Vectors

1. AJC/I/13

The points P and Q have position vectors $\mathbf{i}-\mathbf{j}$ and $3\mathbf{i}+13\mathbf{j}+6\mathbf{k}$ respectively. The plane π_1 contains the point P and the line $\frac{x}{2}=-1-z$, $y=0$.

(i) Find a vector equation of the plane π_1 in scalar product form. [3]

(ii) Find the position vector of the foot of the perpendicular from Q to π_1 . [2]

The line l_1 passes through the points P and Q.

(iii) The line l_2 is the reflection of the line l_1 about the plane π_1 . Find a vector equation of l_2. [3]

The plane π_2 has the equation $\mathbf{r}\bullet\begin{pmatrix} a \\ 6 \\ 4 \end{pmatrix}=b$.

Find the values of a and b such that

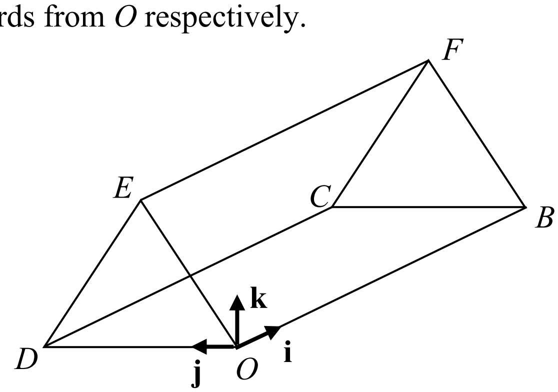
(iv) π_1 and π_2 are parallel and at a distance of $\sqrt{224}$ apart. [3]

(v) π_1 and π_2 are intersecting. [1]

2. AJC/II/2

The diagram shows a roof, with horizontal rectangular base $OBCD$, where OB = 10m and BC = 6m. The triangular planes ODE and BCF are vertical and the ridge EF is horizontal to the base. The planes $OBFE$ and $DCFE$ are each inclined at an angle θ to the horizontal, where $\tan\theta=\frac{4}{3}$.

The point O is taken as origin and vectors **i, j, k**, each of length 1m, are taken along OB, OD and vertically upwards from O respectively.



(i) Show that $\overrightarrow{OF} = 10\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$. [2]

(ii) Find the acute angle between the line OF and the plane BDE, giving your answer to the nearest 0.1° . [4]

3. ACJC/2016/I/111

Referred to the origin O, the position vectors of the points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Given that $\mathbf{a}\times\mathbf{b}=4\mathbf{a}\times\mathbf{c}$, where $\mathbf{b}\neq 4\mathbf{c}$ and $\mathbf{a}$ is a non-zero vector,

(i) show that $\mathbf{b}-4\mathbf{c}=\alpha\mathbf{a}$ where α is a scalar. [1]

(ii) Hence evaluate $|\mathbf{b}\times\mathbf{c}|$, given that the area of triangle OAB is $\sqrt{126}$ and $\alpha=\sqrt{3}$. [2]

(iii) Give the geometrical meaning of $|\mathbf{b}\times\mathbf{c}|$. [1]

It is also given that $\mathbf{b}$ is a unit vector, $|\mathbf{a}|=5$, $|\mathbf{c}|=2$ and $\mathbf{b}-4\mathbf{c}=\sqrt{3}\mathbf{a}$.

(iv) By considering $(\mathbf{b}-4\mathbf{c})\cdot(\mathbf{b}-4\mathbf{c})$, find the angle between $\mathbf{b}$ and $\mathbf{c}$. [3]

4. ASRJC/Prelim/2021/II/3

Three distinct vectors $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$ are each of unit length such that $\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c}=\mathbf{0}$, where λ,μ,γ are non-zero scalars.

(i) Show that $\mu(\mathbf{a}\times\mathbf{b})=\gamma(\mathbf{c}\times\mathbf{a})$ and $\lambda(\mathbf{b}\times\mathbf{a})=\gamma(\mathbf{c}\times\mathbf{b})$. [3]

(ii) By considering the sines of angles between $\mathbf{b}$ and $\mathbf{c}$, $\mathbf{c}$ and $\mathbf{a}$, and $\mathbf{a}$ and $\mathbf{b}$, show that

$$\frac{|\mu|}{\sin\angle COA}=\frac{|\lambda|}{\sin\angle BOC}=\frac{|\gamma|}{\sin\angle AOB}.$$ [4]

5. VJC Prelim/2020/01/Q7

With reference to the origin O, the points A, B, P, Q and R have position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{a}-2\mathbf{b}$, $-2\mathbf{a}-3\mathbf{b}$ and $2\mathbf{a}+\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors and $\mathbf{a}\bullet\mathbf{b}<0$.

Given that $\mathbf{a}$ is a unit vector and the area of triangle PQR is equal to the magnitude of $\mathbf{b}$, show that $\sin\theta=\frac{1}{4}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. Hence, find the value of θ. [5]

M lies on PR such that $PM=\frac{1}{2}MR$. Given that PR is perpendicular to OM, find the magnitude of $\mathbf{b}$, giving your answer correct to 3 decimal places. [5]

6. CJC/2016/I/2

The vectors $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\mathbf{a}=4\mathbf{i}+6p\mathbf{j}-8\mathbf{k} \text{ and } \mathbf{b}=2\mathbf{i}-3\mathbf{j}+4p\mathbf{k}, \text{ where } p>0.$$

It is given that $|\mathbf{a}|=2|\mathbf{b}|$.

(i) Find p. [2]

(ii) Give a geometrical interpretation of $\frac{1}{|\mathbf{b}|}|\mathbf{b}\cdot\mathbf{a}|$. [1]

(iii) Using the value of p found in part **(i)**, find the exact value of $\frac{1}{|\mathbf{b}|}|\mathbf{b}\cdot\mathbf{a}|$. [2]

7. MJC/I/5

With respect to an origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

(i) It is given that B lies on the line segment AC, such that $\overrightarrow{BC} = 2\mathbf{b} - k\mathbf{a}$. State the value of the constant k. Hence, find $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) $\overrightarrow{ON}$ is the projection vector of $\overrightarrow{OB}$ onto $\overrightarrow{OA}$, where $\overrightarrow{ON} = \frac{3}{4}\mathbf{a}$ and $\mathbf{a}.\mathbf{b} = 4$. Show that the length of OA is $\frac{4\sqrt{3}}{3}$. [2]

(iii) Given also that $\mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$, find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [2]

(iv) Given that $OADC$ is a parallelogram and E lies on line segment AC such that $AE : AC = 2 : 3$, find $\overrightarrow{DE}$. [3]

8. CJC/9758/2025/I/8

The plane p passes through the points A, B and C with coordinates $(1,0,2)$, $(2,-1,3)$ and $(-4,-1,0)$ respectively.

(a) Show that a cartesian equation of p is $x - y - 2z = -3$. [3]

The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$, $\lambda \in \mathbb{R}$.

(b) Find the acute angle between l and p. [2]

It is given that a variable point R lies on p and is at a distance of $\sqrt{22}$ from the point Q with coordinates $(3,4,-2)$.

(c) Find the foot of perpendicular from Q to p. [4]

(d) Hence describe geometrically the path traced by R. [2]

9. PJC/II/4

The point A has position vector $\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and the plane, π, has equation $\mathbf{r}\cdot(3\mathbf{i} + 4\mathbf{k}) = 40$. Find

(i) the vector equation of the line l passing through A and normal to the plane, π, [1]

(ii) the position vector of the foot of perpendicular from the point A to the plane, π. [3]

A sphere of radius 2 units has its centre at A. Show that the shortest distance between the sphere and the plane, π, is 3 units. Hence find the position vector of the nearest point on the sphere to the plane. [5]

10. CJC/2010/II/2

The equations of the line l_1 and the plane p_1 are respectively,

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \text{ and } \mathbf{r} = s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \text{ where } \mu, s, t \in \Re$$

(i) Find the acute angle between l_1 and p_1. [3]

(ii) A second plane p_2 has equation $\mathbf{r} \cdot \begin{pmatrix} \alpha \\ 1 \\ \beta \end{pmatrix} = 1$. Given that the two planes p_1 and p_2 intersect at the line l_2: $\dfrac{4x-15}{-2} = y$; $z = \dfrac{5}{2}$, find the values of α and β. [3]

(iii) The plane p_3 with equation $2x + by + z = 1$ is parallel to l_2. Find the value of b. Hence find the distance between l_2 and p_3. [4]

11. TJC Prelim 9758/2024/02/Q4

The line l_1 passes through the point A with coordinates $(1,0,4)$ and is perpendicular to the plane π_1 with equation $2x - y + 4z = -3$.

(a) Find the coordinates of the point B where l_1 meets π_1. [4]

(b) Verify that the point C with coordinates $(5,9,-1)$ lies on π_1. [1]

(c) Find a vector equation of the line l_2 which is a reflection of the line AC in π_1. [3]

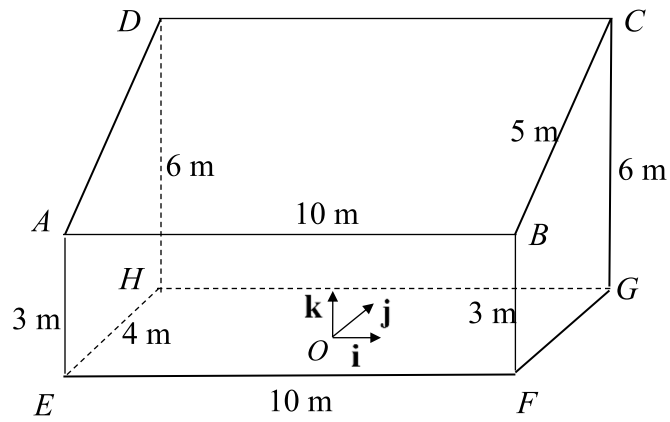
(d) Find a vector equation, in scalar product form, of the plane π_2 which contains l_1 and l_2. [3]

12. MJC/2010/II/2

Three non-zero and non-parallel vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are such that $\mathbf{p} \times \mathbf{q} = 3\mathbf{p} \times \mathbf{r}$. Show that $\mathbf{q} - 3\mathbf{r} = \lambda \mathbf{p}$, where λ is a scalar. [2]

It is also given that $\mathbf{p}$ is a unit vector, $|\mathbf{q}| = 5$, $|\mathbf{r}| = 2$ and the angle between $\mathbf{q}$ and $\mathbf{r}$ is $\cos^{-1}\dfrac{5}{6}$. By considering $(\mathbf{q} - 3\mathbf{r}) \cdot (\mathbf{q} - 3\mathbf{r})$, find the exact values of λ. [4]

13. PJC/2010/II/3



The diagram above shows a partial design of the roof of the gallery of an amphitheatre. $ABCD$ is an inclined rectangular roof, where $AB = 10$ m, and $BC = 5$ m. $EFGH$ is a rectangle on a horizontal ground, where $EF = 10$ m and $EH = 4$ m. The points A and B are 3 m directly above E and F respectively. The points C and D are 6 m directly above G and H respectively.

Point O is the centre of $EFGH$ and is taken as the origin. Perpendicular unit vectors **i**, **j**, **k** are in the direction of $\overrightarrow{EF}$, $\overrightarrow{EH}$ and $\overrightarrow{EA}$ respectively.

(i) Show that $\overrightarrow{AC} = \begin{pmatrix} 10 \\ 4 \\ 3 \end{pmatrix}$ and find a vector equation of the line passing through A and C. [3]

(ii) A spot light is fixed at the top of a vertical pillar erected at point Q with coordinates $(15, 6, 0)$. Find the height of the pillar if the top of the pillar is collinear with A and C. [3]

(iii) The main electrical supply to the gallery runs along the diagonal AC. Find the position vector of point X on AC such that the length of the electric cable that is required to provide electricity from the main electrical supply to an amplifier mounted at D, will be the least. [3]

14. AJC/2011/I/9

The equations of two lines are given as follows:

$$l_1 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \quad \lambda \in \mathbb{R} \qquad l_2 : \mathbf{r} = \mu \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mu \in \mathbb{R}$$

(i) Show that l_1 and l_2 are skew lines. [3]

(ii) If Z is the midpoint between any point on l_1 and any point on l_2, show that the locus of Z is a plane. Write down the equation of this plane p in scalar product form [3]

(iii) S is a point on l_1. The point S ' is the image of point S reflected in plane p. If S ' is also a point on l_2 , find the coordinates of S. [2]

15. AJC/2011/II/5

l is the line of intersection between two perpendicular planes π_1 and π_2. The equations of l and π_2 are given as follows: $l: \mathbf{r} = \lambda \begin{pmatrix} 1 \\ a \\ -1 \end{pmatrix}$, $\lambda \in \mathbb{R}$ $\quad \pi_2: \mathbf{r} \bullet \begin{pmatrix} b \\ 5 \\ -1 \end{pmatrix} = c$ where a, b and c are constants.

(i) State the value of c. [1]

(ii) Show that $5a + b = -1$. [1]

(iii) Given that the vector $\mathbf{i} + 2\mathbf{j}$ is parallel to π_1, find the value of a and b. [3]

(iv) The plane π_3 is parallel to π_2 and is at a distance of 4 units from π_2. Find a vector equation of π_3 in scalar product form. [2]

16. CJC Prelim 9758/2023/02/Q4

The plane Π_1 and the line l have equations

$$\Pi_1: \mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 3 \quad \text{and} \quad l: \frac{x+2}{3} = \frac{4-y}{2} = z - 3$$

respectively.

(a) Find the acute angle between Π_1 and l. [2]

(b) Find the coordinates of the point of intersection between Π_1 and l. [3]

(c) Find the perpendicular distance from $B(10, -4, 7)$ to Π_1. [3]

The plane Π_2 contains l and is perpendicular to Π_1.

(d) Find a cartesian equation of Π_2. [3]

(e) Without using a calculator, find a vector equation of the line which lies in both Π_1 and Π_2. [2]

17. NYJC/2011/I/1

Relative to the origin O, two points A and B have position vectors $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ respectively. The plane π has equation $\mathbf{r} \cdot (2\mathbf{i} + \lambda \mathbf{j}) = \mu$.

(a) If the plane π cuts the line segment AB in the ratio 3:1, find a relation between λ and μ. [3]

(b) If instead the plane π contains the line AB, find the values of λ and μ. [4]

18. NYJC/2011/I/7

Relative to the origin O, the points A and B have position vectors $8\mathbf{i}+6\mathbf{j}-10\mathbf{k}$ and $-10\mathbf{k}$ respectively. The plane π contains A and is parallel to the vector $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ and the line $\mathbf{r}=\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k})$, where $\lambda \in \mathbb{R}$.

(i) Find an equation of plane π in scalar product form. [3]

(ii) Find the position vector of C, the reflection of B in the plane π. [5]

(iii) Find the exact area of triangle OBC. [2]

(iv) Deduce the exact area of triangle ABC, justifying your answer.

19. ACJC/2011/I/4

The equation of the plane p_1 is given by $x-6y+2z=5$.

(i) Show that the line l_1 with equation $\mathbf{r}=\begin{pmatrix}9\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\2\end{pmatrix}$ lies on the plane p_1. [2]

(ii) Find the Cartesian equation of the plane p_2 which is parallel to plane p_1 and contains the point A with coordinates $(1,-1,1)$. [2]

(iii) Line l_2 contains point A and the point B with coordinates (9,1,1). Show that the sine of the acute angle between l_2 and p_1 is $\dfrac{2\sqrt{697}}{697}$. [2]

(iv) Hence, or otherwise, find the exact perpendicular distance between p_1 and p_2.

20. CJC Prelim 9758/2024/02/Q4

The plane p has equation $\mathbf{r}=\begin{pmatrix}1\\3\\4\end{pmatrix}+s\begin{pmatrix}2\\0\\1\end{pmatrix}+t\begin{pmatrix}0\\1\\-1\end{pmatrix}$, and the line l has the equation $\mathbf{r}=\begin{pmatrix}1\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\2\end{pmatrix}$,

where λ, s and t are parameters.

(a) Show that l is perpendicular to p and find the coordinates of the point where l and p intersect. [5]

The line m meets l at the point $A(1,1,3)$ and it meets p at the point $B(1,3,4)$.

(b) The line n is the reflection of m in p. Show that a vector equation of n is $\mathbf{r}=\begin{pmatrix}1\\3\\4\end{pmatrix}+\mu\begin{pmatrix}-4\\2\\5\end{pmatrix}$, $\mu \in \mathbb{R}$. [2]

(c) Find the acute angle between n and p. [2]

(d) Hence or otherwise, find the area bounded by the lines l, m and n. [3]

21. ACJC/2012/I/11

OABC is a rhombus. The points *A* and *C* are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. The point *M* is the centre of the rhombus and angle *AOC* is 60°.

(i) By using scalar products, show that the length of projection of $\overrightarrow{OM}$ on $\overrightarrow{OC}$ is $\frac{3}{4}|\mathbf{c}|$. [4]

(ii) By using a vector product, show that the area of triangle *OMC* can be written as $k|\mathbf{c}|^2$ where k is a constant to be found. [3]

(iii) The point *D* lies on *OC* produced such that $OC : CD = 2:3$. Show that the shortest distance from *D* to the line *OA* can be written as $t|\mathbf{c}|$ where *t* is a constant to be found. [3]

22. RI Promo 9758/2025/Q6

(a) The non-zero vectors **a**, **b**, and **c** are such that $3\mathbf{a}\times\mathbf{b} = 2\mathbf{b}\times\mathbf{c}$, where $3\mathbf{a} \neq -2\mathbf{c}$. Find a linear relationship between **a**, **b**, and **c**. [3]

(b) The variable position vector **v** satisfies the equation

$$\mathbf{v}\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k}) = a\mathbf{i}+\mathbf{j}+\mathbf{k},$$

where a is a non-zero constant.

(i) Show that $a = 3$. [1]

(ii) Hence, by finding the set of position vectors **v**, describe geometrically the set of points represented by the position vectors **v**. [3]

23. MJC/2012/II/5

The line l_1 passes through point *A*, whose position vector is $4\mathbf{i}+5\mathbf{j}-6\mathbf{k}$, and is parallel to the vector $-\mathbf{i}-2\mathbf{j}+3\mathbf{k}$. The plane p_1 has equation $x+2y-3z=4$.

(i) Show that l_1 is perpendicular to p_1. Hence find the coordinates of the foot of perpendicular from *A* to p_1. [3]

The line l_2 is given by $x+1=-y=z-1$.

(ii) Given that the plane p_2 is parallel to the line l_1 and contains the line l_2, show that the equation of p_2 is $x+4y+3z=2$. What is the relationship between the lines l_1 and l_2? [3]

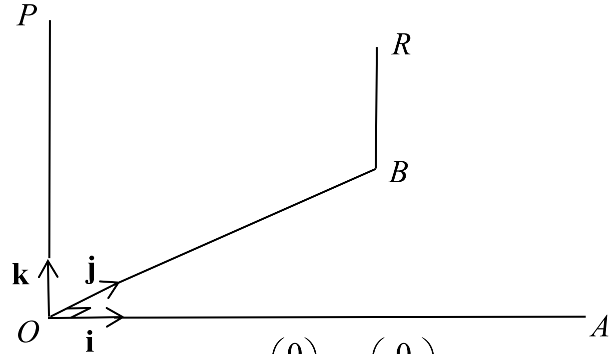
(iii) The planes p_1 and p_2 intersect in a line l_3. Find the vector equation of the line l_3. [1]

24. ACJC Prelim 9758/2025/P2/Q1

It is given that $\overrightarrow{OA} = 3\mathbf{i}+\mathbf{j}+3\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i}-4\mathbf{j}+3\mathbf{k}$. Find the position vector of a point *R* on line *OB* such that *AR* is perpendicular to *OB*. Hence find the position vector of the point A', the reflection of the point *A* in the line *OB*. [4]

25. SAJC/2012/I/9

In the diagram below, OAB is the horizontal base, where OA = 6 units, OB = 4 units, and $\angle AOB = 90°$. Poles OP and BR are placed vertically, where OP = 5 units and $BR = 2$ units. The unit vectors **i, j, k** are parallel to OA, OB and OP respectively. The point O is taken as the origin.



(i) Show that the equation of line PR is $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix}$, where $\lambda \in \mathbb{R}$.

(ii) A point Q divides AB such that AQ:QB = 1:3. Find $\overrightarrow{OQ}$. [2]

(iii) A line l with equation $x = 0;\ \dfrac{1-y}{3} = z - 8$ intersects the line PR at a point X. Find the position vector of X. [2]

(iv) Let $\mathbf{m}$ be a unit vector along AB. Find $|\overrightarrow{QX} \times \mathbf{m}|$. [3]

(v) Give a geometrical meaning of $|\overrightarrow{QX} \times \mathbf{m}|$. Hence, or otherwise, find the area of triangle AXB. [4]

26. CJC/2013/I/9

Referred to the origin O, the points A and B have position vectors given respectively by $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. R is the point that divides AB internally in the ratio $\lambda : \mu$. It is given that $\overrightarrow{OR} = \mathbf{r}$ and $\dfrac{\mathbf{r}\cdot\mathbf{a}}{\mathbf{r}\cdot\mathbf{b}} = \dfrac{\mu|\mathbf{a}|^2 + \lambda\mathbf{a}\cdot\mathbf{b}}{\lambda|\mathbf{b}|^2 + \mu\mathbf{a}\cdot\mathbf{b}}$. Deduce that, when $\lambda : \mu = |\mathbf{a}| : |\mathbf{b}|$, $\dfrac{\mathbf{r}\cdot\mathbf{a}}{\mathbf{r}\cdot\mathbf{b}} = \dfrac{|\mathbf{a}|}{|\mathbf{b}|}$.

Hence, show that the line OR bisects the angle AOB.

27. DHS/2013/I/6

(a) Given that $10\overrightarrow{OF} = 3\overrightarrow{OE} + 7\overrightarrow{OG}$, show that the points E, F and G are collinear. [2]

(b) Relative to the origin O, the points A, B and C have position vectors given by $\mathbf{a} = 12\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$, $\mathbf{b} = 5\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{c} = (1+2\lambda)\mathbf{i} + (5+2\lambda)\mathbf{j} + (-2+\lambda)\mathbf{k}$ respectively. The point P lies on AB produced such that $AP = 2AB$.

(i) Find $\overrightarrow{OP}$. [2]

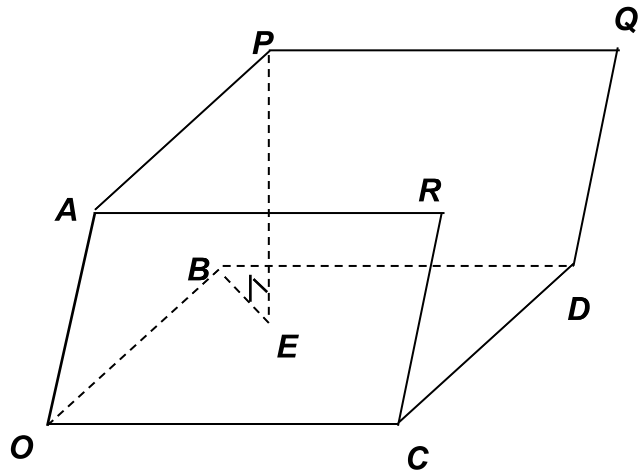
(ii) If AP is perpendicular to BC, find the value of λ. [3]

(iii) Using the value of λ found in part **(ii)**, find the area of triangle ACP. [2]

28. MJC/2013/I/5

The diagram below shows a parallelepiped *OBDCAPQR*. Referred to the origin *O*, the points *A*, *B* and *C* are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The point *E* lies in the plane *OBDC* such that it is directly below the point *P*.

[A parallelepiped is a 6-faced polyhedron whose faces are parallelograms lying in pairs of parallel planes.]



(i) State the normal of the plane *OBDC*. Hence, by considering the triangle *BEP*, find the length of *PE*. Give your answers in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [2]

(ii) Show that the volume of the above parallelepiped is given by $\left|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right|$. [1]

(iii) The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are now given by

$$\mathbf{a} = -10\mathbf{i} + 3\mathbf{j},\ \mathbf{b} = 2\mathbf{i} + 3\mathbf{k} \text{ and } \mathbf{c} = p\mathbf{i} + \mathbf{j} + 5\mathbf{k},$$

where *p* is a constant.

(iv) Given that the volume of the above parallelepiped is 18 units^3, find the possible value(s) of *p*. [3]

(v) A student claims that the following property holds:

$$\left|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right| = \left|\mathbf{b}\cdot(\mathbf{a}\times\mathbf{c})\right|$$

for any non-coplanar vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

Do you agree? Justify your answer. [1]

29. ASRJC Prelim 9758/2025/P2/Q1

Relative to the origin *O*, the position vectors of two points *A* and *B* are $\mathbf{a}$ and $\mathbf{b}$ respectively. The length of $\mathbf{a}$ is *k* units and $\mathbf{b}$ is a unit vector. The angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$ radians.

It is given that the point *M* lies on the line segment *AB* such that *AM*:*AB* is 3:4, find exact area of ΔOAM, giving your answer in terms of *k*. [4]

30. DHS/2013/I/10

The lines l_1 and l_2 have equations $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} b \\ 1 \\ -1 \end{pmatrix}$, where $b > 1$, and $\mathbf{r} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ respectively.

(i) Given that the acute angle between l_1 and l_2 is 30°, find the value of b, giving your answer correct to 2 decimal places. [3]

For the rest of the question, use $b = 3$.

(ii) Find the coordinates of the points A and B where l_1 and l_2 meet the xy–plane respectively. [3]

(iii) The point C has position vector $2\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$. Show that the equation of plane p_1 which passes through the points A, B, and C is given by $x - 2y + 5z = 3$. [3]

(iv) Another plane p_2 with equation $x + 2y + 3z = 2$ meets p_1 in the line l_3. Find the vector equation of l_3. [2]

(v) Explain whether the lines l_2 and l_3 are parallel, skew, coincident or intersecting, justifying your answer clearly. [2]

31. YIJC Prelim 9758/2022/01/Q4

The points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively where $\mathbf{p}$ and $\mathbf{q}$ are non-zero and non-parallel. The points P and Q are fixed and R varies.

(a) Given that $\mathbf{r} \times \mathbf{q} = -\mathbf{q} \times \mathbf{p}$, describe geometrically the set of all possible positions of the point R. [4]

(b) Given instead that $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\mathbf{p} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$, and that $\mathbf{q} \cdot (\mathbf{p} - \mathbf{r}) = 0$, find the relationship between x, y, and z in terms of q_1, q_2 and q_3. Describe the set of all possible positions of the point R in this case. [3]

(c) It is now given that $|\mathbf{q}| = 1$ and C is a point with position vector $\mathbf{c}$ such that $\mathbf{q} \cdot (\mathbf{p} - \mathbf{c}) \neq 0$. Give a geometrical meaning of $|\mathbf{q} \cdot (\mathbf{p} - \mathbf{c})|$. [1]

32. NYJC/2013/I/8

The plane p_1 has vector equation $\mathbf{r} = 4\mathbf{i} + 5\mathbf{j} - \mathbf{k} + s(\mathbf{i} + \mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ where s and t are real parameters. The points A and B, which do not lie in p_1, have position vectors $3\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + 2\mathbf{k}$ respectively. The perpendicular to the plane p_1 from point A meets p_1 at N.

(i) Find the position vector of N. [3]

(ii) Find the exact length of projection of AB onto the plane p_1. [2]

The line l passes through N and is contained in the plane p_1. Another plane p_2 contains the line l and is perpendicular to the line passing through B and N.

(iii) Find the Cartesian equation of p_2. [2]

(iv) Write down a vector which is parallel to l. [1]

33. ACJC/2014/I/1

Referred to the origin O, the points A and B lie in the same plane such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point M on OA is such that $OM : MA = 1:2$ OM, and the point N on OB is such that $ON : OB = 1:3$. The point S is on AB such that $5AS = 4SB$.
Write down the position vector of S in terms of **a** and **b**.
Using vector product, find the ratio of the area of triangle MNS to the area of triangle OAB. [5]

34. HCI/2014/I/8

Relative to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a}+\mathbf{b}$ respectively. The point X is on AB produced such that $AB : AX$ is $1:5$ and the point Y is such that $ACXY$ is a parallelogram. Given that the area of the triangle OAB is 1 square unit and $\mathbf{b}$ is a unit vector.

(i) Find in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vectors of X and Y. Hence show that $OABY$ is a trapezium. [5]

(ii) Give a geometrical meaning of $\left|(\mathbf{a}+\mathbf{b})\cdot\mathbf{b}\right|$. [1]

(iii) Find the area of $ACXY$. Hence find the shortest distance from X to the line that passes through the points A and C. [3]

35. JJC/2014/I/10

Referred to the origin O, the points A and B have position vectors **a** and **b** respectively. It is given that **a** and **b** are perpendicular to each other and have the same magnitude of 3 units each.
Given that A, B and C are collinear,

(i) show that **c** can be expressed as $\mathbf{c} = k\mathbf{b} + (1-k)\mathbf{a}$, where k is a constant. [1]

(ii) Find $|\mathbf{a}\times\mathbf{c}|$, in terms of k, and state its geometrical meaning. [4]

(iii) It is given that the area of triangle OAC is three times the area of triangle OAB. Find the two possible values of k.
Given also that the length of projection of OC onto OA is 12 units, find **c** in terms of **a** and **b**. [5]

36. TMJC Prelim 9758/2024/02/Q2

Referred to the origin O, the points A and B have position vectors **a** and **b** respectively.
The point C is such that O divides the line segment AC in the ratio $2:3$. The point D divides the line segment AB in the ratio $1:4$.

(i) Show that the area of triangle OCD is given by $\lambda|\mathbf{a}\times\mathbf{b}|$, where λ is a constant to be determined. [5]

(ii) It is now given that **a** is perpendicular to **b**. Given that $\overrightarrow{AP}$ is the projection vector of $\overrightarrow{AO}$ onto $\overrightarrow{AB}$, show that $\overrightarrow{AP} = \mu(\mathbf{b}-\mathbf{a})$, where μ is a constant to be determined in terms of $|\mathbf{a}|$ and $|\mathbf{b}|$. [3]

37. PJC/2014/I/12

The planes p_1 and p_2, which meet in the line l, have vector equations

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + s(2\mathbf{i} + 3\mathbf{k}) + t(-4\mathbf{j} + 5\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} \bullet \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} = 24, \text{ respectively.}$$

(i) Find a vector equation of l. [4]

(ii) Find the cartesian equation of the plane p_3 which contains l and the point $(3, -4, 1)$. [3]

38. RJC/2014/I/6

Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. It is given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 5$ and $|3\mathbf{a} - \mathbf{b}| = 10$.

(i) Give the geometrical interpretation of $\left| \mathbf{a} \bullet \dfrac{\mathbf{b}}{|\mathbf{b}|} \right|$. [1]

(ii) Show that $\mathbf{a} \bullet \mathbf{b} = 1$. [2]

(iii) Hence find the shortest distance from A to the line OB, and the area of the triangle OAB. [2]

(iv) Given that $\mathbf{a}$, $2\mathbf{a}+3\mathbf{b}$ and $\mu\mathbf{a} + 2\mathbf{b}$, where μ is a constant, are position vectors of collinear points, find μ. [4]

39. JPJC Prelim 9758/2024/01/Q6

Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively such that angle $AOB = 30°$ and $|\mathbf{a}| = 4|\mathbf{b}|$.

(i) Point C has position vector $m\mathbf{a} + n\mathbf{b}$, where m and n are positive integers. Find the area of triangle ABC in terms of m, n and $|\mathbf{b}|$. [4]

Point D is the mid-point of OA and point E lies on OB such that $OE : EB = 3 : 2$.

(ii) Find the position vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$, giving your answers in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Show that the vector equation of the line BD can be written as $\mathbf{r} = \lambda\mathbf{a} + (1 - 2\lambda)\mathbf{b}$, where λ is a parameter. By finding the vector equation of the line AE in a similar form in terms of a parameter μ, find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of the point F where the lines BD and AE meet. [4]

40. ASRJC Prelim 9758/2023/02/Q1

Relative to the origin O, two fixed points, A and B, have position vectors a and b respectively. The point P has position vector p given by $\mathrm{p} = \lambda\mathbf{a} + (1 - \lambda)\mathbf{b}$ where λ is a parameter such that $0 < \lambda < 1$.

(a) Show that for all real values of λ, the point P is collinear with A and B. [2]

(b) If angle AOB is 90^0, show that the position vector of the foot of perpendicular from O to AB is $\mathbf{a} + \dfrac{|\mathbf{a}|^2}{|\mathbf{a}|^2 + |\mathbf{b}|^2}(\mathbf{b} - \mathbf{a})$. [3]

41. SAJC Prelim/2020/02/Q2

(a) Show that the equation of the line passing through a fixed point with position vector a and parallel to the non-zero vector u, satisfies the equation $(\mathbf{r}-\mathbf{a})\times\mathbf{u}=\mathbf{0}$ where r is the position vector of any point on the line. [2]

(b) Referred to the origin O, the points A and B have position vectors a and b. It is given that $|\mathbf{b}|=\sqrt{3}|\mathbf{a}|$ and $|\mathbf{b}-\mathbf{a}|=\sqrt{7}\,|\mathbf{a}|$.
Using a suitable scalar product, determine if the origin O lies on the circumference of a circle with diameter AB. [4]

42. 2018 AJC Prelim/P2/Q2

(i) Given that $\mathbf{a}$ and $\mathbf{b}$ are non-zero constant vectors, show that the points with position vector $\mathbf{r}$ and satisfying the equation $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$ lie on a line. [3]

The line L_1 has equation $\left[\mathbf{r}-\begin{pmatrix}2\\1\\-1\end{pmatrix}\right]\times\begin{pmatrix}0\\1\\1\end{pmatrix}=\mathbf{0}$, and the line L_2 has equation $\mathbf{r}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\mu\begin{pmatrix}m\\0\\1\end{pmatrix}$,

where m is a constant and μ is a real parameter.

(ii) Find the vector equations of the parallel planes p_1 and p_2, in scalar product form, such that p_1 contains L_1 and p_2 contains L_2, leaving your answers in terms of m. [3]

(iii) Find the acute angle between the plane p_1 and the line which contains both the points $(2,1,-1)$ and $(-1,2,3)$, leaving your answer in terms of m. [2]

(iv) It is now given that L_1 and L_2 is contained in a common plane p_3. Using your answer in part **(iii)**, or otherwise, find the cartesian equation of the plane p_3. [1]

43. 2018 DHS Prelim/II/2

With reference to the origin O, the position vectors of point A and B are $\mathbf{a}$ and $\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors.

(i) State the shortest distance from B to the line passing through O and A. [1]

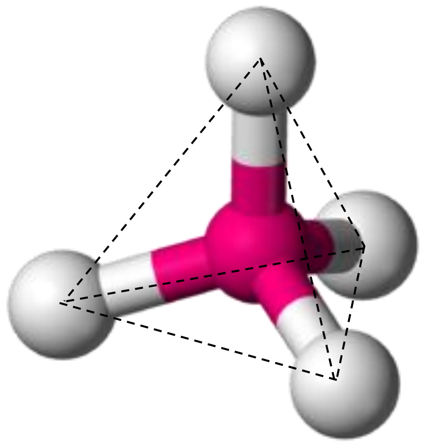
(ii) Given that $\mathbf{c}=\mathbf{a}+\mathbf{b}$, state the geometrical meaning of $|\mathbf{b}\times\mathbf{c}|$ and show that $|\mathbf{b}\times\mathbf{c}|=|\mathbf{a}\times\mathbf{b}|$.
[2]

(iii) Given that $\mathbf{a}\times 2\mathbf{b}=\mathbf{d}\times 3\mathbf{a}$, find a linear relationship between $\mathbf{a},\mathbf{b}$ and $\mathbf{d}$. [2]

44. 2018 IJC Prelim/P1/Q11

In Chemistry, the molecular structure of chemical compounds is often of interest to chemists as this will aid them in predicting the chemical properties of the compound.

In studying the molecular structure of *silicon tetrachloride*, it is found that this compound takes the form of a regular tetrahedron, that is, it consists of a *silicon* atom at the centre with four *chlorine* atoms symmetrically positioned at the corners of the regular tetrahedron (see diagram).



Suppose the centers of the *chlorine* atoms are at the points A, B, C and D with coordinates $(5,-2,5)$, $(5,4,-1)$, $(-1,-2,-1)$ and $(7,-4,-3)$ respectively, where $ABCD$ forms a regular tetrahedron.

(i) Verify that triangle ABC is an equilateral triangle. [2]

(ii) Find a vector that is perpendicular to the plane containing triangle ABC. [1]

(iii) π_1 is a plane that is perpendicular to $\overrightarrow{AB}$ and passes through the mid-point of the line segment AB. Find the cartesian equation of π_1. [2]

(iv) π_2 is a plane that is perpendicular to $\overrightarrow{BC}$ and passes through the mid-point of the line segment BC. Given that π_1 and π_2 meet in the line l, find a vector equation for l. [3]

(v) The position of the *silicon* atom is at the point G, where G is equidistant from A, B, C and D. Find the coordinates of G. [3]

(vi) The angle AGD is also known as the bonding angle of the compound. Find the bonding angle. Show your workings clearly. [2]

45. 2022 ASRJC Prelim/II/2

Relative to the origin O, the points A, B and C have position vectors **a**, **b** and **c** respectively. It is given that λ and μ are non-zero numbers such that $\lambda\mathbf{a} + \mu\mathbf{b} - \mathbf{c} = \mathbf{0}$ and $\lambda + \mu = 1$.

(i) Show that the points A, B and C are collinear. [3]

The angle between a and b is known to be obtuse and that $|\mathbf{a}| = 2$.

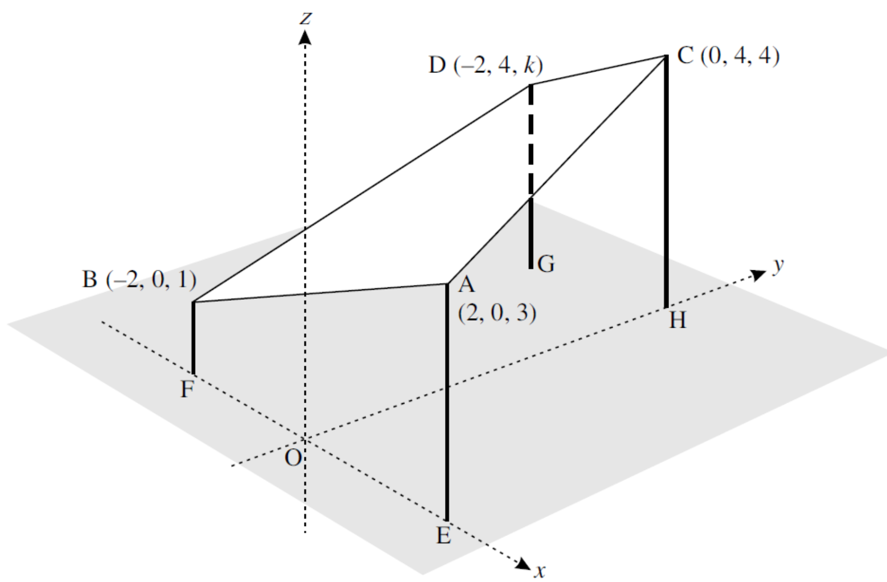
(ii) If k denotes the area of triangle OAB, show that $(\mathbf{a} \cdot \mathbf{b})^2 = 4\left(|\mathbf{b}|^2 - k^2\right)$. [3]

D is a point on the line segment AB with position vector d.

(iii) It is given that area of triangle OAB is 6 units^2, $|\mathbf{b}| = 10$ and that AOD is 90^0. By finding the value of $\mathbf{a} \cdot \mathbf{b}$, find **d** in terms of **a** and **b**. [4]

46. 2023 SAJC Prelim/01/Q9

A tentage for an outdoor activity is built by attaching a piece of polyester tent fabric to the tops of four vertical poles AE, BF, DG & CH, where E, F, G and H are positioned at ground level on the x-y plane. Coordinates of A, B, C and D are as shown in the diagram below, with lengths measured in meters. The length of the pole DG is k meters.



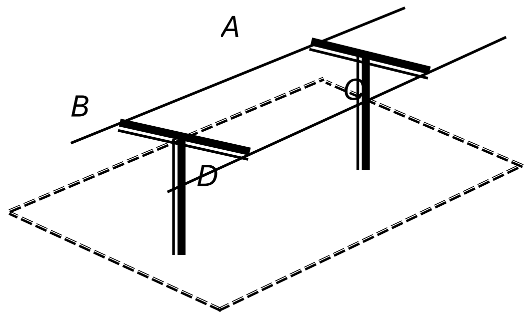
(i) Find the cartesian equation of the plane $ABDC$. [3]

(ii) Find the acute angle that plane $ABDC$ makes with the horizontal. [2]

(iii) Find the value of k. [2]

(iv) Show that the shape of the polyester tent fabric $ABDC$ is a trapezium. Hence, or otherwise, find its exact area in the form of $a^{\frac{3}{2}}$ units^2, where a is an integer to be determined. [5]

47. ACJC Prelim 9758/2024/01/Q11



At a ski resort, engineers are installing cables for a new cable car system to transport skiers to ski slopes. The system involves installing cables running between support towers. Cables are laid in straight lines and the widths of cables can be neglected. The cable *AB* is used to transport skiers up the slope and another parallel cable *CD* is used to transport skiers down the slope. Straight lines are used to represent the cables and a plane $\prod$ is used to model the ski slope.

The cable *AB* has vector equation $\mathbf{r}=\begin{pmatrix}5\\-5\\7\end{pmatrix}+\lambda\begin{pmatrix}-2\\2\\3\end{pmatrix}$ where $\lambda\in\mathbb{R}$ and $-5<\lambda<15$.

The parallel cable *CD* passes through the point $(3,-18,10)$. The cartesian equation of the ski slope $\prod$ is $x-2y+3z=5$.

(i) Find the distance between the cables *AB* and *CD*. [2]

(ii) The length of the cable from point *A* to point *B* is 100 units. Find the length of the projection of *AB* on the ski slope $\prod$. [3]

(iii) Find the coordinates of a point *P* on the cable *AB* which is at a perpendicular distance of $2\sqrt{14}$ units from the ski slope $\prod$. [3]

(iv) There is a viewing gallery on the mountain that overlooks the ski slope. The engineers wish to install a huge glass window plane at the viewing gallery. The window plane is perpendicular to the ski slope and is parallel to the cable *AB*. Given that $(10, 10, 20)$ is a point on the window plane, find the cartesian equation of the window plane. [3]

(v) Due to a power outage while testing the system, a cable car got stuck on the cable *AB* at the point *Q* with coordinates $(-7, 7, 25)$. The maintenance team wishes to reach the point *Q* as quickly as possible. Find the coordinates of the point on the ski slope closest to the point *Q* from where the team should launch their operation. [3]

48. CJC Prelim 9758/2024/02/Q1

(a) The points P, Q and R have position vectors $\mathbf{j}+\mathbf{k}$, $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $-2\mathbf{i}+\mathbf{j}+4\mathbf{k}$ respectively. Find the exact length of projection of $\overrightarrow{QR}$ on $\overrightarrow{PQ}$. [3]

(b) Two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $|\mathbf{a}|=\lambda|\mathbf{b}|$ and $|\mathbf{a}+\lambda\mathbf{b}|=2|\mathbf{a}-\lambda\mathbf{b}|$, where λ is a positive constant. Using a suitable scalar product, show that $\cos\theta=\dfrac{3}{5}$, where θ is the acute angle between $\mathbf{a}$ and $\mathbf{b}$. [3]

49. DHS Prelim 9758/2024/01/Q8

The line l_1 passes through the point A (1, 2, 4) and point B (−1, −1, 3). The line l_2 has equation $\underset{\sim}{r}=\begin{pmatrix}3\\-5\\2\end{pmatrix}+t\begin{pmatrix}6\\-7\\2\end{pmatrix}$, $t\in\mathbb{R}$.

(a) Explain why l_1 and l_2 are skew lines. [2]

(b) Find an equation of the plane, in scalar product form, that includes the midpoint of AB and the line l_2. [3]

The plane π_1 has equation $2x+7y+5z=18$. The point C lies on l_1 such that the foot of perpendicular of C onto π_1 has coordinates (3, 1, 1).

(c) Find the coordinates of C. [3]

The plane π_2 has equation $3x-4y+\lambda z=\mu$ and line l_1 does not intersect π_2.

(d) Find the value of λ. Hence find the acute angle between the planes π_1 and π_2. [3]

(e) If the distance between π_2 and l_1 is 2 units, find the exact values of μ. [3]

50. TJC Prelim 9758/2024/01/Q5

The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. C is the point on line OB such that AC is perpendicular to OB.

(a) By using a suitable scalar product, or otherwise, show that $\overrightarrow{OC}=\dfrac{(\mathbf{a}\cdot\mathbf{b})}{|\mathbf{b}|^2}\mathbf{b}$. [3]

(b) Give a geometrical interpretation of $\dfrac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{b}|}$. [1]

(c) It is given that $\mathbf{a}=\begin{pmatrix}-3\\3\\-1\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}1\\h\\0\end{pmatrix}$. Given also that the length of the line segment AB is 5 units and angle AOB is an obtuse angle, find the exact value of h. [4]

Answers

1	(i) $r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = -2$ (ii) $\overrightarrow{ON} = \begin{pmatrix}-1\\1\\-2\end{pmatrix}$ (iii) $\underset{\sim}{r} = \begin{pmatrix}-5\\-11\\-10\end{pmatrix} + \lambda\begin{pmatrix}3\\5\\5\end{pmatrix}, \lambda \in \mathbb{R}$ (iv) $a = 2$, $b = 108$ or -116 (v) $a \neq 2,\ b \in \Re$
2	(ii) 50.7°
3	(ii) $\dfrac{\sqrt{378}}{2}$ (iv) $\theta = 128.7^\circ$
5	165.5° $\lvert \underset{\sim}{b} \rvert = 0.340$
6	(i) 1 (iii) $\dfrac{42}{\sqrt{29}}$
7	(i) $k = 2$; $\overrightarrow{OC} = 3\mathbf{b} - 2\mathbf{a}$ (iii) $\theta = \dfrac{\pi}{3}$ or 60^0 (iv) $\begin{pmatrix}-2\\2\\-2\end{pmatrix}$
8	(b) $\theta = 56.4°(1\text{ d.p.})$, (c) $(2,5,0)$, (d) Circle that lies on *p* with centre $(2,5,0)$ and radius 4 on plane *p*.
9	(ii) $\mathbf{q} = \begin{pmatrix}4\\1\\7\end{pmatrix}$, $\mathbf{p} = \dfrac{1}{5}\begin{pmatrix}11\\5\\23\end{pmatrix}$
10	(i) 38.1° (ii) $\alpha = 2,\ \beta = -2.6,\ b = 1,\ \dfrac{3\sqrt{6}}{2}$
11	(a) $(-1,1,0)$ (c) $\mathbf{r} = \begin{pmatrix}5\\9\\-1\end{pmatrix} + \mu\begin{pmatrix}8\\7\\3\end{pmatrix}$ (d) $\mathbf{r} \cdot \begin{pmatrix}31\\-26\\-22\end{pmatrix} = -57$
12	$\lambda = \pm\sqrt{11}$
13	(i) $\mathrm{r} = \begin{pmatrix}-5\\-2\\3\end{pmatrix} + \lambda\begin{pmatrix}10\\4\\3\end{pmatrix}$ (ii) 9m (iii) $\overrightarrow{OX} = \dfrac{3}{5}\begin{pmatrix}-5\\-2\\6\end{pmatrix}$
14	(ii) $p:\ \mathbf{r} \bullet \begin{pmatrix}0\\1\\-1\end{pmatrix} = 1$ (iii) $\left(-\dfrac{1}{4}, \dfrac{1}{2}, -\dfrac{3}{2}\right)$
15	(i) 0 (iii) $a = -1$ and $b = 4$ (iv) $\mathbf{r} \bullet \begin{pmatrix}4\\5\\-1\end{pmatrix} = 4\sqrt{42}$ or $\mathbf{r} \bullet \begin{pmatrix}4\\5\\-1\end{pmatrix} = -4\sqrt{42}$
16	(a) 19.1° (b) $(4,0,5)$ (c) $\sqrt{6}$ units (d) $x + 5y + 7z = 39$

	(e) $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$, $\mu \in \mathbb{R}$
17	(i) $5+4\lambda=\mu$ (ii) $\lambda=-1$ **,** $\mu=1$
18	(i) $\mathbf{r}\cdot\begin{pmatrix} 4 \\ 3 \\ -5 \end{pmatrix}=100$; (ii) $\begin{pmatrix} 8 \\ 6 \\ -20 \end{pmatrix}$; (iii) 50 sq. units (iv) 50
19	(ii) $x-6y+2z=9$, (iii) $\dfrac{4}{\sqrt{41}}$
20	(a) $\left(\frac{1}{3},\frac{7}{3},\frac{13}{3}\right)$ (c) 1.11 rad or 63.4° (d) 2 units^2
21	(ii) $k=\dfrac{\sqrt{3}}{8}$, (iii) $t=\dfrac{5\sqrt{3}}{4}$
22	(a) $\mathbf{b}=k\left(3\mathbf{a}+2\mathbf{c}\right), k\in\mathbb{R}\setminus\{0\}$, (b) (ii) The set of **v** forms a straight line passing through $(-1,3,0)$ and parallel to $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$.
23	(i) (2, 1, 0), (ii) skew lines, (iii) $l_3:\mathbf{r}=\begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix}+\gamma\begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}, \gamma\in\mathbb{R}$, (iv) $\alpha=2$ and $\beta\neq-14$
24	$\overrightarrow{OR}=\dfrac{2}{5}\begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix}$, $\overrightarrow{OA'}=\dfrac{1}{5}\begin{pmatrix} 5 \\ -21 \\ -3 \end{pmatrix}$
25	(ii) $\overrightarrow{OQ}=\begin{pmatrix} 9/2 \\ 1 \\ 0 \end{pmatrix}$, (iii) $\overrightarrow{OX}=\begin{pmatrix} 0 \\ -8 \\ 11 \end{pmatrix}$, (iv) 14.855, 53.6 units^2
26	(b)(i) $\overrightarrow{OP}=\begin{pmatrix} -2 \\ 0 \\ -10 \end{pmatrix}$ (ii) $\lambda=1$ (iii) Area of $\Delta ACP=\dfrac{1}{2}\sqrt{18696}=68.4$ unit^2
27	(i) Normal of plane $OBDC=\mathbf{b}\times\mathbf{c}$ $PE=\dfrac{\lvert\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\rvert}{\lvert\mathbf{b}\times\mathbf{c}\rvert}$ (iii) $p=\pm2$ Yes since $\lvert\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\rvert=\lvert\mathbf{b}\cdot(\mathbf{a}\times\mathbf{c})\rvert=$ Volume of the given parallelpiped

29	$\dfrac{3}{16}k$
30	(i) $b=3.07$ (2 d.p.) (ii) $A(9,3,0), B(5,1,0)$ (iv) l_3 : $\mathbf{r}=\begin{pmatrix}2.5\\-0.25\\0\end{pmatrix}+\beta\begin{pmatrix}-8\\1\\2\end{pmatrix},\ \beta\in\mathbb{R}$ (v) l_2 and l_3 do not intersect, skew lines, not parallel.
31	(a) The set of all possible positions of the point R form the line (OR R is any point on the line) passing through P and parallel to the vector $\underset{\sim}{q}$. (b) The set of all possible positions of the point R form the plane (OR R is any point on the plane) that contains the point P(1, -2, 3) with a normal vector $\mathbf{q}$ (or perpendicular to **q)** (c) It is the shortest distance from point C to the plane in (b). **OR** It is the length of projection of $\overrightarrow{CP}$ onto $\mathbf{q}$.
32	(i) $\begin{pmatrix}3\\3\\-2\end{pmatrix}$ (ii) $\sqrt{11}$ (iii) $2x+3y-4z=23$ (iv) $\begin{pmatrix}3\\2\\3\end{pmatrix}$
33	$\overrightarrow{OS}=\dfrac{5\mathbf{a}+4\mathbf{b}}{9}$; 2:9
34	(i) $\overrightarrow{OX}=5\mathbf{b}-4\mathbf{a}$; $\overrightarrow{OY}=4(\mathbf{b}-\mathbf{a})$ Since $\overrightarrow{OY}=4(\mathbf{b}-\mathbf{a})\Rightarrow\overrightarrow{OY}=4\overrightarrow{AB}$ $\Rightarrow\overrightarrow{OY}$ is parallel to $\overrightarrow{AB}\Rightarrow OABY$ is a trapezium. (ii) $\lvert(\mathbf{a}+\mathbf{b})\cdot\mathbf{b}\rvert$ is the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OB}$. (iii) 10; 10
35	(i) $9\lvert k\rvert$; area of a parallelogram with sides OA and OC. (ii) ±3 (iii) $\mathbf{c}=-3\mathbf{b}+4\mathbf{a}$
36	(i) $\dfrac{3}{20}$, (ii) $\mu=\dfrac{\lvert\mathbf{a}\rvert^2}{\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2}$
37	(i) $r=\begin{pmatrix}8\\16\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\-2\\1\end{pmatrix},\lambda\in\mathbb{R}$ (ii) $9x-2y+5z=40$
38	(i) $\dfrac{4}{5}\sqrt{14};2\sqrt{14}$ (iv) $\dfrac{5}{3}$
39	(i) $(m+n-1)\lvert\mathbf{b}\rvert^2$ (ii) $\overrightarrow{OD}=\dfrac{1}{2}\mathbf{a}$ $\overrightarrow{OE}=\dfrac{3}{5}\mathbf{b}$ (iii) $\dfrac{2}{7}\mathbf{a}+\dfrac{3}{7}\mathbf{b}$

42	□p2: $\underset{\sim}{r} \bullet \begin{pmatrix} 1 \\ m \\ -m \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ m \\ -m \end{pmatrix} = -1 - m$ (iii) $\theta = \sin^{-1}\left\| \dfrac{3+3m}{\sqrt{26+52m^2}} \right\|$ (iv) $x - y + z = 0$
43	(i) $\left\|\mathbf{b} \times \hat{\mathbf{a}}\right\|$ (ii) Area of a parallelogram with sides OB and OC (iii) $\mathbf{a} = k\left(2\mathbf{b} + 3\mathbf{d}\right), k \in \mathbb{R}$
44	(ii) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (iii) $y - z = -1$ (iv) l: $\underset{\sim}{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, where $\lambda \in \mathbb{R}$ (v) (4, -1, 0)
45	$\mathbf{d} = \dfrac{4}{5}\mathbf{a} + \dfrac{1}{5}\mathbf{b}$
46	(i) $x + y - 2z = -4$. (ii) $\theta = 35.3^o$ (iii) $k = 3$ (iv) $6^{\frac{3}{2}}$ units^2
47	(i) 13.1 (ii) 98.1 (iii) P $(7, -7, 4)$ (iv) $12x + 9y + 2z = 250$ Foot of perpendicular is $\left(-\dfrac{21}{2}, 14, \dfrac{29}{2}\right)$
48	(a) $\dfrac{5}{7}\sqrt{14}$ units
49	(b) $\underset{\sim}{r} \bullet \begin{pmatrix} -43 \\ -30 \\ 24 \end{pmatrix} = 69$ (c) (5, 8, 6) (d) $\lambda = 6$, $\theta = 83.3^o$ *or* 1.45rad (e) $\mu = 19 - 2\sqrt{61}$ or $19 + 2\sqrt{61}$
50	(c) $h = 3 - 2\sqrt{2}$