16. Capacitors

(Extension to the topic of "Electric fields" and "Current and Circuits")

Content Page

LEARNING OBJECTIVES

Capacitance

(a) Define capacitance as the ratio of the charge stored to the potential difference and use $C = \frac{Q}{V}$ to solve problems.

(b) Recall that the electric potential energy stored in a capacitor is given by the area under the graph of potential difference against charge stored, and use this and the equations $U = \frac{1}{2}QV$, $U = \frac{1}{2}\frac{Q^2}{C}$, and $U = \frac{1}{2}CV^2$ to solve problems.

RC circuits

(c) Solve problems using the formulae for the combined capacitance of two or more capacitors in series and in parallel.

(d) Describe and represent the variation with time, of quantities like current, charge and potential difference, for a capacitor that is charging or discharging through a resistor, using equations of the form $x = x_0 e^{-\frac{t}{\tau}}$ or $x = x_0\left[1 - e^{-\frac{t}{\tau}}\right]$, where $\tau = RC$ is the time constant.

16.1 CAPACITANCE

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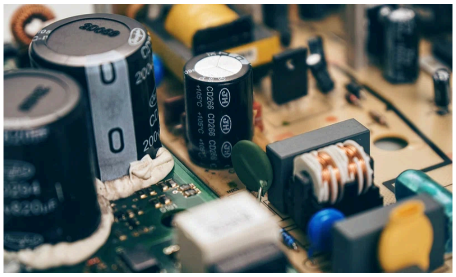
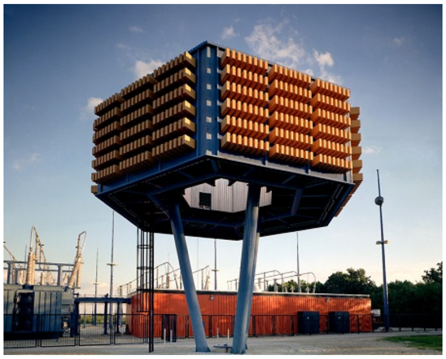
Most electronic devices, such as radios, computers and MP3 players, make use of components called capacitors. Capacitors are used to store energy in electrical and electronic circuits and then they gradually release this energy if there is a power failure, so that the computer will operate long enough to save valuable data. Fig. 16.1*a* shows a variety of shapes and sizes of capacitors.





(source: https://history.fnal.gov/historical/art_arch/captree.html)

(a) **(b)**

Fig. 16.1

Capacitors are usually quite small but Fig. 16.1*b* shows a giant capacitor, specially constructed to store electrical energy at the Fermilab particle accelerator in the United States.

Every capacitor has two leads, each connected to a metal plate. To store energy, these two plates must be given equal and opposite electric charges. Between the plates is an insulating material called the dielectric (note: dielectric is not required for H2 Physics). Fig. 16.2 shows a simplified version of the construction of a capacitor and the spiral 'Swiss-roll' form of capacitor in practice.

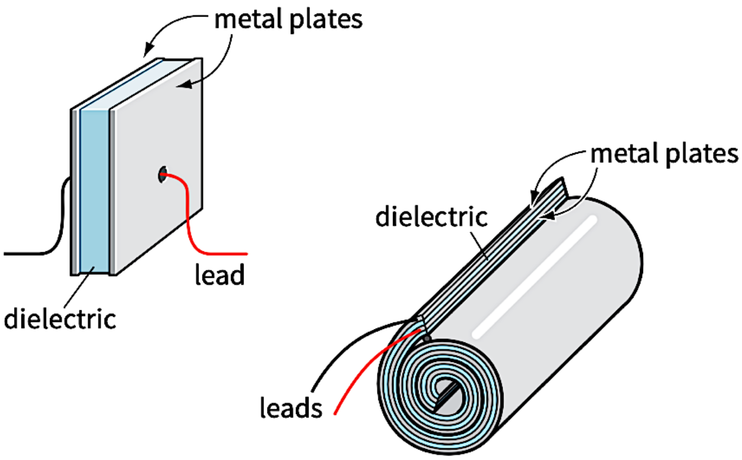


Fig. 16.2

16.1.1 Charging and discharging a capacitor

Charging the capacitor

Consider a pair of parallel plates separated by a vacuum gap and connected to a power supply. Connecting the capacitor to a supply pulls charge $+Q$ from one plate and transfers it to the other, leaving behind charge $-Q$ (In reality, electrons are moved onto one plate making it negatively charged while the other plate becomes positively charged). The supply does work in separating the charges.

Fig. 16.3 shows the separation of charges and the flow of electrons round the circuit.

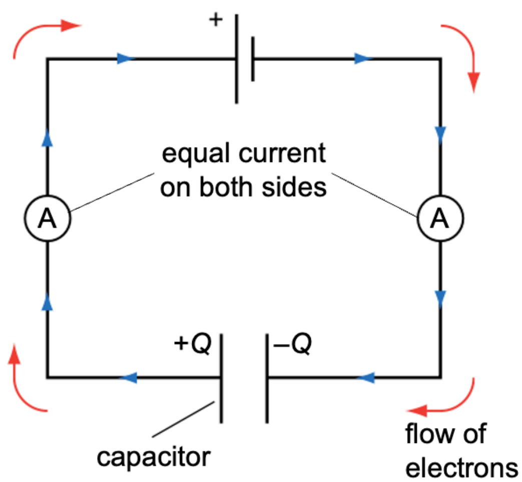


Fig. 16.3

- Current stops when the potential difference (p.d.) across the capacitor is equal to the e.m.f. of the supply. We then say that the capacitor is "fully charged".

- Since the two plates now store equal and opposite charges, the total charge on the capacitor is zero. When we talk about the "charge stored" by a capacitor, we refer to the **magnitude of the charge *Q* stored on each plate**.

- To make the capacitor plates store more charge, we can use a supply of higher e.m.f.

Discharging the capacitor

If we connect the leads of the charged capacitor together, electrons flow back towards the positive plate around the circuit and the capacitor is discharged.

We can also connect the charged capacitor to other electrical components, as shown in Fig. 16.4. When the switch closes, electrons will flow through the components and the capacitor discharges and its p.d. decreases. At any instant,

$$V_C = V_R + V_L$$

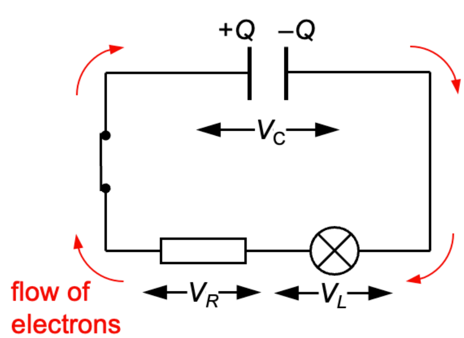


Fig. 16.4

16.1.2 The meaning of capacitance

Fig. 16.5 shows a capacitor marked with the value 2200 μF which is known as the capacitance. The capacitance of a capacitor is defined as

> Capacitance is the charge per unit potential difference across the plates where the charge refers to the charge on one plate.

MUST MEMORISE!

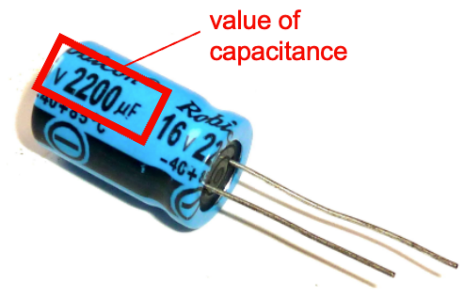


Fig. 16.5

The mathematical relation, from the definition, is

$$C = \frac{Q}{V} \qquad \text{(eq. 16.1)}$$

where C is the capacitance of the capacitor,
Q is the magnitude of the charge on each of the plate,
V is the potential difference across the capacitor.

The SI derived unit of capacitance is the **farad** (F). From eq. 16.1,

$$1\ \text{F} = 1\ \text{C V}^{-1}$$

In practice, a farad is a large unit. Few capacitors have a capacitance of 1F. Capacitors usually have their values marked in picofarads (pF), nanofarads (nF) or microfarads (μF).

Besides the capacitance value, there are other markings on capacitors:

- **Highest safe working voltage.** If you exceed this value (16 V for the capacitor in Fig. 16.5), charge may leak across between the plates and the dielectric will cease to be an insulator.

- **Polarity.** Some capacitors must be connected correctly in a circuit. They have an indication to show which end must be connected to the positive or negative end of the supply (see the blank band with a negative sign in Fig. 16.5). Failure to connect correctly will damage the capacitor, and can be extremely dangerous.

16.1.3 Energy stored in a capacitor

When you charge a capacitor, you use a power supply to push electrons onto one plate and off the other. The power supply does work on the electrons, so their potential energy increases. You recover this energy when you discharge the capacitor.

In order to charge a capacitor, work must be done to push electrons onto one plate and off the other (Fig. 16.6*a*). At first, there is only a small amount of negative charge on the left-hand plate. Adding more electrons is relatively easy, because there is not much repulsion. As the charge on the plate increases, the repulsion between the electrons on the plate and the new electrons increases, and a greater amount of work must be done to increase the charge on the plate. Fig. 16.6*b* shows how the p.d. V increases as the amount of charge Q increases. It is a straight line because $V = \frac{1}{C}Q$ (from eq. 16.1).

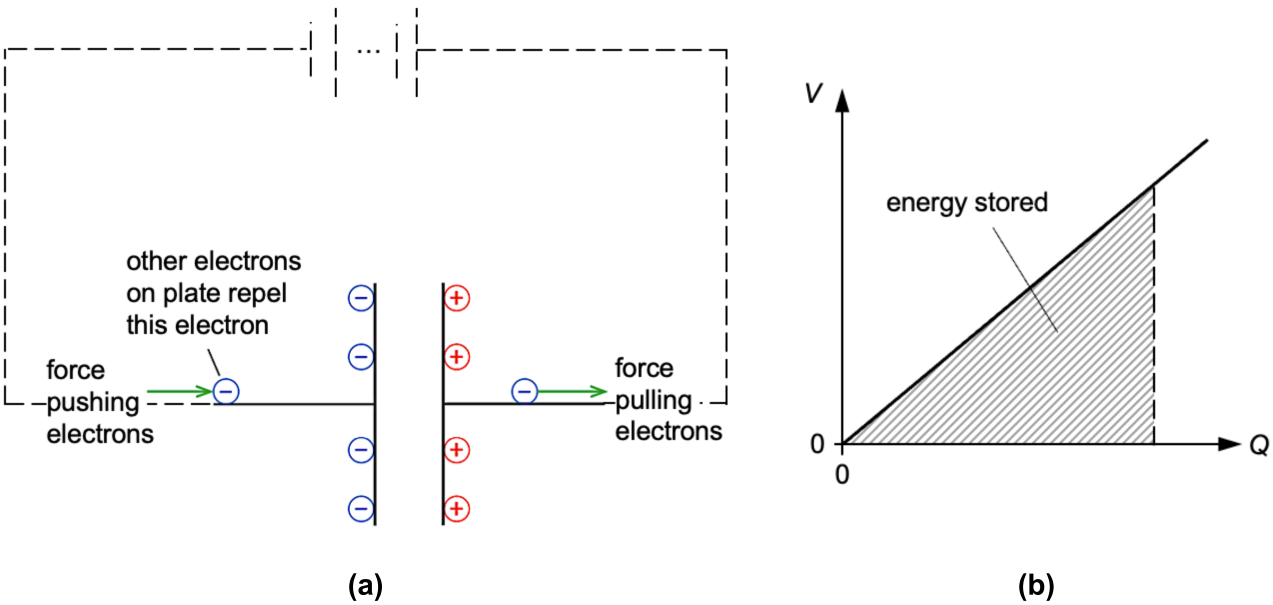


Fig. 16.6

The work done in charging a capacitor to a particular p.d. is the area under the V-Q graph. This energy transferred in stored in the electric potential store as electric potential energy U given by

$$U = \frac{1}{2}QV \qquad \text{(eq. 16.2}a\text{)}$$

Using eq. 16.1,

$$U = \frac{1}{2}CV^2 \qquad \text{(eq. 16.2}b\text{)}$$

$$U = \frac{1}{2}\frac{Q^2}{C} \qquad \text{(eq. 16.2}c\text{)}$$

Example 1 A 2000 μF capacitor is charged to a p.d. of 10 V.

(a) Determine the amount of charge stored by the capacitor.

(b) Calculate the energy stored by the capacitor.

Solution

(a) Charge stored $Q = CV = (2000 \times 10^{-6})\,(10) = 0.020$ C

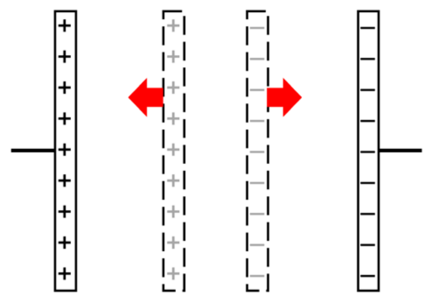
(b) Energy stored by the capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}(2000 \times 10^{-6})(10)^2 = 0.10$ J

Note: This is a small amount of energy compared with the energy stored by a rechargeable battery, typically of the order of 10 000 J.

Example 2

A capacitor of capacitance *C* comprising two parallel plates of area *A* separated by a distance *d*. The capacitor is charged to a p.d. of *V* so that each plate carries a charge of *Q* and it is then disconnected from the circuit.

Explain how the charge *Q*, electric field *E*, p.d. *V*, capacitance *C* and potential energy *U* will change when the plates of the disconnected capacitor is pulled further apart.



Solution

quantity	**change**	**explanation**
charge *Q*	constant	The charge remain constant since it is disconnected.
electric field *E*	constant	• The charge creates a specific number of electric field lines between the plates. Since it is disconnected, the charge is constant and so the total number of field lines is constant. • Since the plates remains the same size, the field lines have nowhere to spread out, so their density stays exactly the same, regardless of how far apart the plates are moved.
p.d. between plates *V*	increase	Since electric field $E = \dfrac{\text{potential difference}}{\text{separation of plates}}$ = constant, p.d. increases when the separation of the plates increases.
capacitance *C*	decrease	When the p.d. increases without a change in amount of charges, the capacitance decreases (eq. 16.1).
potential energy *U*	increase	Since $U = \dfrac{1}{2}\dfrac{Q^2}{C}$, the constant *Q* and decreased *C* lead to an increase potential energy. *[Alternatively] Another way to look at this is the plates attract each other. Work is done to pull them further apart which increases the potential energy.*

Check your understanding: How will the p.d., capacitance and potential energy change when the plates of the disconnected charged capacitor is brought closer?

16.1.4 Capacitance of isolated bodies

It is not just capacitors that have capacitance – all bodies have capacitance.

In dry conditions, a person may become charged up simply by rubbing against a synthetic fabric. The person will be at a high voltage and store a significant amount of charge. When the person touches a earthed metal object, discharging in the form of a tiny spark occurs.

If we consider a conducting sphere of radius r insulated from its surroundings and carrying a charge Q it will have a potential at its surface of V, where

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Since $C = Q/V$, the capacitance of a charged conducting sphere is

$$C_{\text{sphere}} = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}} = 4\pi\varepsilon_0 r$$

For Earth, with a radius of 6.4×10^6 m, its capacitance is approximately 0.7 mF.

16.2 RC CIRCUITS

(c) Solve problems using the formulae for the combined capacitance of two or more capacitors in series and in parallel.
(d) describe and represent the variation with time, of quantities like current, charge and potential difference, for a capacitor that is charging or discharging through a resistor, using equations of the form $x = x_0 e^{-\frac{t}{\tau}}$ or $x = x_0 \left[1 - e^{-\frac{t}{\tau}}\right]$, where $\tau = RC$ is the time constant.

16.2.1 Capacitors in series and in parallel

Capacitor in parallel

When two capacitors of capacitance C_1 and C_2 are connected in parallel, they are equivalent to a single capacitor with larger plates of capacitance C_{total}. The bigger the plates, the more charge that can be stored for a given voltage, and hence the greater the capacitance.

We can derive the equation for capacitors connected in parallel by considering the charge on the two capacitors shown in Fig. 16.7.

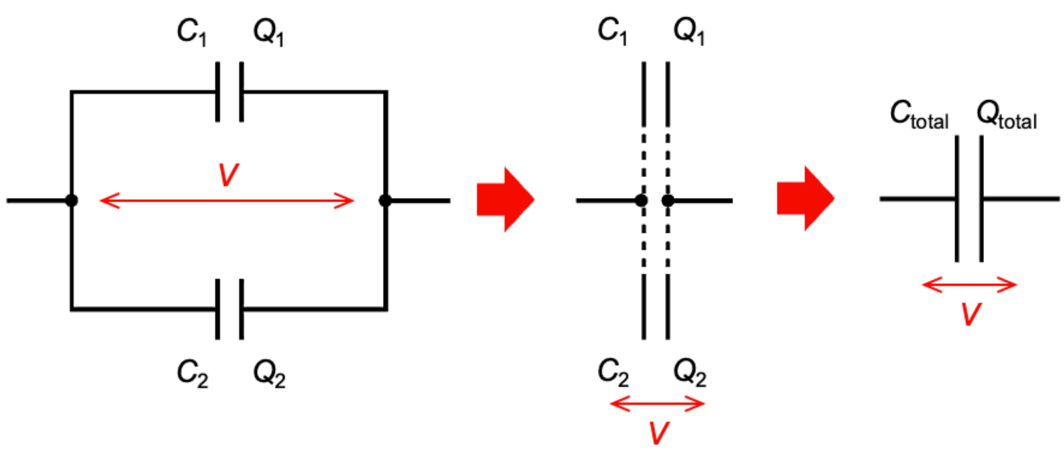


Fig. 16.7

Since the p.d. across each capacitor is V, $Q_1 = C_1 V$ and $Q_2 = C_2 V$.

The total charge in the two capacitors is $Q_{total} = Q_1 + Q_2 = C_1 V + C_2 V = (C_1 + C_2)V$

For the equivalent single capacitor,

$$C_{total} = \frac{Q_{total}}{V} = \frac{(C_1 + C_2)V}{V}$$

$$\therefore C_{total} = C_1 + C_2$$

The general equation of the total capacitance of two or more capacitors is

$$C = C_1 + C_2 + \cdots \qquad \text{(eq. 16.3)}$$

Refer to "data and formulae"

Capacitor in series

Consider two capacitors of capacitance C_1 and C_2 are connected in series (Fig. 16.8).

- **Charge.** When the voltage is first applied, charge $+Q$ arrives on the left-hand plate of C_1. This repels charge $+Q$ off the right-hand plate, leaving it with charge $-Q$. Charge $+Q$ now arrives on the left-hand plate of C_2, and this in turn results in charge $-Q$ on the right-hand plate.

  The capacitors are initially uncharged and, from the conservation of charge, it must remain so at the end. Since there is charge $+Q$ at one end and $-Q$ at the other, the requirement is satisfied.

Key concept: Capacitors connected in series store the same charge.

- **p.d of each capacitor.** The p.d. across the capacitors are $V_1 = \dfrac{Q}{C_1}$ and $V_2 = \dfrac{Q}{C_2}$.

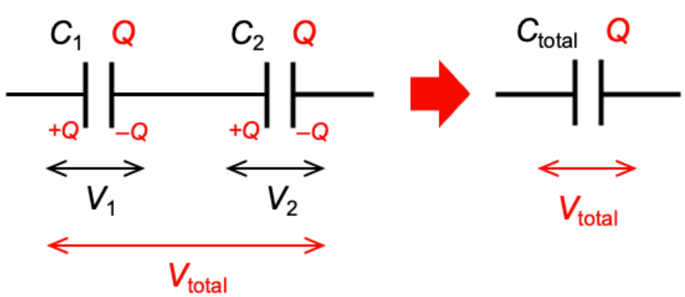


Fig. 16.8

The total p.d. across the two capacitors is $V_{total} = V_1 + V_2$.

For the equivalent single capacitor,

$$C_{total} = \frac{Q}{V_{total}} = \frac{Q}{V_1 + V_2} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}}$$

$$\frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\therefore \frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

The general equation of the total capacitance of two or more capacitors is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \qquad \text{(eq. 16.4)}$$

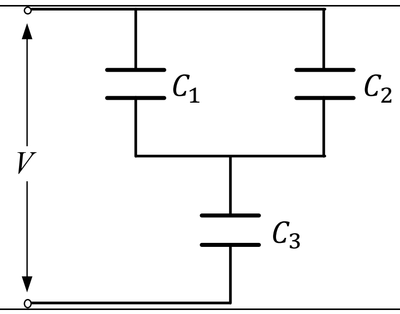
Refer to "data and formulae"

Example 3

(a) Determine the equivalent capacitance for the combination of capacitances shown in the figure, across which potential difference V is applied and

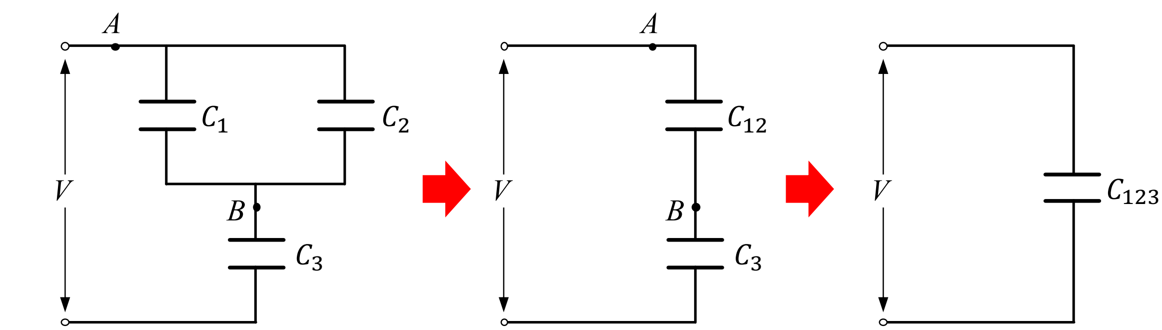
$C_1 = 12.0\ \mu\text{F},\ \ C_2 = 5.30\ \mu\text{F},\ \ C_3 = 4.50\ \mu\text{F}.$

(b) The potential difference applied to the input terminals V = 12.5 V. Determine the charges on each capacitor.



Solution

(a) Capacitors connected in series or parallel can be replaced with their equivalent capacitor.



C_1 and C_2 are *in parallel*

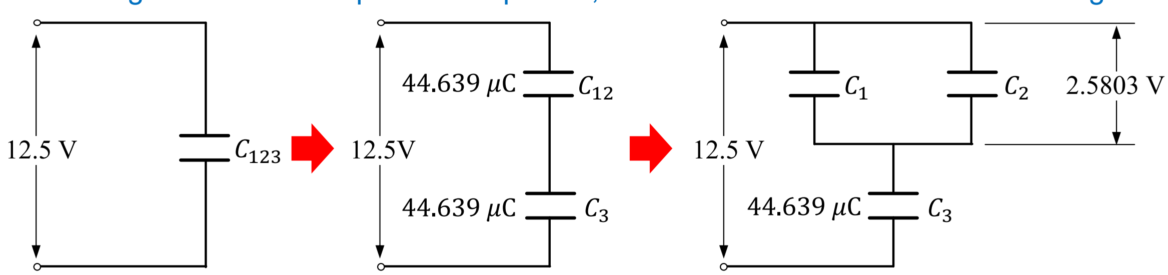
$$C_{12} = C_1 + C_2 = 12.0 + 5.30 = 17.3\ \mu\text{F}$$

C_{12} and C_3 are *in series*

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$C_{123} = \left(\frac{1}{17.3} + \frac{1}{4.50}\right)^{-1} = 3.57\ \mu\text{F}$$

(b) Starting from the final equivalent capacitor, work backwards to calculate the charge.



Charge on C_{123}
$= C_{123}V$
$= 3.5711 \times 12.5$
$= 44.639\ \mu\text{C}$

C_{12} and C_3 are *in series*, so charge on each capacitor is equal to the charge on C_{123}.

Potential difference of C_{12}
$$= \frac{q}{C} = \frac{44.639 \times 10^{-3}}{17.3 \times 10^{-3}} = 2.5803\ \text{V}$$

C_1 and C_2 are *in parallel*, so p.d. of each capacitor is equal to the p.d. of C_{12}.

Charge on C_1
$= CV$
$= 12.0 \times 2.5803$
$= 30.964\ \mu\text{C}$

Charge on C_2
$= 5.30 \times 2.5803$
$= 13.676\ \mu\text{C}$

Charge on $C_1 = 31.0\ \mu\text{C}$, charge on $C_2 = 13.7\ \mu\text{C}$ and charge on $C_3 = 44.6\ \mu\text{C}$.

16.2.2 Sharing charge and energy

If a capacitor is charged and then connected to a second capacitor, what happens to the charge and the energy that it stores?

Fig. 16.9 shows two initially uncharged capacitors of capacitance C_1 and C_2 connected in a circuit comprising a two-way switch. When the switch is closed at "a", capacitor C_1 is charged to p.d. V and contains charge Q. When the switch is closed at "b", the charged capacitor C_1 is connected to the uncharged C_2.

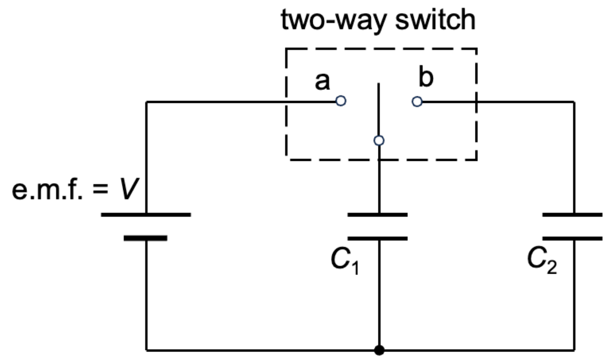


Fig. 16.9

Since the capacitors are connected together in parallel, so

- their combined capacitance C is equal to the sum of their individual capacitances (eq. 16.3),
- the charge Q in C_1 will be redistributed between the two capacitors AND the total charge remains at Q,
- they must have the same final p.d. V' across them at equilibrium.

Example 4

A 300 mF capacitor is charged to 10 V and disconnected from the power supply. It is then connected to an uncharged 100 mF capacity. Determine, at steady state,

(a) the p.d. across each capacitor,

(b) the charge on each capacitor,

(c) the energy stored by the combination of capacitors,

(d) the change in energy stored by the combination of capacitors.

Solution

(a) Initial charge in the first capacitor $Q = CV = (300 \times 10^{-3})(10) = 3.0$ C

Equivalent capacitance $C' = 300 + 100 = 400$ mF

Charge on equivalent capacitor = initial charge in 300 mF = 3.0 C

p.d. across equivalent capacitor $V' = Q / C' = 3 / 400 \times 10^{-3} = 7.5$ V

p.d. across each capacitor is 7.5 V (since they are connected in parallel).

(b) Charge on 100 mF capacitor = $(300 \times 10^{-3})(7.5) = 2.25$ C

Charge on 300 mF capacitor = $(100 \times 10^{-3})(7.5) = 0.75$ C

(c) [Method 1]

Energy stored in 300 mF capacitor $= \frac{1}{2}C_1V'^2 = \frac{1}{2}(300 \times 10^{-3})(7.5)^2 = 8.4375$ J

Energy stored in 300 mF capacitor $= \frac{1}{2}C_2V'^2 = \frac{1}{2}(100 \times 10^{-3})(7.5)^2 = 2.8125$ J

Total energy stored = 8.4375 + 2.8125 = 11.25 = 11.3 J

[Method 2]

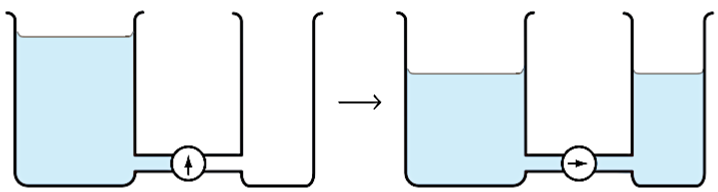
Energy stored in the combination $= \frac{1}{2}C'V'^2 = \frac{1}{2}(400 \times 10^{-3})(7.5)^2 = 11.25 = 11.3$ J

(d) Initial energy stored in 300 mF capacitor $= \frac{1}{2}C_1V^2 = \frac{1}{2}(300 \times 10^{-3})(10)^2 = 15$ J

Change in energy stored = 11.25 – 15 = –3.75 J

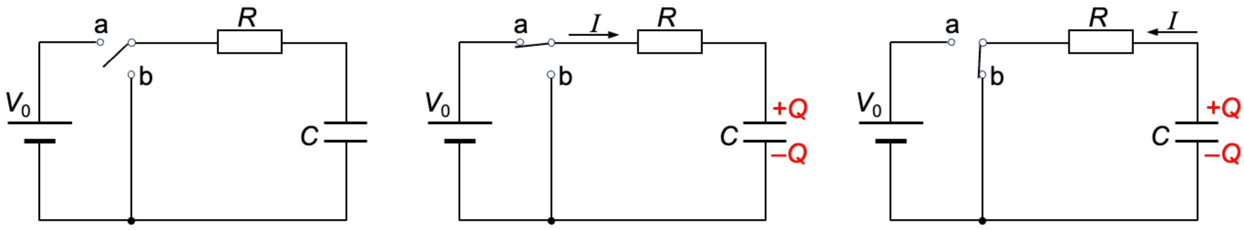
Discussion

An analogy to the situation is representing the capacitors by containers of water. A wide container (high capacitance C_1) is filled to a certain level (p.d.). It is then connected to a container with a smaller capacitance (C_2), and the levels equalise (i.e. p.d. is the same for each). Notice that the potential energy of the water has decreased, because the height of its centre of gravity above the base level has decreased. Energy is transferred to the internal store because there is friction during the flow within the moving water and between the water and the container.



16.2.3 RC circuits with constant e.m.f. source

A RC circuit contains both resistance R, capacitance C and a source of e.m.f. V_0 (Fig. 16.10*a*). The two-way switch allows the circuit to charge or discharge the capacitor.



(a) **(b)** Charging capacitor **(c)** Discharging capacitor

Fig. 16.10

Charging a capacitor

When the switch is at position "a" (Fig. 16.10*b*), there is an increasing charge stored in the capacitor. The maximum charge Q_0 stored in the capacitor when its p.d. is equal to the e.m.f. of the source (i.e., $Q_0 = CV_0$). We will state without proof the variation with time t of the amount of charge Q on the capacitor is

$$Q = Q_0\left[1 - e^{-\frac{t}{\tau}}\right] \qquad \text{(eq. 16.5}a\text{)}$$

Refer to "data and formulae"

where τ is the time constant and $\tau = RC$.

Dividing eq. 16.5*a* by the capacitance C, and e.m.f. $V_0 = \dfrac{Q_0}{C}$,

$$\frac{Q}{C} = \frac{Q_0}{C}\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$\therefore V = V_0\left[1 - e^{-\frac{t}{\tau}}\right] \qquad \text{(eq. 16.5}b\text{)}$$

Differentiate eq. 16.5*c* with respect to t,

$$\frac{dQ}{dt} = \frac{Q_0}{RC}e^{-\frac{t}{\tau}} = \frac{V_0}{R}e^{-\frac{t}{\tau}}$$

$$\therefore I = I_0 e^{-\frac{t}{\tau}} \qquad \text{(eq. 16.5}c\text{)}$$

where I_0 is the current at t = 0 where the current in the circuit is at its maximum.

Fig. 16.11 shows the variation with time t of the charge in the capacitor Q, the p.d. across the capacitor V and the current I in the circuit (when $t = \tau$, the term $e^{-t/\tau} = 1/e \approx 0.368$).

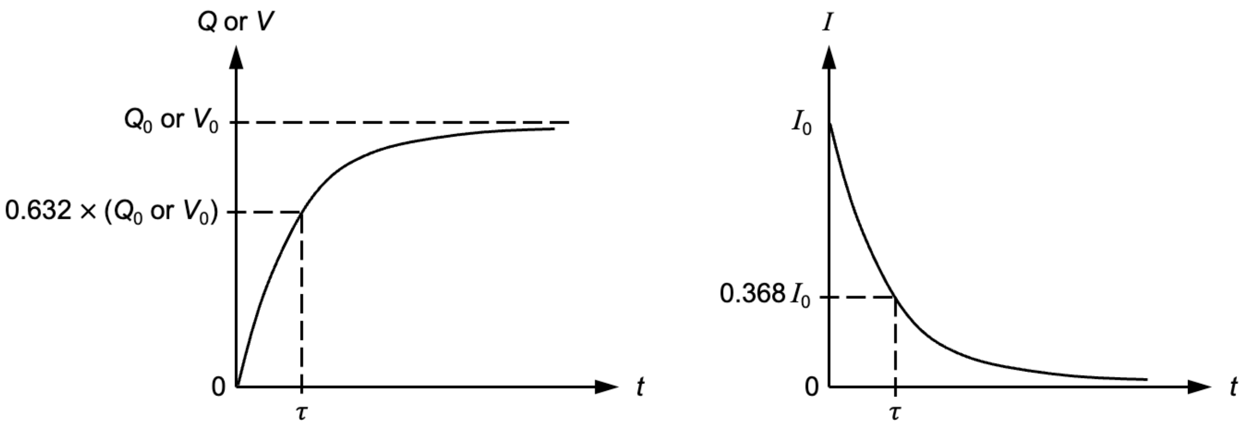


Fig. 16.11

Discharging a capacitor

When the switch is at position “b” (Fig. 16.10*c*), the capacitor discharges through the resistor.

Initially (time $t = 0$), the charge stored in the capacitor is Q_0 and its p.d. is $Q_0 = CV_0$. We will state without proof the variation with time t of the amount of charge Q on the capacitor is

$$Q = Q_0 e^{-\frac{t}{\tau}} \qquad \text{(eq. 16.6}a\text{)}$$

Refer to “data and formulae”

Dividing eq. 16.6*a* by the capacitance C, and e.m.f. $V_0 = \dfrac{Q_0}{C}$,

$$\frac{Q}{C} = \frac{Q_0}{C} e^{-\frac{t}{\tau}}$$

$$\therefore V = V_0 e^{-\frac{t}{\tau}} \qquad \text{(eq. 16.6}b\text{)}$$

Differentiate eq. 16.6*c* with respect to *t*,

$$\frac{dQ}{dt} = -\frac{Q_0}{RC} e^{-\frac{t}{\tau}} = -\frac{V_0}{R} e^{-\frac{t}{\tau}} \Rightarrow I = -I_0 e^{-\frac{t}{\tau}}$$

The negative sign in the current shows that it flows in the opposite direction to the current when the capacitor is charging. When the capacitor is fully discharged (at $t \to \infty$), current becomes 0.

$$\therefore I = I_0 e^{-\frac{t}{\tau}} \qquad \text{(eq. 16.6}c\text{)}$$

Fig. 16.12 shows the variation with time t of the charge in the capacitor Q, the p.d. across the capacitor V and the current I in the circuit (when $t = \tau$, the term $e^{-t/\tau} = 1/e \approx 0.368$).

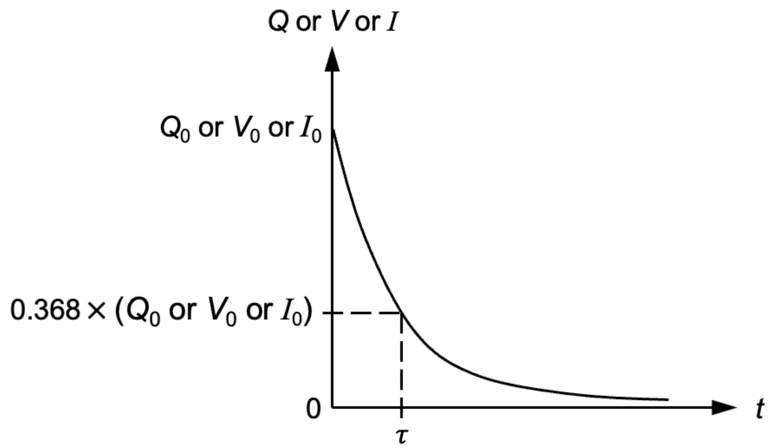
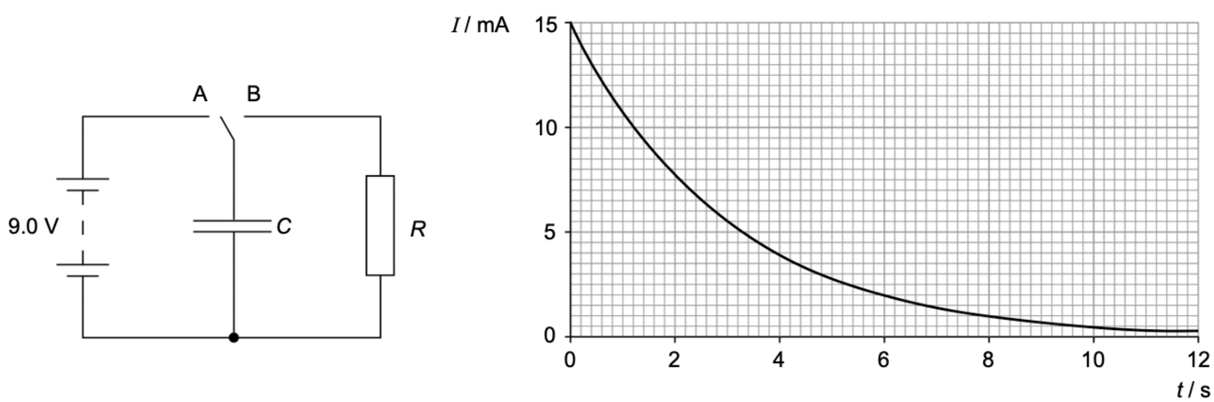


Fig. 16.12

Example 5 A circuit is used to investigate the discharge of a capacitor. The switch is connected to A to fully charge the capacitor of capacitance C. The switch is then connected to B and the variation with time t of the current I in the circuit is shown.



(a) Deduce the resistance R of the resistor.

(b) Explain why the current decreases as the capacitor discharges.

(c) Determine the capacitance C of the capacitor.

Solution

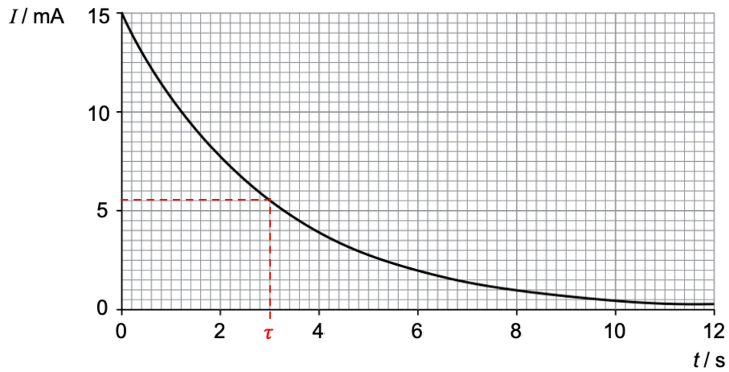
(a)
- When the capacitor is fully charged, its p.d. equals the e.m.f. of the battery.
- When the switch is first connected to B, p.d. across R = p.d. of capacitor and the current is maximum.
- Using Ohm's law,

$$R = \frac{V}{I} = \frac{9.0}{15 \times 10^{-3}} = 600\ \Omega$$

(b) As the capacitor discharges, fewer charges are left on the plates, so p.d. across it decreases. Since resistance stays the same, the decreasing voltage leads to a drop in current (Ohm's law).

(c) [Method 1 – using time constant τ]

Since $I = I_0 e^{-\frac{t}{\tau}}$, after time $t = \tau$, current decreases to $I = I_0 e^{-\frac{\tau}{\tau}} = \frac{1}{e} I_0 = \frac{1}{e} \times 15 = 5.52$ mA



$\tau = RC \Rightarrow 3 = 600 \times C$

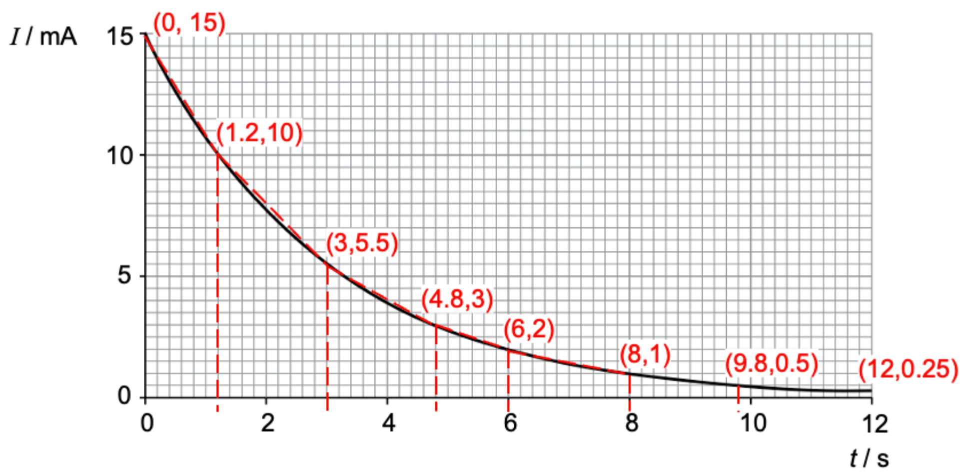
Capacitance $C = 5.0 \times 10^{-3}$ F = 5.0 mF

[Method 2 – using area under the graph]

The area under the *I-t* graph shows the total charge released by the capacitor during discharge.

The initial charge stored in the capacitor = total charge released.

Divide the graph into the following trapeziums,



Area under graph Q_0

$$= \left[\frac{1}{2}(15+10)(1.2-0) + \frac{1}{2}(10+5.5)(3-1.2) + \frac{1}{2}(5.5+3)(4.8-3) + \frac{1}{2}(3+2)(6-4.8) + \frac{1}{2}(2+1)(8-6) + \frac{1}{2}(1+0.5)(9.8-8) + \frac{1}{2}(0.5+0.25)(12-9.8)\right] \times 10^{-3}$$

$= 0.044775$ C

Capacitance $C = Q / V = 0.044775 / 9.0 = 4.975 \times 10^{-3}$ F = 5.0 mF (2 s.f.)

16.3 SOME APPLICATIONS OF CAPACITORS

Smoothing of rectified alternating current

In rectification, to produce a steady direct current or voltage from an alternating current or voltage, a smoothing capacitor is necessary.

Fig. 16.13 shows a circuit with a single capacitor with capacitance C is connected in parallel with a load resistor of resistance R. The capacitor **charges up** from the rectified input voltage as it rises. When the rectified voltage drops it **discharges** gradually through the resistor. The capacitor charges up and discharge in a fixed period. The resulting output voltage V_{out} or output current I_{out} against time is smoothened in a 'ripple' shape.

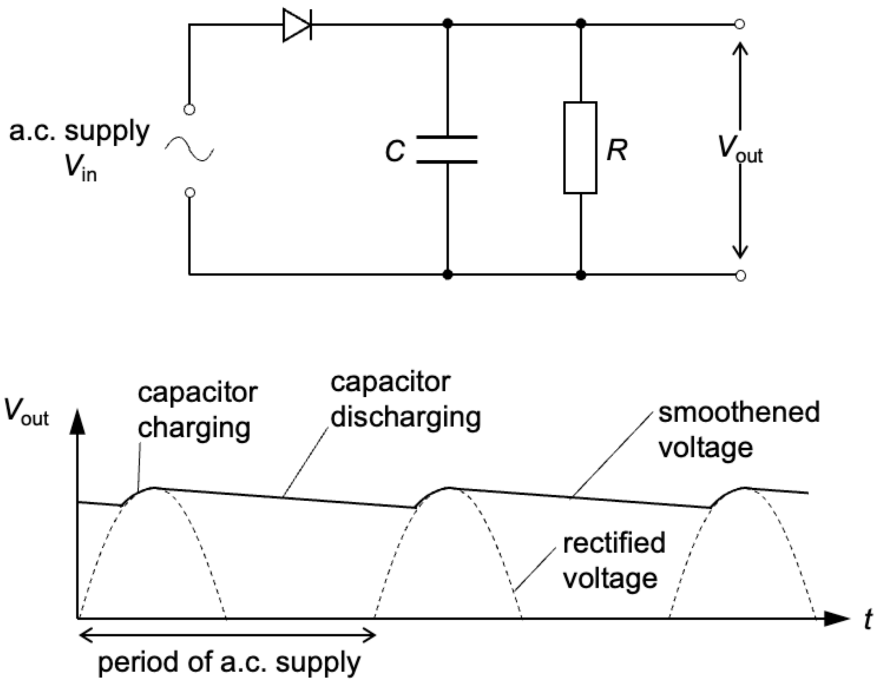


Fig. 16.13

The less the rippling effect, the smoother the rectified current and voltage output. The amount of smoothing is controlled by adjusting the time constant $\tau = RC$ such that the capacitor discharges slowly. This can be achieved by using a capacitor with greater capacitance C and/or a resistor with larger resistance R to ensure that τ is greater than the time interval between the adjacent peaks of the output signal.

Oscillator

One application of an RC circuit is the relaxation oscillator which control indicator lights that flash at a frequency determined by the values for *R* and *C*. Fig. 16.14 shows a relaxation oscillator consists of a power supply, a resistor, a capacitor and a neon lamp.

The neon lamp acts like an open circuit (infinite resistance) until the p.d. across the neon lamp reaches a specific voltage. At this voltage, the neon in the lamp breaks down and it acts like a short circuit (zero resistance), and the capacitor discharges through the neon lamp and produces a bright flash. After the capacitor fully discharges through the neon lamp, it begins to charge again, and the process repeats.

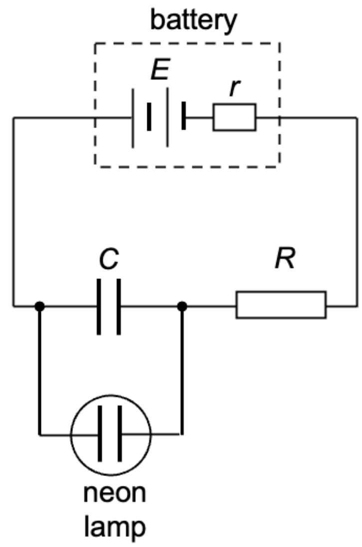


Fig. 16.14

Example 6

Determine the time interval between flashes in the neon lamp in Fig. 16.14 given that

- the e.m.f. *E* of the power supply is 100 V and the internal resistance *r* is 1.0 Ω,
- the capacitance *C* is 50 mF,
- the resistance *R* is 100 Ω,
- the breakdown voltage of the neon lamp is 80 V,
- the time it takes for the capacitor to discharge is negligible.

Solution

[Method 1]

Without the lamp,

maximum p.d. across the capacitor = 100 V

maximum charge = $(50 \times 10^{-3})(100) = 5$ C

When the neon lamp discharge,

p.d. across lamp and the capacitor = 80 V

charge on the capacitor = $(50 \times 10^{-3})(80) = 4$ C

Using eq. 16.5*a* and total resistance in circuit is 101 Ω,

$$4 = 5\left[1 - e^{-\frac{t}{101 \times 0.05}}\right]$$

$$e^{-\frac{t}{5.05}} = 0.2$$

$$t = -5.05 \ln 0.2 = 8.13 \text{ s}$$

[Method 2]

Without the lamp, maximum p.d. across the capacitor = 100 V.

Using eq. 16.5*b* and total resistance in circuit is 101 Ω,

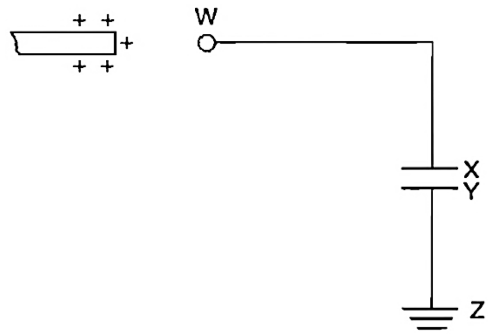
$$80 = 100\left[1 - e^{-\frac{t}{101 \times 0.05}}\right]$$

$$t = 8.13 \text{ s}$$

EXERCISE 1

You should take about 40 minutes to complete this exercise.

1.1 An uncharged capacitor is connected between earth Z and a terminal W. A positively charged rod is brought close to W.



Which of the following describes the movement of charge?

A Electrons move from W to X and from Y to Z.

B Electrons move from W to X but not from Y to Z.

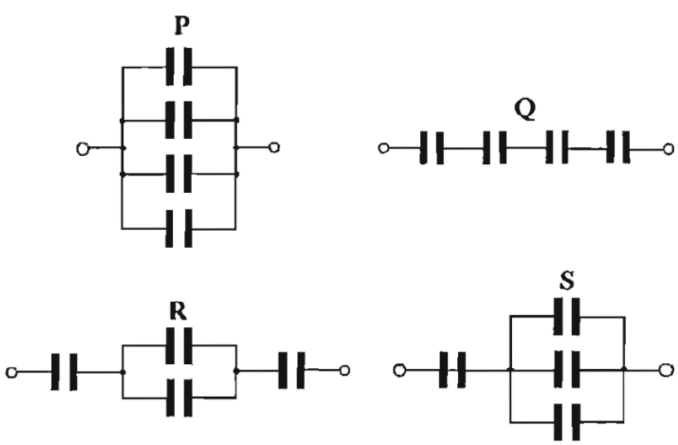
C Electrons move from X to W and from Z to Y.

D Electrons move from X to W but not from Z to Y.

1.2 A 20 μF capacitor is charged by a constant current of 10 mA. If the capacitor is initially uncharged, how long does it take for the potential difference across the capacitor to reach 300 V?

A 6.0×10^{-4} s **B** 0.60 s **C** 15 s **D** 6.0×10^{5} s

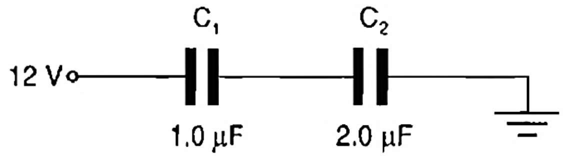
1.3 Four identical capacitors are connected as shown.



Which of the following lists the arrangements in order of **decreasing** capacitance?

A PQRS **B** PSRQ **C** QRSP **D** QSRP

1.4 Two capacitors are connected in series as shown.

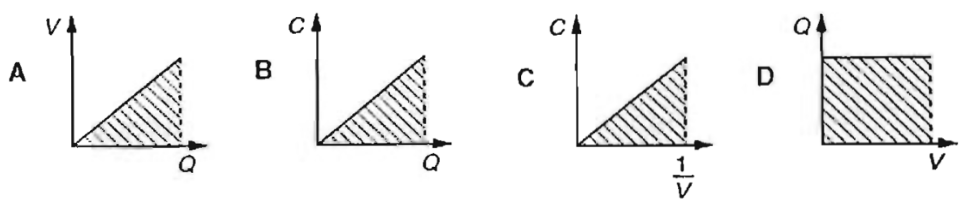


What is the charge carries by each of these capacitors?

	charge on C_1 / μC	charge on C_2 / μC
A	4.0	4.0
B	4.0	8.0
C	8.0	4.0
D	8.0	8.0

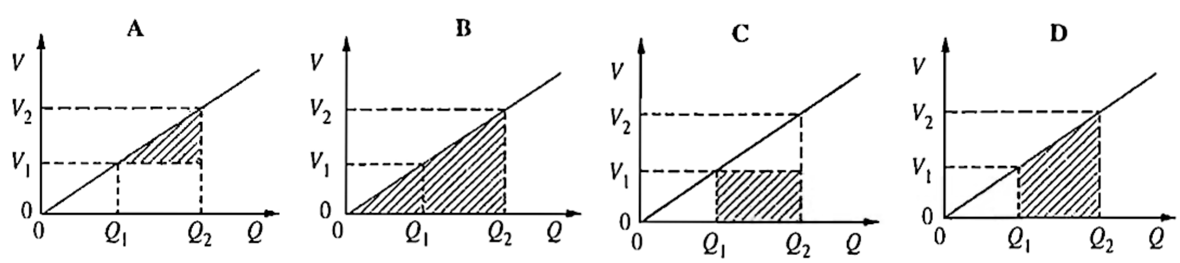
1.5 The energy stored in a capacitor of capacitance *C*, carrying charge *Q* with potential difference *V* between its plates. may be obtained by calculating the area under an appropriate graph.

Which graph shows the correct relationship between a pair of the quantities *C*, *Q* and *V,* and in addition shows a shaded area which corresponds to the energy stored in the capacitor?

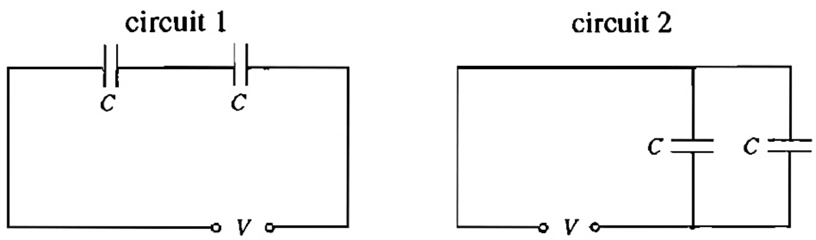


1.6 When the potential difference *V* between the plates of a capacitor is increased from V_1 to V_2, the charge *Q* on the plates increases from Q_1 to Q_2.

On the graphs, which shaded area represents the increase in the energy stored in the capacitor?



1.7 The diagrams show two ways of connecting two identical capacitors of capacitance C in circuits each with a supply of p.d. V.



What is the value of $\dfrac{\text{total energy stored in the capacitors in circuit 1}}{\text{total energy stored in the capacitors in circuit 2}}$?

A 0.25 **B** 0.50 **C** 2.0 **D** 4.0

1.8 A capacitor is charged to a voltage V and then discharged through a small d.c. motor. As the capacitor discharges, the motor raises a mass through a height h. The experiment is repeated for several values of V.

A constant fraction of the capacitor energy is converted to gain of gravitational potential energy.

Which graph would be expected to give a straight line?

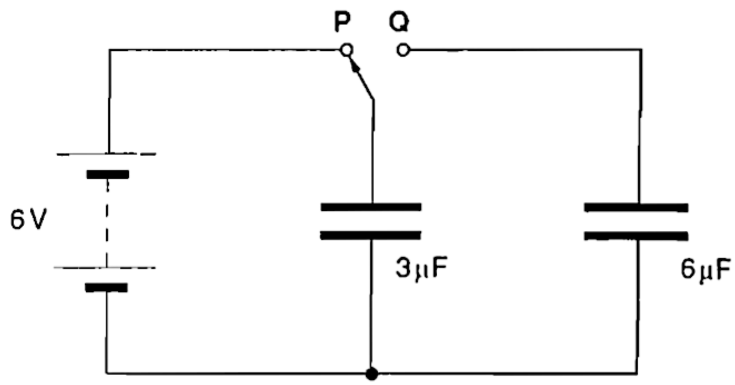
A h against V^2

B h against V

C h against $\sqrt{V}$

D h against $1/\sqrt{V}$

1.9 In the circuit shown, a capacitor of capacitance 3 μF is charged from a battery of e.m.f. 6 V with the switch connected to terminal **P**.



The switch is now connected to **Q**. This charges the 6 μF capacitor from the 3 μF one.

What is the new potential difference across the combination?

A 1 V **B** 2 V **C** 4 V **D** 6 V

1.10 A capacitor of capacitance C_1 is charged to a potential difference of 100 V and then disconnected. When a second uncharged capacitor of capacitance C_2 is connected in parallel across it, the new p.d. is 60 V.

What is the value of the ratio $\frac{C_1}{C_2}$?

A 0.60 **B** 0.67 **C** 1.50 **D** 1.67

Answer key

1.1	C	**1.2**	B	**1.3**	B	**1.4**	D	**1.5**	A
1.6	D	**1.7**	A	**1.8**	A	**1.9**	B	**1.10**	C