

9

PROBABILITY

SYLLABUS

- Learn to apply addition and multiplication of probabilities
- Understand mutually exclusive events and independent events
- Use tables of outcomes, Venn diagrams, tree diagrams and permutation and combination technique to calculate probabilities.
- Calculate conditional probabilities in simple cases
- Use of:
 - $P(A') = 1 - P(A)$
 - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 - $P(A | B) = \frac{P(A \cap B)}{P(B)}$

LECTURE 4

Lesson Outline

- Guided self-practice questions of how and why each solving approach is applied

4 SELF-PRACTICE QUESTIONS WITH GUIDED SOLUTIONS



After attempting the guided questions in this section, you may go to the Maths Google Site at <https://sites.google.com/acjc.edu.sg/acjcmaths/jc1-class-of-2025/class-of-2025-h2-maths> for complete solutions.

Guided Self-Practice Question 1

Two boxes, one white and one black, are used in a game. The boxes contain balls labelled '2', '5' and '10'. The balls are identical except for their numbers. The table below shows the number of balls in each box.

	Number of balls labelled		
	'2'	'5'	'10'
White box	5	2	1
Black box	5	1	2

In the first round of the game, a player draws 3 balls randomly from the white box. If the numbers on the 3 balls add up to 9 or more, the player enters the second round of the game to draw another 3 balls randomly from the black box.

- Find the probability that the player draws 3 different numbers in the first round. [1]
- Show that the probability that sum of the numbers add up to 12 and 6, in the first and second round respectively, is $\frac{25}{1568}$. Hence, find the probability that the sum of the numbers drawn from the two rounds adds up to 18. [4]

[RVHS/2019/II/Q6]

Solution**(i) Method 1: Using “Combination”**

Number of ways to draw 3 different numbers in the first round = $5 \times 2 \times 1$

= 10 ways

Total number of ways to select **any** 3 numbers
 $= \binom{8}{3} = 56$ ways

Since **all** the 3 objects selected has to be distinct, using $\binom{5}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}$, is also acceptable.

However, it is **not** encouraged to use the method of always ‘choosing **1**’,

i.e. $\binom{a}{1} \cdot \binom{b}{1} \cdot \binom{k}{1}$, since in most of the questions, the objects selected usually are **not all** distinct.

P(player draws 3 different numbers in the first round)

$$= \frac{\boxed{10}}{\boxed{56}}$$

$$= \frac{5}{28}$$

In probability, we are calculating
 ‘number of possible ways’ **out of** ‘total number of ways without any restriction’.

Method 2: Using “Fractions”

Possible cases are: ‘2’, ‘5’, ‘10’

‘2’, ‘10’, ‘5’

⋮

‘10’, ‘5’, ‘2’

P(for the case '2', '5', '10')

$$= \frac{\boxed{5}}{8} \times \frac{\boxed{2}}{7} \times \frac{\boxed{1}}{6}$$

$$P(\text{for the case '2', '10', '5'}) = \frac{5}{8} \times \frac{1}{7} \times \frac{2}{6}$$

⋮

$$P(\text{for the case '10', '5', '2'}) = \frac{1}{8} \times \frac{2}{7} \times \frac{5}{6}$$

Since each case gives the same answer, thus,

P(player draws 3 different numbers in the first round)

$$\frac{\boxed{5}}{8} \times \frac{\boxed{2}}{7} \times \frac{\boxed{1}}{6} \times \boxed{3} != \frac{5}{28}$$

It is necessary to find the total number of cases. To begin, list out a few possible cases. Make use of ‘factorial’ to find the number of ways for n distinct “objects” to permute. In this case, there are 3 distinct “objects”, ‘2’, ‘5’ and ‘10’. Hence, there are 3! ways, thus 6 cases. The cases are considered different since the orders are different.

(ii) Method 1: Using “Combination”

Drawing 3 balls from the white box, to obtain a total sum of 12, there should be one ‘2’ and two ‘5’s. Similarly, from the black box, to obtain a total sum of 6, there should be three ‘2’s.

Number of ways to obtain a sum of 12 from the first round

$$= \binom{5}{1} \times \binom{2}{2}$$

As not **all** the 3 objects selected are distinct, $\binom{5}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}$ will lead to multiple counting. For selection of one ‘2’ and two ‘5’s, use $\binom{5}{1} \cdot \binom{2}{2}$.

Number of ways to obtain a sum of 6 from the second round

$$= \binom{5}{3}$$

Since the 3 objects selected are **NOT** distinct, $\binom{5}{1} \cdot \binom{4}{1} \cdot \binom{3}{1}$ will lead to multiple counting. To select three ‘2’s, use $\binom{5}{3}$.

Required probability

= P(sum from first round is 12 **and** sum from second round is 6)

$$= \frac{\binom{5}{1} \times \binom{2}{2}}{\binom{8}{3}} \times \frac{\binom{5}{3}}{\binom{8}{3}}$$

$$= \frac{25}{1568} \quad (\text{shown})$$

In probability, we are calculating ‘number of possible ways’ **out of** ‘total number of ways without any restriction’.

Hence, P(sum of the numbers drawn adds up to 18)

= P(sum of each of 1st and 2nd round is 9) +

P(sum of 1st round is 12 and sum of 2nd round is 6)

Since both rounds need two ‘2’s and one ‘5’, the 3 ‘objects’ selected in each round are **not all** distinct, $\binom{5}{1} \cdot \binom{4}{1} \cdot \binom{2}{1}$ will again lead to multiple counting. To select two ‘2’s and one ‘5’, use $\binom{5}{2} \cdot \binom{2}{1}$.

Since the 3 objects selected are **not** distinct, $\binom{5}{1} \cdot \binom{4}{1}$ will lead to multiple counting. To select two ‘2’s, use $\binom{5}{2}$.

$$= \frac{\binom{5}{2} \times \binom{2}{1}}{\binom{8}{3}} \times \frac{\binom{5}{2} \times \binom{1}{1}}{\binom{8}{3}} + \frac{25}{1568} = \frac{125}{1568}$$

Method 2: Using “Fractions”

Possible cases are:

‘2’, ‘5’, ‘5’

‘5’, ‘2’, ‘5’

‘5’, ‘5’, ‘2’

Listing seems easy here since there are only 3 cases. Alternatively, using factorial, with 2 repeated ‘5’s out of the 3 numbers, there are $\frac{3!}{2!}$ ways.

$$P(\text{for the case '2', '5', '5' from the white box}) = \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6}$$

⋮

$$P(\text{for the case '5', '5', '2' from the white box}) = \frac{2}{8} \cdot \frac{1}{7} \cdot \frac{5}{6}$$

Since each case gives the same answer, thus,

P(sum of numbers from 1st round is 12)

$$= \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6} \times \frac{3}{2}!$$

Now, for obtaining three ‘2’s from the black box,

$$P(\text{for the case '2', '2', '2'}) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$$

Since all ‘2’s are the same, there are no other cases.

Required probability

= P(sum from first round is 12 **and** sum from second round is 6)

$$= \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6} \times \frac{3}{2}! \times \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{25}{1568} \quad (\text{shown})$$

Hence, P(sum of the numbers drawn adds up to 18)

= P(sum of each of 1st and 2nd round is 9) +

P(sum of 1st round is 12 and sum of 2nd round is 6)

$$= \frac{5}{8} \times \frac{4}{7} \times \frac{2}{6} \times \frac{3}{2}! \times \frac{5}{8} \times \frac{4}{7} \times \frac{1}{6} \times \frac{3}{2}! + \frac{25}{1568}$$

$$= \frac{125}{1568}$$

Again, listing seems easy here since there are only 3 cases. Alternatively, using factorial, with 2 repeated ‘2’s out of the 3 numbers, there are $3!/2!$ ways for both 1st and 2nd rounds.

■

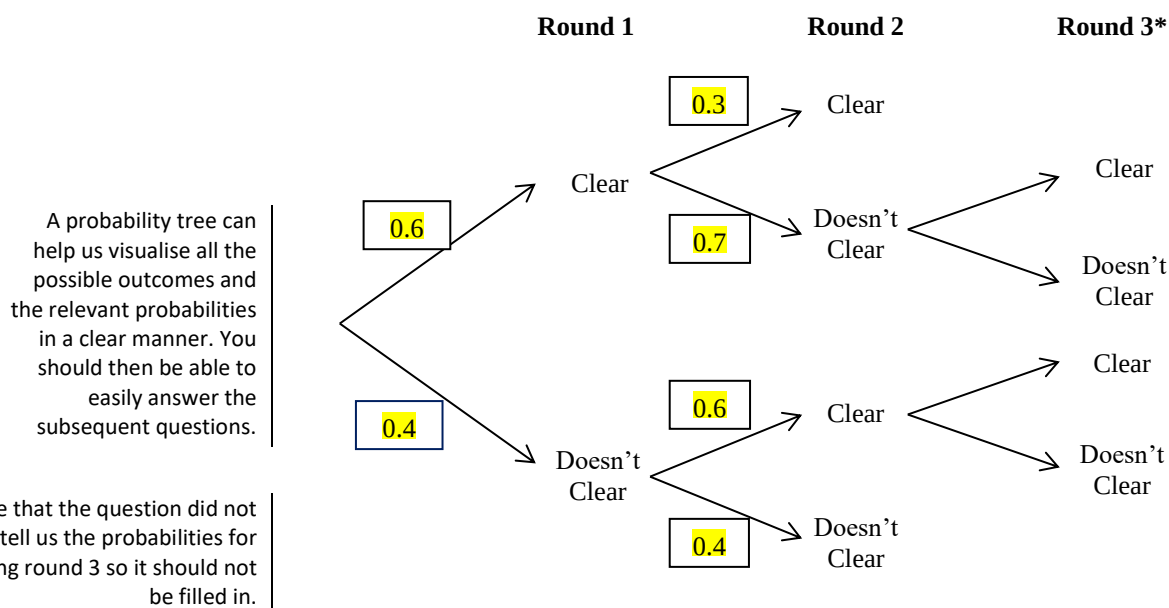
Guided Self-Practice Question 2

A computer game consists of at most 3 rounds. The game will stop when a player clears 2 rounds or does not clear 2 consecutive rounds. The probability that a player clears round 1 is 0.6. The conditional probability that the player clears round 2 given that he clears round 1 is half the probability that he clears round 1. The conditional probability that the player clears round 2 given that he does not clear round 1 is the same as the probability that he clears round 1.

- (i) Find the probability that a player plays 3 rounds. [1]
 (ii) Find the probability that a player clears round 1, given that he does not clear round 2. [2]
 (iii) The total probability that a player plays 3 rounds and clears round 3 is 0.2. Find the probability that a player clears exactly 2 rounds. [2]
- [TMJC/2019/II/Q6(i)-(iii)]

Solution

- (i) Fill in the boxes with the relevant probabilities in the probability tree in the next page.
 (Recall: For each pair of branches, the probabilities add up to 1...)



$$\begin{aligned}
 &P(\text{a player plays 3 rounds}) \\
 &= P(\text{clears round 1 but doesn't clear round 2}) \\
 &\quad + P(\text{doesn't clear round 1 but clears round 2}) \\
 &= (0.6)(0.7) + (0.4)(0.6) \\
 &= 0.66
 \end{aligned}$$

- (ii)
- $P(\text{clears round 1} | \text{doesn't clear round 2})$

$$= \frac{P(\text{clears round 1 and does not clear round 2})}{P(\text{does not clear round 2})}$$

$$= \frac{(0.6)(0.7)}{(0.6)(0.7) + (0.4)(0.4)} = \frac{21}{29} = 0.724 \text{ (3 s.f.)}$$

Use the conditional probability formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)},$$

then fill in the corresponding probabilities in the next step by referring to your probability tree.

- (iii)
- $P(\text{a player clears exactly 2 rounds})$

$$= P(\text{clears round 1 and round 2})$$

$$+ P(\text{clears round 1, doesn't clear round 2, clears round 3})$$

$$+ P(\text{doesn't clear round 1, clears round 2 and round 3})$$

$$= (0.6)(0.3) + 0.2 = 0.38$$

"The total probability that a player plays 3 rounds and clears round 3 is 0.2. Find the probability that a player clears exactly 2 rounds."

Think about what this probability of 0.2 represents...what does it mean when "a player plays 3 rounds and clears round 3"?

Refer to your probability tree...this refers to the case the player:

I) "clears round 1, doesn't clear round 2 and clears round 3"; or

II) "doesn't clear round 1, clears round 2 and clears round 3"



Guided Self-Practice Question 3

In a school survey, a group of 80 students are asked about how much time per week (to the nearest hour) they spend on their co-curricular activities (CCA). The readings are shown below:

	CCA (hours)		
	3 or less	4 to 6	7 or more
Boy	17	20	10
Girl	18	$15 - k$	k

A student is selected random from the group. Defining the events as follows:

G : The student is a girl.

L : The student spends 6 hours or less weekly.

M : The student spends 4 hours or more weekly.

Find the following probabilities in terms of k .

(i) $P(L' \cup M')$ [2]

(ii) $P(G | L')$ [1]

Given that $P(L \cap M) = \frac{2}{5}$, find the value of k . Hence determine if L and

M are independent, justifying your answer. [3]

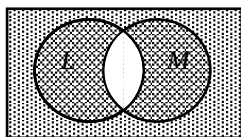
If it is given instead that the events G and $L \cap M$ are mutually exclusive, find the value of k . [1]

[DHS/2019/II/Q7 (modified)]

Solution

Recall that $P(A) = \frac{n(\text{event } A)}{n(\text{all in the universal set})}$

Consider two options for finding $P(L' \cup M')$ by considering Venn diagram



$$\begin{aligned} \text{(i)} \quad P(L' \cup M') &= \frac{80 - n(L \cap M)}{80} \\ &= \frac{80 - (35 - k)}{80} \\ &= \frac{45 + k}{80} \end{aligned}$$

Alternatively,

$$\begin{aligned} P(L' \cup M') &= P(L') + P(M') - P(L' \cap M') \\ &= \frac{10 + k}{80} + \frac{35}{80} - 0 = \frac{45 + k}{80} \end{aligned}$$

Or use the formula
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Recall formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\begin{aligned} \text{(ii)} \quad P(G|L') &= \frac{P(G \cap L')}{P(L')} \\ &= \frac{k}{k + 10} \end{aligned}$$

$L \cap M$ means time spent is between 4 hours and 6 hours (including 4 and 6)

$$\begin{aligned} \text{From the table, } P(L \cap M) &= \frac{20 + (15 - k)}{80} \\ &= \frac{35 - k}{80} \end{aligned}$$

To show that events A and B are independent, can show any of the following:

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A|B) = P(A)$

(c) $P(B|A) = P(B)$

Any of (a), (b) or (c) with the complements of A and/or B .

Since it is given that $P(L \cap M) = \frac{2}{5}$, then $\frac{35 - k}{80} = \frac{2}{5}$.

Solving for k , $k = 3$.

To determine if L and M are independent:

It is given that $P(L \cap M) = \frac{2}{5}$.

Now just need to evaluate $P(L)P(M)$

$$P(L)P(M) = \frac{67}{80} \times \frac{45}{80} = \frac{603}{1280} \neq \frac{2}{5}$$

Since $P(L \cap M) \neq P(L)P(M)$, L and M are **not** independent

Alternatively,

$$P(L) = \frac{70 - k}{80} = \frac{67}{80} \text{ and } P(L|M) = \frac{35 - k}{45} = \frac{32}{45} \neq \frac{67}{80}.$$

Since $P(L) \neq P(L|M)$, L and M are **not** independent

Since $P(G \cap (L \cap M)) = 0$

$$\Rightarrow 15 - k = 0$$

$$\therefore k = 15$$

If A and B are mutually exclusive, then $P(A \cap B) = 0$.
Intuitively, this means that no member of set A can be found in set B , and vice versa. ■

Guided Self-Practice Question 4

A game has three sets of ten symbols, and one symbol from each set is randomly chosen to be displayed on each turn. The symbols are as follows.

Set 1 + + + + × × × ○ ○ ★

Set 2 + + + × ○ ○ ○ ○ ★ ★

Set 3 + + × × × × ○ ○ ○ ★

For example, if a + symbol is chosen from set 1, a ○ symbol is chosen from set 2 and a ★ symbol is chosen from set 3, the display would be + ○ ★.

- (i) Find the probability that, on one turn,
- ★★★ is displayed, [1]
 - at least one ★ symbol is displayed, [2]
 - two × symbols and one + symbol are displayed, in any order. [3]
- (ii) Given that exactly one of the symbols displayed is ★, find the probability that the other two symbols are + and ○. [4]

[9758/2014/II/Q10]

Solution

(i)(a) Required probability

$$= P(\star \text{ in set 1}) \cdot P(\star \text{ in set 2}) \cdot P(\star \text{ in set 3})$$

$$= \frac{1}{10} \cdot \frac{2}{10} \cdot \frac{1}{10}$$

$$= \frac{1}{500}$$

Consider the probability of ★ appearing in each of the sets. The choice of symbol from one set is *independent* of the choice from any other sets.

(b) Required probability

$$= 1 - P(\text{no } \star \text{ displayed})$$

$$= 1 - \frac{9}{10} \cdot \frac{8}{10} \cdot \frac{9}{10}$$

$$= \frac{44}{125}$$

The direct method of at least one ★ will require three cases: $P(1 \star \text{ displayed})$, $P(2 \star \text{ displayed})$, and $P(3 \star \text{ displayed})$. Consider the complement instead.

$$(c) \quad P(\times \times +) = \frac{3}{10} \cdot \frac{1}{10} \cdot \frac{2}{10} = \frac{3}{500}$$

$$P(\times + \times) = \frac{3}{10} \cdot \frac{3}{10} \cdot \frac{4}{10} = \frac{18}{500}$$

$$P(+ \times \times) = \frac{4}{10} \cdot \frac{1}{10} \cdot \frac{4}{10} = \frac{4}{250}$$

$$\text{Hence required probability} = \frac{3}{500} + \frac{18}{500} + \frac{4}{250} = \frac{29}{500}$$

Need to consider the number of ways two $\times$ and one $+$ can be arranged. Can the complement method work here?

(ii) $P(\text{exactly one } \star)$

$$= P(\star ? ?) + P(? \star ?) + P(? ? \star)$$

$$= \frac{1}{10} \cdot \frac{8}{10} \cdot \frac{9}{10} + \frac{9}{10} \cdot \frac{2}{10} \cdot \frac{9}{10} + \frac{9}{10} \cdot \frac{8}{10} \cdot \frac{1}{10}$$

$$= \frac{153}{500}$$

The question stated

“**Given** that exactly one of the symbols displayed is $\star$, find the probability that the other two symbols are $+$ and $\circ$.”

Look out for the word “**given**”, which usually implies conditional probability! Recall formula for conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Find $P(B)$ first since it is easier to identify. In this case, B = event that exactly one of the symbols is $\star$. $?$ refers to any symbol other than $\star$.

$P(\text{exactly one } \star, \text{ one } +, \text{ one } \circ \text{ in any order})$

$$= P(\star + \circ) + P(\star \circ +) + P(+ \circ \star) + P(+ \star \circ)$$

$$+ P(\circ \star +) + P(\circ + \star)$$

Identify the cases in the intersection $A \cap B$ and evaluate their probabilities.

$$= \left(\frac{1}{10} \cdot \frac{3}{10} \cdot \frac{3}{10} \right) + \left(\frac{1}{10} \cdot \frac{4}{10} \cdot \frac{2}{10} \right) + \left(\frac{4}{10} \cdot \frac{4}{10} \cdot \frac{1}{10} \right) + \left(\frac{4}{10} \cdot \frac{2}{10} \cdot \frac{3}{10} \right) \\ + \left(\frac{2}{10} \cdot \frac{2}{10} \cdot \frac{2}{10} \right) + \left(\frac{2}{10} \cdot \frac{3}{10} \cdot \frac{1}{10} \right)$$

$$= \frac{71}{1000}$$

Hence, required probability

$$= P(\text{exactly one } \star, \text{ one } +, \text{ one } \circ \text{ in any order} \mid \text{exactly one } \star)$$

$$= \frac{\frac{71}{1000}}{\frac{153}{500}} = \frac{71}{306}$$

Put the probabilities found above into the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$ to evaluate the answer.

■

Guided Self-Practice Question 5

In this question, give each of your answers as an **exact** fraction in its lowest term.

- (a) In a certain sample space, it is known that events A and B are independent. Given that $P(A \cup B) = \frac{3}{4}$ and $P(A' \cap B) = \frac{2}{15}$, where A' is the complement of event A , find
- (i) $P(B)$, [3]
- (ii) $P(A \cap B | A \cup B)$. [2]
- (b) A teacher is to form two groups of 4 students from a class of 3 boys and 5 girls for Mathematics consultation session. Three of the girls, Ivy, Tamie and Cassy are good friends from the class. Find the probability that
- (i) the boys are together in the same group, [2]
- (ii) either Ivy or Tamie is in the same group as Cassy. [3]

[NJC/2010/II/Q8]

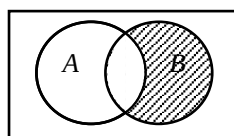
Solution

$$(a)(i) P(A) = P(A \cup B) - P(A' \cap B)$$

$$= \frac{3}{4} - \frac{2}{15}$$

$$= \frac{37}{60}$$

Draw a Venn Diagram to help you visualise the given probabilities...



Since events A and B are independent, events A' and B are also independent,

$$P(A' \cap B) = P(A')P(B) = \frac{2}{15}$$

$$P(B) = \frac{2}{15} \cdot \frac{1}{P(A')} = \frac{2}{15} \cdot \frac{60}{23}$$

$$= \frac{8}{23}$$

Alternatively,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A)P(B)$$

$$\Rightarrow \frac{3}{4} = \frac{37}{60} + P(B) \left(1 - \frac{37}{60}\right)$$

$$\Rightarrow P(B) = \frac{\frac{3}{4} - \frac{37}{60}}{\frac{23}{60}} = \frac{8}{23}$$

If two events are independent, their complement events will also be independent, i.e. if A and B are independent events, then,

A' and B'

A' and B

A and B'

are also independent events.

Also, if two events X and Y are independent, then,

$$P(X \cap Y) = P(X)P(Y)$$

Use the conditional probability formula

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)},$$

then simplify the numerator in the next step.

$$\begin{aligned} \text{(ii)} \quad & P(A \cap B | A \cup B) \\ &= \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} \\ &= \frac{P(A \cap B)}{P(A \cup B)} \\ &= \frac{\frac{37}{60} \cdot \frac{8}{23}}{\frac{3}{4}} \\ &= \frac{296}{1035} \end{aligned}$$

Hint: take note of possible double-counting (Refer to Example 21 of Chapter 8: Permutations and Combinations Lecture Notes)

$$\begin{aligned} \text{(b)(i)} \quad & P(\text{all 3 boys in the same group}) \\ &= \frac{\text{No. of ways the boys are together in the same group}}{\text{No. of ways with no restrictions}} \\ \text{No. of ways the boys are together in the same group} &= {}^5C_1 {}^4C_4 \\ \text{No. of ways with no restrictions} &= \frac{{}^8C_4 {}^4C_4}{2!} \\ \therefore P(\text{all 3 boys in the same group}) &= \frac{1}{7} \\ \\ \text{(ii)} \quad & P(\text{either Ivy or Tamie is in the same group as Cassy}) \\ &= \frac{\text{No. of ways either Ivy or Tamie is in the same group as Cassy}}{\text{No. of ways with no restrictions}} \\ \text{Case 1: Ivy is in the same group as Cassy (Tamie is not)} &= {}^5C_2 {}^3C_3 \\ \text{Case 2: Tamie is in the same group as Cassy (Ivy is not)} &= {}^5C_2 {}^3C_3 \\ \text{Case 3: Ivy, Tamie and Cassy are all in the same group} &= {}^5C_1 {}^4C_4 \\ \text{No. of ways either Ivy or Tamie is in the same group as Cassy} &= {}^5C_2 {}^3C_3 + {}^5C_2 {}^3C_3 + {}^5C_1 {}^4C_4 \\ \text{No. of ways with no restrictions} &= \frac{{}^8C_4 {}^4C_4}{2!} \quad (\text{same as in b(i)}) \\ \therefore P(\text{either Ivy or Tamie is in the same group as Cassy}) &= \frac{{}^5C_2 {}^3C_3 + {}^5C_2 {}^3C_3 + {}^5C_1 {}^4C_4}{\frac{{}^8C_4 {}^4C_4}{2!}} = \frac{25}{35} = \frac{5}{7} \end{aligned}$$

■

Guided Self-Practice Question 6

For events A , B and C it is given that $P(A) = 0.45$, $P(B) = 0.4$, $P(C) = 0.3$ and $P(A \cap B \cap C) = 0.1$. It is also given that events A and B are independent, and that events A and C are independent.

- (i) Find $P(B|A)$. [1]
- (ii) Given also that events B and C are independent, find $P(A' \cap B' \cap C')$. [3]
- (iii) Given instead that events B and C are **not** independent, find the greatest and least possible values of $P(A' \cap B' \cap C')$. [4]

[9758/2015/II/Q9]

Solution

- (i) Since A and B are independent,

$$P(B|A) = P(B) = 0.4$$

If A and B are independent, we have

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A|B) = P(A)$

(c) $P(B|A) = P(B)$

- (ii) Since A and B are independent,

$$P(A \cap B) = P(A) \cdot P(B) = 0.18$$

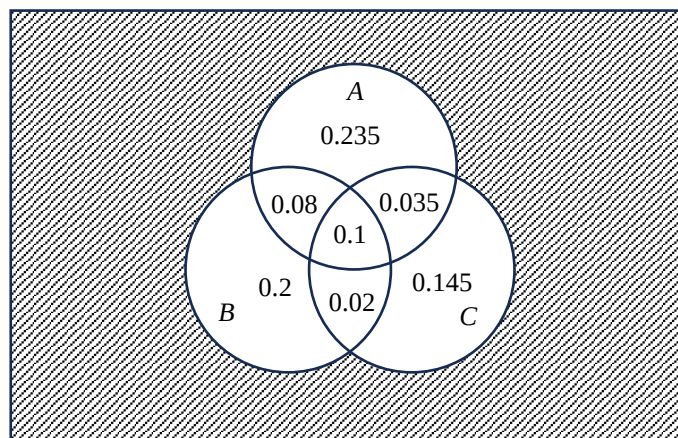
Since A and C are independent,

$$P(A \cap C) = P(A) \cdot P(C) = 0.135$$

Since B and C are independent,

$$P(B \cap C) = P(B) \cdot P(C) = 0.12$$

Refer to the Venn Diagram. $P(A' \cap B' \cap C')$ refers to the shaded region.



Draw a Venn Diagram to help you visualise the given probabilities.

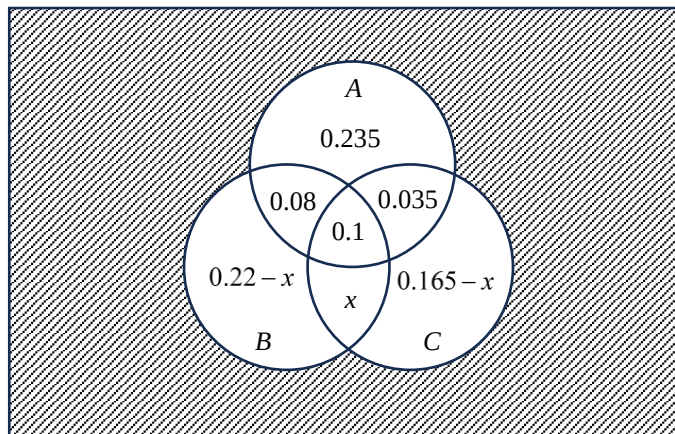
Fill in the probabilities found in the Venn Diagram.

It is given that $P(A \cap B \cap C) = 0.1$, and with the other probabilities given/found above, you can obtain these probabilities shown.

$$\begin{aligned} P(A' \cap B' \cap C') &= 1 - P(A \cup B \cup C) \\ &= 1 - (0.45 + 0.2 + 0.02 + 0.145) \\ &= 0.185 \end{aligned}$$

- (iii) Refer to the Venn Diagram. $P(A' \cap B' \cap C')$ refers to the shaded region.

Since B and C are not independent, this is the region that we cannot determine (unlike (ii))! Just let this value be x for now.



Express $P(A' \cap B' \cap C')$ in terms of x .

Since B and C are not independent, we let $P(B \cap C) = x + 0.1$.

$$\begin{aligned} P(A' \cap B' \cap C') &= 1 - P(A \cup B \cup C) \\ &= 1 - (0.45 + (0.22 - x) + x + (0.165 - x)) \\ &= 0.165 + x \end{aligned}$$

Least possible $P(A' \cap B' \cap C')$ occurs when x is smallest.

Least possible $P(A' \cap B' \cap C')$ occurs when $x = 0$.

$$\therefore \text{least possible } P(A' \cap B' \cap C') = 0.165$$

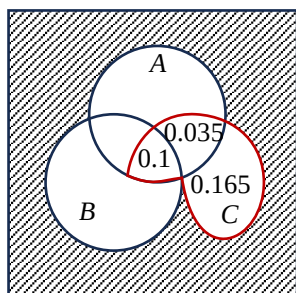
Greatest possible $P(A' \cap B' \cap C')$ occurs when x is maximised.

Greatest possible $P(A' \cap B' \cap C')$ occurs when $x = 0.165$.

$$\therefore \text{greatest possible } P(A' \cap B' \cap C') = 0.165 + 0.165 = 0.33$$

Alternative explanation:

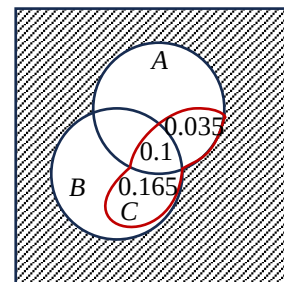
Since we do not know the value of x , it means that x can vary! How can x vary then? Look at the following two diagrams.



For least $P(A' \cap B' \cap C')$, shaded region must be smallest. The 'circle' representing $P(C)$ must occupy the largest space in the shaded region.

For greatest $P(A' \cap B' \cap C')$, shaded region must be largest.

Again, to make this happen, we vary x such that the circle representing $P(C)$ occupy the smallest space in the shaded region.



ANNEX

Practice Questions on Conditional Probability

- 1 Susan took two tests. The probability of her passing both tests is 0.6. The probability of her passing the first test is 0.8. Find the probability of her passing the second test given that she passed the first test.
- 2 A bag contains red and blue marbles. Two marbles are drawn without replacement. The probability of selecting a red marble and a blue marble is 0.28. The probability of selecting a red marble on the first draw is 0.5. What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was red?
- 3 Find the probability that the total score on two dice is greater than 9, given that the score on the first die is 5.
- 4 Events A and B are such that $P(A) = \frac{1}{3}$, $P(B|A) = \frac{1}{4}$ and $P(A' \cap B') = \frac{1}{6}$. Find
 - (i) $P(A \cup B)$
 - (ii) $P(B)$ [N01/II/11(a)]
- 5 A fair die is thrown three times. Events A , B and C are defined as follows:
 A : the total score is an odd number
 B : a six appears at the first throw
 C : the total score is 13.
 (a) Calculate $P(A)$ and $P(A|B)$ and state whether A and B are independent events.
 (b) Calculate $P(C)$ and $P(C|B)$ and state whether B and C are independent events.
[N82/II/2(a)]

Answers

- 1 0.75
- 2 0.56
- 3 $\frac{1}{3}$
- 4 (i) $\frac{5}{6}$ (ii) $\frac{7}{12}$
- 5 (a) $\frac{1}{2}$, $\frac{1}{2}$, independent (b) $\frac{7}{72}$, $\frac{1}{6}$, not independent