac{1}{2}\left\lvert \left(\underset{\sim}{a} \times \frac{1}{7}\underset{\sim}{a}\right) + \left(\underset{\sim}{a} \times \frac{4}{7}\underset{\sim}{b}\right)\right\rvert$ $= \frac{1}{2}\left\lvert \frac{1}{7}(\underset{\sim}{a} \times \underset{\sim}{a}) + \frac{4}{7}(\underset{\sim}{a} \times \underset{\sim}{b})\right\rvert$ $= \frac{1}{2}\left\lvert \underset{\sim}{0} + \frac{4}{7}(\underset{\sim}{a} \times \underset{\sim}{b})\right\rvert \qquad (\because \underset{\sim}{a} \times \underset{\sim}{a} = \underset{\sim}{0})$ $= \frac{2}{7}\lvert \underset{\sim}{a} \times \underset{\sim}{b}\rvert$, where $k = \frac{2}{7}$ (shown).
11 (iv) [1]	The length of projection of OA onto the line OB.
11(v) [3]	**<u>Method 1</u>** Perpendicular distance from A to OB $= \sqrt{(\lvert \underset{\sim}{a}\rvert)^2 - (\lvert \underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}\rvert)^2}$ $= \sqrt{(\lvert \underset{\sim}{a}\rvert)^2 - \left(\frac{1}{2}\lvert \underset{\sim}{a}\rvert\right)^2} \quad \left(\lvert \underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}\rvert = \lvert \underset{\sim}{a} \bullet \underset{\sim}{b}\rvert = \frac{1}{2}\lvert \underset{\sim}{a}\rvert \text{ (given)}\right)$ $= \sqrt{\lvert \underset{\sim}{a}\rvert^2 - \frac{1}{4}\lvert \underset{\sim}{a}\rvert^2}$ $= \sqrt{\frac{3}{4}\lvert \underset{\sim}{a}\rvert^2}$ $= \frac{\sqrt{3}}{2}\lvert \underset{\sim}{a}\rvert$  **<u>Method 2</u>** Perpendicular distance $= \lvert \underset{\sim}{a}\rvert \sin\theta$ (need to find θ) From $\lvert \underset{\sim}{a} \cdot \underset{\sim}{b}\rvert = \frac{1}{2}\lvert \underset{\sim}{a}\rvert$, we have $\left\lvert \lvert \underset{\sim}{a}\rvert \lvert \underset{\sim}{b}\rvert \cos\theta \right\rvert = \frac{1}{2}\lvert \underset{\sim}{a}\rvert$ $\Rightarrow \cos\theta = \frac{1}{2}$ (since $\lvert \underset{\sim}{b}\rvert = 1$) $\Rightarrow \theta = 60°$ Perpendicular distance $= \lvert \underset{\sim}{a}\rvert \sin 60° = \frac{\sqrt{3}}{2}\lvert \underset{\sim}{a}\rvert$

Qn	Solution
12 (i) [1]	At any point in time, the inflow **rate** is equal to the outflow **rate** (both 5 litres per minute). Therefore, the volume of the mixture in the tank remains constant.
12 (ii) [1]	Salt inflow rate = Salt concentration of brine × Brine inflow rate $\frac{dQ_{in}}{dt} = \frac{dQ_{in}}{dV} \times \frac{dV}{dt}$ $= 0.2 \times 5$ $= 1 \text{ kg / min (shown)}$
12 (iii) [2]	Mass of salt in tank at time t is Q kg, Vol of mixture = 100 litres Salt concentration in tank at time t $= \frac{Q}{100}$ kg/litre Salt outflow rate = Salt Concentration in tank × Outflow rate of mixture $\frac{dQ_{out}}{dt} = \frac{dQ_{out}}{dV} \times \frac{dV}{dt}$ $= \frac{Q}{100} \times 5$ $= \frac{Q}{20}$ $\frac{dQ}{dt} = \frac{dQ_{in}}{dt} - \frac{dQ_{out}}{dt} = 1 - \frac{Q}{20}$ (shown)
12 (iv) [5]	$\frac{dQ}{dt} = 1 - \frac{Q}{20} = \frac{20-Q}{20}$ $\int \frac{1}{20-Q}\, dQ = \int \frac{1}{20}\, dt$ $-\ln\lvert 20-Q\rvert = \frac{1}{20}t + c$ $\ln\lvert 20-Q\rvert = -\frac{t}{20} - c$ $\lvert 20-Q\rvert = e^{-\frac{t}{20}-c}$ $20 - Q = Ae^{-\frac{t}{20}}$ When $t = 0$, $Q = 0$ (initially pure water). $20 - 0 = Ae^{0} \Rightarrow A = 20$ $\therefore Q = 20 - 20e^{-\frac{t}{20}}$

<table>
<tr>
<td>12
(v)
[2]</td>
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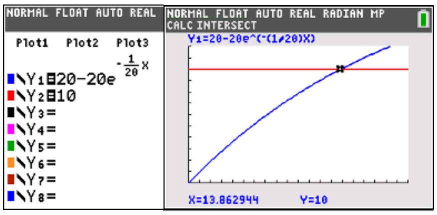
Salt concentration in tank $=0.1$ kg/litre
Mass of salt $=0.1\times 100=10$ kg
Sub $Q=10$: $10=20-20e^{-\frac{t}{20}}$
<u>Method 1 (GC Graph)</u>
From GC, $t=13.863$
$t=13.9$ (3 s.f.)


<u>Method 2 (Algebraic)</u>
$10=20-20e^{-\frac{t}{20}}$
$e^{-\frac{t}{20}}=\frac{1}{2}$
$-\frac{t}{20}=\ln\frac{1}{2}$
$t=-20\ln\frac{1}{2}=13.9$ (3 s.f.)
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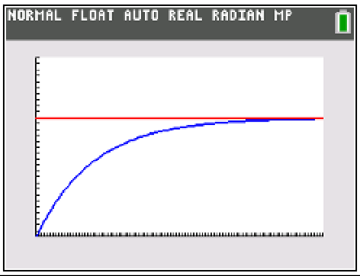
<tr>
<td>12
(vi)
[1]</td>
<td>
$Q=20-20e^{-\frac{t}{20}}$
When $t\to\infty$, $e^{-\frac{t}{20}}\to 0$, $Q\to 20$
Q increases and approaches 20.


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